The Traverses of a Post-Keynesian Model of a Corn-Credit Economy

by

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B.Ec. (Hons.) (Monash)

Submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

University of Tasmania
October 2002

DECLARATION

This thesis contains no material which has been accepted for the award of any other higher degree or graduate diploma in any tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person except where due reference is made in the text of this thesis.

C.L. Richardson

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1st October 2002

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ABSTRACT

In this thesis, five dynamic Post-Keynesian corn-credit models are constructed as testbeds for numerical analysis experiments that generate traverse paths through historical time using computer simulation. The "corn-credit economy" is a synthesis of theoretical insights gleaned chiefly from the writings of Joan Robinson, Knut Wicksell, Michal Kalecki, John Maynard Keynes, Gunnar Myrdal, and Adolph Lowe.

The first Model A has a fixed corn price, money wage and interest rate. Each such *constant* is replaced by equations to become a *variable* in Models B, C and D, respectively, e.g. money expenditure directed at a given supply of foodcorn determines its price. The final Model E uses a conventional demand function for the same purpose.

Each 31st December, capitalist farmers decide the *flow* of seedcorn invested, then store the balance of that year's harvest as foodcorn. On 1st January, this lagged seedcorn flow becomes the opening *stock* of seedcorn, which is sown by workers employed to raise the next crop. Likewise, the lagged foodcorn flow becomes the opening *stock* of foodcorn available for sale to consumers while the new crop is being tended.

The workers' fortnightly money wages, together with all profit and interest incomes, are partly saved but mainly spent on foodcorn released weekly from the granaries. The structural-form equation (common to all models) that determines the volume of seedcorn invested is crucial, since the reduced forms show that investment decisions drive the evolution of all other economic variables.

In turn, seedcorn invested is itself driven by a time-series of "profitability gaps" between the realised and required rates of return on capital stock, in a process of circular and cumulative causation. A constant "reaction coefficient" determines the rate of capital accumulation as a fraction of the profitability gap. Such positive feedback or path dependence is so pronounced in the complex structural form of Model E that no reduced form can be derived.

All models are solved numerically for a 100-year equilibrium stationary state, then a smooth exponential growth path is generated for all but Model E. These stationary and steady states are used as reference time paths or "basecases", from which specimen traverse paths are made to depart by perturbing the model's parameters.

A sensitivity matrix is constructed for Model E by initiating numerous convergent traverses connecting its initial with a final stationary state. This matrix shows the long-period effect of a change in each parameter upon every endogenous variable. By tracing "chains of causation" made visible by the matrix, these cause-effect elasticities confirm that Keynes's *paradox of thrift* and Rowthorn's *paradox of costs* operate in Model E. The most powerful parameters are the foodcorn demand elasticities, indicating a strong influence of sovereign consumers on the price, profitability and production of corn.

Certain key variable-pairs are scatter-plotted to see whether some conventional relationships used for comparative static analysis hold over the range of reaction coefficient values defining a viable corn-credit economy. Instead of the familiar curves, these scatter-plots reveal a sequence of well-defined patterns, resembling the evolution of a spiral galaxy such as our own Milky Way.

Due to the observed traverse behaviour of Model E being quite violent, a public sector is added and the modified Model E* is used to discover a policy mix that tames the *laisser faire* instability of a demographic shift from zero to positive workforce growth. This policy-constrained or "instrumental traverse", as defined by Adolph Lowe, successfully guides the economy onto a tranquil steady-state growth path with near-full employment.

ACKNOWLEDGEMENTS

This research could not have been undertaken without the encouragement and advice of my supervisor, Dr Jerry Courvisanos; the love and understanding of my wife, Rona Richardson; a generous PhD Scholarship from the University of Tasmania; and the support of my colleagues and fellow research students in the School of Economics. I thank them all most sincerely.

Generations of Australian postgraduate students in economics have been helped out by Emeritus Professor G.C. Harcourt, AO, Fellow of Jesus College, Cambridge. I have benefited greatly from Geoff's detailed comments and suggestions concerning my initial traverse modelling efforts, then again around the time of the UWA-ANU PhD Conference.

At the UWA-ANU PhD Conference in Economics and Business held in Perth during November 2001, Dr Mardi Dungey was the discussant for my paper based on early drafts of Appendix B and parts of Chapter 3. I appreciate greatly the time Mardi devoted to making both written and verbal comments, which even extended to performing some ingenious "reality checks" on the profitability gap theory using empirical data on investment. Professor Bob Gregory kindly provided further advice and some "spot-on" criticisms. I also thank Professor Ken Clements and other session participants for their questions and comments.

As the work progressed, I made presentations at three Research Seminars of the School of Economics. Participants materially assisted with their questions, comments and criticisms, led by my Head of School, Professor Ranjan Ray. Thanks also are due to my associate supervisor, Dr Hugh Sibly, and to Drs William Coleman and Paul Blacklow, who gave freely of their time and expertise.

I also wish to thank Professor Larry Forbes and Associate Professor David Blest of the School of Mathematics and Physics for discussions that gave me the confidence (though not the licence) to propose the Mathematical Conjecture in Chapter 8.

I'm delighted to report that not once, before or during my career as an economist, have I ever encountered a librarian who was in any way unhelpful or discourteous. As information specialists, librarians support the world of scholarship on their Atlas shoulders. They are my favourite people and those servicing the Newnham Campus of the University of Tasmania, led by Lana Wall, are a credit to their honourable and demanding profession. Liz Pugh, Jill Wells, Susan Bell, and Ian Bollard gave special assistance.

There are some top-drawer administrators I need to thank for their efficiency and ever-helpful attitude towards me and my work. They include Sue Abel and Tracy Kostiuk of the School of Economics, Vanessa Folvig and Louise Ransom of the Research Higher Degrees Unit, Dee Robinson and Maria Goodsir of the School of Management, and Jane Smith of the Faculty of Commerce.

Finally, the computer-intensive nature of my research meant that I had to call on the information technology expertise of Greg Hind, Colin Kitto and Thi-Thi Khin from time to time. They always fixed the problem or provided the answer.

Again, my thanks to you all. The errors and omissions that remain are down to me.

In fond remembrance,

this thesis is dedicated to my

two most influential teachers

Professor Tom Asimakopulos

(1930 – 1990)

and

Professor Keith Frearson, DFC

(1922 – 2000)

Scholars and Gentlemen

TABLE OF CONTENTS **Page**

4 – A Fixprice Corn-Credit Economy

5 - Flexing the Corn Price, Money Wage and Interest Rate

6 – A Flexprice Corn-Credit Economy

Appendices

LIST OF TABLES

LIST OF FIGURES

PREFACE

"Continue with continuation – into the future."

Sir John Hicks[1]

In the final sentence of his last paper, published *post mortem*, Hicks exhorts his fellow economists to move forward on the basis of dynamic process (or sequence) analysis. Hicks (1956, p 143) says that "continuation theory" is concerned with the linking up of successive single time periods into a sequence, which ties in nicely with Michal Kalecki's observation that the long period has no independent existence.

My thesis attempts to go with the flow recommended by Sir John and makes heavy use of a piece of Classical dynamic analysis that was named and further developed by him: the Traverse. In 1965 Hicks conceived the traverse as a smooth and convergent time path linking two Neoclassical equilibrium growth trajectories, the initial one having been disrupted by a structural or "qualitative" change, viz. faster workforce growth. Yet within five years, he had switched to a Neo-Austrian approach having far richer dynamics.

One of the best works based on Hicks's Second Traverse Analysis is *Out of Equilibrium*, a 1998 treatise on the numerical simulation of a model Neo-Austrian monetary production economy, by Mario Amendola and Jean-Luc Gaffard. Many passages surely would have found favour with Joan Robinson, Nicholas Kaldor and contemporary Post-Keynesians. In their *Introduction*, for instance, the authors state that an "out-of-equilibrium process" must be "traced out step by step" through "real irreversible time" because qualitative change "implies rebuilding a different structure" and "cannot be deduced from a comparison of alternative states of an economy".

Economics today, it seems to me, consists of a Neoclassical mainstream and at least eight schools of heterodox thought – including Post-Walrasians engaged in pushing the Lausanne School paradigm beyond static general equilibrium analysis. The fascinating thing is that, despite deriving disparate results from their incompatible comparative static analyses, competing schools often report similar findings when they use dynamic models. For a start, none in my sample of 39 traverse models reported disconfirming David Ricardo's "On Machinery" traverse analysis, which has stood like a rock since 1821, while all around it the Iron Law of Wages melted, Say's Law crumbled and the Gold Standard fell.

Richard Day (1993, p 23) reports that complex dynamics arises robustly in market adjustment and iterative price mechanisms; disequilibrium business cycle theories; Classical growth theory; optimal growth and equilibrium business cycle theory; overlapping generations models; adaptive optimising or recursive programming models; and system dynamics models. This spans practically the full gamut of competing schools of thought.

If we heed Hicks's advice to continue with continuation, all schools of economic thought should experience further convergence by utilising significant resources of computing power to simulate nonlinear dynamic models through historical time and apply the pan-disciplinary methodology of complexity analysis. This is what I have attempted in my thesis, working within the developing Post-Keynesian paradigm and upholding a tradition which began with William Petty in 1662: Start with a Corn Model.

[1] 'The Unification of Macroeconomics', *Economic Journal*, 1990, p 538

CHAPTER ONE

THE TRAVERSE, CORN-CREDIT ECONOMIES AND POST-KEYNESIAN ECONOMICS

1.1 Introduction

In this thesis an experimental approach is utilised to study the short-term movements and long-term evolution of an abstract agrarian capitalist economy. This model economy has competitive markets for food, labour and credit – hence also price, wage and interest rate flexibility. Numerical analysis experiments are performed to elucidate the consequences of a parameter change. These parametric perturbations cause the model economy to depart from an initially smooth time path of development, one displaying either zero economic growth or a constant positive exponential growth rate.

Although simple in structure, the deterministic economic model developed for this thesis is too nonlinear and recursive to be solved using standard analytical methods. Therefore computer simulation is used to generate solution spreadsheets in which each column represents one *year* of calendar time. Each row represents a parameter or an endogenous *variable*, hence all prices, quantities, aggregates, indices, ratios, etc. form annual time series that can be tabulated and plotted.

Joan Robinson (1962, p 23) is careful to distinguish between two concepts of "time" used in dynamic analyses. A *logical time* argument "... proceeds by specifying a sufficient number of equations to determine its unknowns, and so finding values for them that are compatible with each other." An *historical time* argument "... specifies a particular set of values obtaining at a moment of time ... and shows how their interactions may be expected to play themselves out." The abstract economy modelled in this thesis does not exist in logical time, but in historical time, so that "... today is an ever-moving break between the past and the unknown future", as Robinson (1978, p 10) later added.

In this chapter the central "traverse" concept is first defined and five stages of construction are laid out for the economic model, which serves as a testbed for performing traverse experiments. Two long-period dynamic trajectories are differentiated: the "equilibrium" stationary state of zero growth and the "fully-adjusted" steady state of constant positive growth. These are the two experimental "basecases", i.e. reference time paths from which specimen traverses depart – and to which they may (or may not) return.

The importance of one-commodity models of the "pure corn economy" throughout the history of economic thought is noted and the more recent theoretical construct of a "pure credit economy" is defined, before combining both into the testbed "corn-credit economy" model of this thesis. Then the Post-Classical "approach" to economic analysis is outlined and one group of adherents (the Post-Keynesian "school") is discussed as their paradigm provides the theoretical underpinnings of this thesis. Finally, the research objectives are laid out and the organisation of the thesis into subsequent chapters is presented.

1.2 The Traverse

Conventionally, dynamic economic models have been analysed with reference to only two long-period time paths, viz. the "stationary state" of zero growth (or "simple reproduction") and the "steady state" of constant positive exponential growth (or "extended reproduction"). Both are fully-adjusted dynamic paths, i.e. real output follows the same stationary "flatline" (or steady "growthline") indefinitely, when plotted against time. The stationary state features a constant positive *level* of real output, while the steady state has a constant positive *growth rate* of real output.

The traverse is the out-of-adjustment time path an economy follows if dislodged from its original dynamic trajectory. Christian Gehrke and Harald Hagemann (1996, p 140) state that "The analysis of an economy which originally has been in a steady-state equilibrium but which was disturbed by a change in one of the exogenous determinants of growth … is one of the most challenging problems in economics." They also quote Robert Solow (1984, p 21) as having once quipped that the traverse "… is the easiest part of skiing but the hardest part of economics." This degree of inherent difficulty explains why most economists working in this field choose to analyse the *least intractable* traverse, that connecting an initial with some final fully-adjusted economic growth regime, i.e. two stationary states with different levels of output and employment, or two steady states having different growth rates.

In reality, however, such smooth initial and final growth paths are uncommon, partly because of the economy's endogenous cyclical behaviour and partly because there is an incessant stream of traverse-inducing perturbations, such as innovations, demographic shifts, mineral discoveries, new legislation, oil shocks, civil unrest, and wars. So, with most real-world economies undergoing business cycles and being "in traverse" most of the time, the more that is understood about these dynamic processes of adjustment, the higher will be the quality of investor, financier, producer, consumer, corporate, government, and multilateral planning to regularise unstable traverses or ameliorate their deleterious effects.

In this thesis, traverse phenomena are investigated experimentally *via* "perturbations", i.e. parameter-changes that throw the dynamic model out of its initial fully-adjusted state. The corn-credit economy's subsequent development then is observed as a "century" of simulated historical time passes. As the disequilibrium traverse process loses energy and peters out, one *may* observe the economy to be converging onto some fresh, final, fully-adjusted, long-period dynamic path. If instead it *diverges* (or the amplitude of its first cycle is so great that the economy collapses), the existence of a finite-duration traverse path between two dynamic states cannot be demonstrated.

The traverse concept is used by J R Hicks (1965) to describe the (smooth or cyclically convergent) time path that he expects an economy to follow in moving from an initial to a final stationary or steady state. These two growth regimes were, after all, the most mathematically tractable dynamic paths for which *analytical* solutions could be obtained, in the years before nonlinear recursive complexity models could be built and subjected to *numerical* analysis using computer simulation. However, some traverses of the nonlinear corn-credit models developed herein are cyclically regular or divergent, so that a final state of stationary or steady growth is not necessarily established. These might be termed "perpetual traverses" to distinguish them from "terminating traverses".

An implicit traverse concept was introduced into modern economics literature by Michal Kalecki (1933) then utilised by Adolph Lowe (1955) and Joan Robinson (1956). Many years later the traverse was *named* and explicitly brought into the theoretical mainstream by J R Hicks (1965) and John Hicks (1970, 1973). However, the first acknowledged traverse analysis is to be found more than a century earlier in "On Machinery", a new chapter concerning the machines-labour substitution process, which David Ricardo (1821) added to the third edition of his *Principles of Political Economy and Taxation.* In addition, Joseph Halevi and Peter Kriesler (1998) persuasively argue that an embryonic consideration of traverse phenomena also appeared in Volume 2 of *Capital* by Karl Marx (1885). Andrew Kliman (2001) reinforces this interpretation by presenting evidence that Marx viewed his simple and extended reproduction schemes as models of *unbalanced* growth. The "unbalanced growth" analyses of François Perroux (1955), William Lewis (1955), Gunnar Myrdal (1957), Albert Hirschman (1958), Simon Kuznets (1959), and Walter Rostow (1960) effectively are portraits of dynamic disequilibrium traverse processes in the context of development economics.

Following Kriesler (1989, pp 1-2), the traverse is defined as "the dynamic (out of equilibrium) adjustment path in historical time" that an economy will be observed to follow, once it has departed from a fully-adjusted growth path for any reason. Jerry Courvisanos (1996, pp 51 &

67, fn 29) shows that this notion of an *observed* traverse is implicit in Kalecki's work on investment theories. In Chapter 2 it is argued that the observed traverse concept – which also was adopted by Robinson – should be contrasted with the *instrumental* traverse of Lowe and the *Neoclassical* and *Neo-Austrian* traverses of Hicks.

For both Lowe and Hicks, the traverse process peters out once the economy has absorbed the perturbation(s) and regained a fully-adjusted dynamic path. On those rare occasions when Kalecki and Robinson directly refer to the traverse, they tend to call it "the transition period" – which may be of indefinite duration. The fact that most real-world economies operate in such "transitional states" of dynamic disequilibrium most of the time is nicely summed up in Kriesler's (1989, p 10) dictum: "Life is a Traverse".

1.3 Stages of Model Construction

There are five stages in constructing this minimal, but progressively more flexible (hence also more complex) recursive dynamic model: A, B, C, D, and E. At each construction stage, the parameters of this developing "cycles, distribution and growth" (CDG) model are perturbed to generate specimen observed traverses. This leads up to Chapter 6, which is highly "traverse-intensive" because it is there that the dynamic behaviour of the final *laisser faire* model is more fully investigated.

In Chapter 4, Model A is pure "fixprice", having a *constant* corn price, money wage and interest rate. "Flexing" is the act of replacing these price constants with *variables*, whose values thereafter are determined by independent equations. In Chapter 5, Model B flexes the corn price, Model C also flexes the money wage, and Model D additionally flexes the interest rate. All models are monetised using "dollars" as the unit of account and standard of value, in accordance with Knut Wicksell's (1898, 1906) theoretical construct of the "pure credit economy", discussed below.

In Chapter 6, the pure "flexprice" Model E introduces a superior method of determining the corn price – one based on the demand and supply curves for foodcorn, rather than on the classical economists' assumption that workers do not save. Model E is the fifth and *final* stage in this nested sequence – "nested" in the sense that D forms the core of E, C lies at the heart of D, and so on back to the germinal fixprice system, Model A.

The nesting process involves adding extra equations at each construction stage, so as to progressively endogenise four of the fully-fixprice Model A's parameters. These four constants – identified by Roman, rather than Greek, letters – are the corn price (P), money wage (w), interest rate (i), and foodcorn retained (Qf), i.e. that volume of foodcorn destined for consumption within farmer households, rather than for sale to all other consumers. None of these four parameters survives at the *ultimate* stage: the fully-flexprice Model E, which contains nothing but Greek-letter constants.

A feature of Model E is that only three initial values – given by the economy's history – are needed to determine its 55 endogenous variables (albeit 32 of them national accounting type "aggregates") over an indefinite future period of simulated historical time. These three parameters are the model's opening volume of seedcorn invested (Qiz), the money wage (wz) and the nominal interest rate (iz), where "z" stands for "year zero", the base year for all models. Model E also has six independent equations (with 14 Greek-letter constants as its remaining parameters) and one equilibrium condition.

Model A is consistent with Neo-Keynesian economics – the "Neoclassical Synthesis" of Keynes and Walras, a.k.a. "hydraulic" or "bastard" Keynesianism – because all prices are "sticky" and the full burden of adjustment following any parameter change is borne by quantity variables. Models B through E are consistent with Post-Classical economics, i.e. that constellation of schools of thought subscribing to the *dynamic surplus approach* – as opposed to the *static allocation approach* underpinning all Neoclassical schools. These contrasting approaches are discussed below, and also by such economists as Piero Sraffa (1960), Edward Nell (1967), and Vivian Walsh & Harvey Gram (1980). Post-Keynesian economists make up the largest and most active school embracing the dynamic surplus approach. The fact that all four models feature competitive, fully-flexible prices refutes the common misconception that Keynesian results can be derived *only* from systems exhibiting some degree of rigidity in prices, wages, interest rates, or in mark-ups over prime costs.

Model E represents a one-commodity closed monetary production corn-credit economy having no government and developing in path-dependent fashion over 100 "years" of simulated historical time, with no constraints on its stock of land. It is a purely deterministic model into which demonstration data are inserted, i.e. all parameters – the 14 constants and three initial values – are specified numerically. No claims of realism are made, for Model E is purely an abstract theoretical construct designed to lay bare the economic mechanism and serve as an experimental testbed.

1.4 Traverses from the Stationary and Steady States

The first step in investigating dynamic behaviour in the CDG models described above is to spark off a traverse from the stationary-state basecase, i.e. the model's flatline solution time path. From this initial state of long-period zero-growth dynamic equilibrium, one or more parameters are changed ("perturbed") and the resulting traverse – not always towards a final fully-adjusted state – is observed and evaluated, using time series of key endogenous variable values, both tabulated and plotted.

Long-period dynamic stationary-state equilibrium prevails whenever the *profitability expectations* held by entrepreneurs continue being realised as one year follows another. This does not replace, but supplements, the Neoclassical definition of equilibrium, which prevails whenever *prices* (of corn, labour and credit, in this model) flex in such a manner as to equate demand with supply, thus "clearing" all markets within the economy. Both forms of equilibrium characterise the initial flatline stationary-state solution time path at all stages of model construction.

When not in dynamic equilibrium, the corn-credit economy must be in disequilibrium *per definitione*. It may be in traverse (i.e. following a time path of adjustment) or it might have reached some disequilibrium, though fully-adjusted, growth path. An example of the latter is regular cycling ("limit cycles"), a disequilibrium time path which, nonetheless, is fully-adjusted in the sense that it will persist indefinitely if left undisturbed. It is not misleading to refer to this particular regime as the "cyclical state" or the "cycleline" time path. A smooth trendline fitted to the cycle data may exhibit stationary or steady growth, yet this fact would possess no independent theoretical significance.

The next step is to ignite a traverse from the steady-state basecase, i.e. the model's growthline time path. This classic economic growth trajectory is not a dynamic *equilibrium* regime in these corn-credit models although, like the cyclical state, it is a *fully-adjusted* growth path. Dynamic equilibrium is achieved only when the expected, realised and normal profit rates equalise and remain that way over time. An economy cannot be in dynamic equilibrium if the profitability expectations of its entrepreneurs are not being confirmed, for this situation would lead them to alter their investment behaviour. The common misconception that the steady state is a dynamic equilibrium growth regime arises from the mathematically convenient (but economically inappropriate) translation of stationary-state variable *levels* into steady-state growth *ratios*.

The steady state of growth is, in fact, a very special kind of *dynamic disequilibrium* situation – a unique growth path along which the difference between the expected profit rate and the normal profit rate (i.e. the opportunity cost of capital) remains strictly constant and positive. (By contrast, this difference in profitability must be zero in the stationary state and must be fluctuating regularly between high and low values in the cyclical state.) It is the disequilibrium *prospect* of positive pure profitability that drives the capitalist entrepreneurs, both individually and collectively, to achieve and maintain the constant positive growth rate of investment and output that characterises the steady state.

The term "fully adjusted" means "no traverse process is underway". Thus, the initial and final growth paths (which are connected by the adjustment path of the traverse) are fully adjusted, not only when they constitute stationary or steady states, but even when they feature the endless recurrence of *limit cycles* around either of these trendlines.

The term "dynamic equilibrium" means that, along the relevant time path, profitability expectations are being realised. Thus, entrepreneurs cannot possibly be in equilibrium (as defined above) along any dynamic path *other than* the stationary state of zero growth. In the corn-credit models of this thesis, the stationary-state dynamic equilibrium is *solved for*, but all other time paths must be *generated* – including that of the classic steady state.

The stationary, steady and cyclical states correspond to the stable, fully-adjusted, long-period growth paths that should exist in well-specified CDG models. While it is recognised that long-period equilibrium will hardly ever characterise any real-world economy, economic theorists need to demonstrate that equilibrium solutions do exist for their systems of dynamic simultaneous equations, whether recursive or not. Being able to solve an economic model for its stationary state is equivalent to demonstrating the *existence* of an equilibrium. Dynamic *stability* properties are relevant: it is important to know under what conditions an economy in disequilibrium traverse converges on or diverges from a fully-adjusted, long-period time path. The *uniqueness* and *optimality* properties of equilibria are of lower-order importance for research involving the use of stationary-state equilibrium time paths as basecase comparators for traverse experiments.

1.5 The Pure Corn Economy

Simple models of a pure corn economy occupy a special niche in the history of economic thought. The particular attraction of corn models is that they avoid commodity aggregation and capital theoretic problems, while laying bare the underlying causal economic relationships with extreme clarity. The strand begins with Petty (1662) and progresses *via* Cantillon (1732), Quesnay (1758), Smith (1776), Ricardo (1815), Torrens (1821), Malthus (1823), Hodgskin (1825), Sismondi (1827), Ramsay (1836), Senior (1836), Marx (1867), McCulloch (1870), Marshall (1890), and Dmitriev (1898). It then proceeds on down to Sraffa (1960), Robinson (1960), Hicks (1965), Spaventa (1970), Robinson & Eatwell (1973), Hollis

& Nell (1975), Casarosa (1978), Walsh & Gram (1980), Marglin (1984), Roemer (1988), Kaldor (1996), and Kurz & Salvadori (2000).

One can see from these citations that the construction and analysis of corn models almost ceased between 1871 and 1960. This was the period during which the "marginalist revolution" of Jevons (1871), Menger (1871) and Walras (1874) was being absorbed into the new Neoclassical canon and having its significance critically assessed by the competing Classical (and fledgling Post-Classical) schools of economic thought. Significantly, over two-fifths of the discipline's stock of corn models has been developed since 1960, the year that Sraffa published his seminal treatise, *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory* and re-invigorated the original "commodity surplus" approach to economic analysis.

However, most corn models represent *barter exchange* economies, in which production is viewed as "exchange with nature". All multisectoral models in which the price of some *numéraire* commodity (not necessarily corn) is denoted as Pn = 1 also represent barter exchange economies. Setting Pn = 1 allows the prices of all remaining (n – 1) commodities to be expressed relative to the *numéraire*.

In such barter models, the saving and investment decisions are inseparable – indeed, they are identical – being taken in terms of corn (not money) by a single social class. In the Classical economists' vision, this is the class of capitalist tenant farmers who retain (i.e. save ≡ invest) a large proportion of each harvest to create a "wage fund" of foodcorn (plus a stock of seedcorn) as circulating capital to sustain the workforce while next season's crop is being raised. After recovering their opening capital stock from each harvest, farmers are left with a "corn surplus", which then is distributed as "corn rent" to the landlords and as "corn profit" to themselves. Thus, most corn output has been bartered for the services of labour and land, but that portion of the commodity surplus remaining in the hands of the farmers can be expressed as a *rate* of profit: corn profits as a proportion of their opening corn capital stock, comprising the wage fund and the seedcorn advances.

Capitalist tenant farmers are the heroes of the Classical parable. They are the only social class dedicated to spending out of their incomes in a productive, future-orientated manner. What they do not consume out of their corn profits, they save/invest to maintain (stationary state) or to enlarge (steady state) their advances of corn wages and seedcorn, hence also next year's crop.

What happens next is central to the Classical economists' dynamic vision of economic growth and income distribution among the three great classes of society involved in corn production. Adam Smith was optimistic, forecasting *increasing* returns due to specialisation and division of labour in the manufacturing sector – provided there were growing markets at home and/or overseas for commodities like pins, textiles, footwear, and crockery. But Ricardo, Malthus and most of their contemporary Classical economists saw wages being entirely consumed by needy workers and rents dissipated in riotous living by louche, luxury-loving landlords, leaving only the thrifty farmers and other capitalists to devote most of their profits to expanding the economy's seedcorn and wage fund – hence also its future cultivation, manufacturing and employment possibilities. Then, as corn production extends into new (or intensifies on old) lands, rents rise and profits fall, due to the inexorable law of *diminishing* returns.

Eventually, the steadily growing economy slows and finally stops. It has asymptoted to its dismal *stationary state* of constant real output, characterised by maximum rents, subsistence wages and some minimum rate of profit – just sufficient to keep the typical capitalist farmer continuing his tenancy rather than joining the working class. (The fact that some minimum rate of profit must continue to be earned in the stationary state to sustain farming families is true at all developmental stages of the corn-credit model of this thesis.)

Some modern economists have added investment sectors producing fixed capital equipment to the earlier models that featured circulating corn capital only, e.g. the corn/tractors economy of Hicks (1965). Others have introduced consumption sectors producing both necessities and luxuries, e.g. the elegant corn/gold economy of Luigi Pasinetti (1977). The dismal predictions of the Ricardo/Malthus corn models have been reversed because technical progress (resulting in higher-yielding seedcorn and/or higher-productivity labour) has overcome diminishing returns to land. Yet the stationary state of zero growth and the steady state of constant positive exponential growth – seem to have been retained for the sake of theoretical continuity and mathematical tractability.

1.6 The Pure Credit Economy

By contrast with the vast majority of barter specimens, the CDG corn-credit model constructed for this thesis represents a particular type of *monetary* production economy. Wages, profits and interest are paid in "money", conceived as a *unit of account* and a *standard of value*. The value of every transaction is measured in "dollars" and recorded in double-entry ledger accounts, a practice that is further justified in Chapter 4. This "money of account", therefore, is neither a *medium of exchange* nor a *store of value*. The monetised corn model of this thesis can be viewed as one representation of the "pure credit economy", a useful construct introduced to the literature by Knut Wicksell (1898, 1906).

According to Keynes, the ultimate "primitive" of monetary theory is the "unit of account" concept, not the "medium of exchange" notion. In his *Treatise on Money: The Pure Theory of Money*, Keynes (1930a, p 3) pointed out that

> Money of account ... is the primary concept of a theory of money.
>
> A money of account comes into existence along with debts, which are contracts for deferred payment, and price lists, which are offers of contracts for sale or purchase. Such debts and price lists ... can only be expressed in terms of a money of account.
>
> Money itself ... derives its character from its relationship to the money of account, since the debts and prices must first have been expressed in terms of the latter.

Humankind first utilised a money-of-account to record debts and publish price lists, as in the Sumerian city-states. Only later did various communities adopt rare, divisible and transportable goods (such as cowrie shells and the precious metals) as their preferred, storable money-of-exchange.

The Wicksellian pure credit economy concept allows theorists to develop models (of production, not merely exchange) that feature a money-of-account recorded in debit and credit ledgers by employers. The book-keeping entries are made in terms of "dollars", rather than "corn" or some other commodity. So, from the very outset (viz. Model A), the series of five corn-credit models has been constructed to resemble a modern monetary economy, in which wages are paid in money, corn is priced in dollars and profitability is struck as the ratio of dollars of money profit to "dollars-worth" of capital stock.

Wicksell distinguished between pure cash, mixed cash-credit and pure credit economies. In the "pure cash economy", he analysed the stock of money (M dollars) as a circulating medium of exchange comprising gold and fully-convertible currency notes. Wicksell took the "quantity theory" identity ($M\ V \equiv P\ T$ dollars), fixed the velocity of circulation (V) and the number of transactions (T), then assumed that causation ran from left to right. Thus M determined P, the absolute price level (measured as dollars/transaction).

In the mixed "cash-credit economy", however, transactors also hold cheque account deposits with banks. To analyse this "currency-deposit economy", Wicksell developed his celebrated "cumulative process", driven by a gap between the "natural" and "money" rates of interest. He regarded Henry Thornton's (1802) work, *The Paper Credit of Great Britain*, as seminal.

The natural rate (r% pa) equates desired saving with intended investment, while the money rate (i% pa) is what banks charge on loans and pay on deposits. Should the return on capital exceed the cost of capital (r > i% pa), planned investment will exceed planned saving (I > S dollars pa). That is, entrepreneurs seeking to finance new projects wish to borrow more from banks than savers are depositing. By making more loans, banks create more deposits so, with V and T both constant, the consequent growth in the economy's stock of money (gM% pa) produces a cumulative price inflation (gP% pa) until the interest rate gap is closed. In the opposite case (r < i% pa), both gM and gP will turn negative as loans/deposits contract and a cumulative deflationary process sets in until r = i% pa once more.

In the "pure credit economy", Wicksell pushed his theory to its logical limit: now all transactions are settled by transfers of deposits on the books of banks. There is no precious metal or paper currency ("inside money") to fulfil the medium of exchange function, so money reverts to its historic original nature as "outside money", a unit of account and a standard of value.

Wicksell (1898, pp 110-11) explains that this makes it impossible to view the supply of money as independent of the demand for money. Regardless of how much outside or "credit money" may be demanded from the banks, that is the amount which they are in a position to lend: "The banks have merely to enter a figure in the borrower's account to represent a credit granted or a deposit created. When a cheque is then drawn and subsequently presented to the banks, they credit the account of the owner of the cheque with a deposit of the appropriate amount (or reduce his debit by that amount)." There is thus no room for any quantity theory of money in the pure credit economy; the supply of money is furnished by the demand for it. No monetary base of circulating currency exists to discipline the cumulative process of credit creation/destruction, so an infinity of price-quantity (P-M) combinations are possible.

A different "sheet anchor" for the level of dollar prices must be found; candidates include the money wage (w dollars/worker pa) and the nominal interest rate (i% pa). Robinson (1971, p 90) points out that Keynes favoured the "money-wage rate" and today's neo-Wicksellian central bankers invariably keep price inflation within acceptable limits by fine-tuning a short-term interest rate variously called "cash rate", "funds rate", "bank rate", or "discount rate".

1.7 Modelling a Corn-Credit Economy

In all the Post-Keynesian models of this thesis, it is the farmers who control the physical investment decision. They are the entrepreneurs who decide how many sacks of "seedcorn" to retain in their barns for planting after each end-of-year harvest, allocating the balance (sacks of "foodcorn") to their granaries, for sale at weekly markets held during the new crop year. Throughout that year, the farmers meet their fortnightly payrolls by writing ledger account entries into paybooks carried by their workers, who spend these money wages at the weekly foodcorn markets. Workers are assumed to do no money saving in Models B, C and D, whereas the fixprice Model A needs no worker saving assumption. In these four models, the farmers also retain a fixed quantity of foodcorn, for consumption within their own households.

There are no pure landlords. Rather, each farming family owns a large tract of inherited land of uniform fertility. Each farm's hired workforce is permitted to reside on their employer's land and there exist indefinite possibilities for expanding the crop at constant unit cost. In Alfred Marshall's (1890) terminology, there is neither an intensive nor an extensive "margin of cultivation" that could necessitate payments of ground rent.

Model C is the last one in which workers, most of whom are employed by small farmers, retain confidence that their paybook entries (backed by their employers' personal credit) will be accepted by all foodcorn vendors at the weekly markets. So, in Models D and E, farmers must *finance* their wage bills, by borrowing and paying interest on money loans extended by "bankers". These loans are simply ledger entries, written in favour of all farmers by some of their fellow farmers. The richest farmers have become part-time bankers because only *their* credit is trusted by all. This is due to their (perceived) ability to liquidate the vast stocks of corn held in their own granaries, as and when required, at the ruling corn price.

In the fifth and final Model E, farmers, bankers and workers all determine their own purchases of foodcorn by spending out of their money incomes from last year's profits and this year's wages and interest on bank deposits. Thus each consumer competes with all others "on a level playing field" for a share in the physical volume of foodcorn reaching the weekly markets. That part of household income not spent for consumption becomes money saving, used to expand the household sector's interest-bearing bank deposits. Bankers distribute to eligible households (based on the size of each one's bank deposits) most of the interest income they receive for making money loans to farmers so they can meet their fortnightly wage bills. (The bankers keep the balance of interest as recompense for their labour and exposure to lending risks.)

The investment function used in these models has the accumulation of circulating capital (the economy's aggregate stock of seedcorn) being determined by the "profitability gaps" that may open up from time to time as the years roll by. Such gaps can occur between the money profit rates that farmers *expect* to realise, and the normal profit rates that they face. The normal profit rate is the *known* money opportunity cost of holding capital in the form of seedcorn, rather than in the form of money. It comprises the going interest rate plus a risk premium associated with agricultural production.

Along any long-period stationary-state equilibrium growth path, a time-series of these profitability gaps would show them all to be zero. Every year's expected (and realised) profit rates then would be equal to the risk-inclusive normal profit rate or opportunity cost of capital or "required rate of profit" or "target rate of return" that farmers seek to earn. They then have no incentive to alter the quantity of seedcorn annually retained, thus the stationary state will persist indefinitely. This profitability gap theory of investment is further explained in Chapter 3 and in Appendix B.

As these are monetary, rather than barter, models the real (corn) wage rate and real (corn) profit rate – indeed all "real" magnitudes – are precisely what they appear to be, viz. mere derived variables, constructed by *ex post* division of the relevant money values by an index of the corn price level. This reflects an important difference between the Post-Keynesian and Neoclassical world views. There is a "Neoclassical dichotomy" (which most mainstream economic theory respects) between the determination of real and monetary variables, with primacy being awarded to the former because "money is a veil". Thus supply and demand barter trading in the "labour market" is said to determine the real wage rate; in the "loanable funds" market, the real interest rate; and in the "capital" market, the real profit rate. The quantity theory of money is invoked to explain the absolute (i.e. nominal or "dollar") levels of these relative factor prices, and all commodity prices as well.

In contrast, Post-Keynesian theory holds that the labour market determines the money wage rate; that the money market determines the nominal interest rate; and that the overall profit rate is some positive function of the economy's overall growth rate of capital accumulation. Post-Keynesianism is one school of economic thought within the dynamic Post-Classical approach, which is discussed next.

1.8 Post-Classical Economics

Sraffa (1960) may have been the first to formally distinguish the Post-Classical *surplus approach* from the Neoclassical *allocation approach*, a dichotomy later confirmed by Nell (1967), Walsh & Gram (1980), Eichner (1985), Hagemann (1992), Lavoie (1992), Duménil & Lévy (1993), and several other authors. Post-Classical economists subscribe to the view that a continuing dynamic process of "circular reproduction" generates a "commodity surplus" for distribution among competing "social classes" by the price mechanism. Neoclassical economists, however, believe that static "production functions" continuously transform scarce "factors of production" into an array of "commodities" to be allocated by the price mechanism among competing consumers.

There is no concept of social class in the Neoclassical *schema*, nor is there any physical surplus of commodities – only "consumer surpluses" of excess utility and "producer surpluses" of excess profits. The latter comprise pure economic rents, which are destined to be competed away in long-run equilibrium. Individual economic agents earn the real "marginal revenue product" (MRP) of whatever "resource endowment" of scarce productive factors each happens to own. This unexplained opening distribution of real wealth – in the form of land, labour and capital (physical, financial and human) – is considered to be a *datum* of interest only to historians, political scientists and sociologists. Likewise, the "tastes and preferences" that define the fixed "utility function" of each sovereign consumer are given – an object of inquiry for anthropologists and psychologists. The process of *exchange* is paramount. Consumption occurs after "trading with people" – as in Roy Radford's (1945) prisoner-of-war camp – while production simply constitutes "trading with nature".

Post-Classicals tend to give their allegiance to the particular "paradigm" (Thomas Kuhn, 1962) or "research program" (Imre Lakatos, 1970) that underpins their own particular school of economics. By contrast, Neoclassicals of *all* schools (and there are just as many) subscribe to a *single unified paradigm*: the elegant general equilibrium (GE) model created by Walras (1874), Arrow & Debreu (1954) and Debreu (1959). A comprehensive treatment is to be found in Arrow & Hahn (1971).

However, there are strong indications that a synthesis presently is under construction by economists working within at least eight heterodox schools, not all of whom subscribe to the Post-Classical dynamic surplus approach, e.g. the Neo-Austrians and Post-Walrasians. This is evidenced by the growing number of contributions from adherents of one school who have incorporated insights from one or more competing schools into their economic analyses. A representative selection appears in Table 1.1 below, in which the numbers enclosed within square brackets identify the schools for which a synthesis is being attempted.

Table 1.1 – Convergence of Heterodox Schools of Economic Thought

1. Keynesian	Weintraub (1966) [1,2] Minsky (1982) [1,2] Dillard (1980) [1,6] Dillard (1984) [1,4] Davidson (1996) [1,2,5,6] Wray (1996) [1,4]
2. Post-Keynesian	Kaldor (1972) [2,1,5] Bhaduri & Robinson (1980) [2,1,3,4] Rowthorn (1981) [2,3] Gordon (1994) [2,4,6] Lavoie (1995) [2,3,4] Arestis (1996) [2,6] Courvisanos (1996) [2,5] Berr (1999) [2,1,3] Lavoie & Seccareccia (2001) [2,4] Courvisanos & Verspagen (2002) [2,6]
3. Neo-Ricardian	Steedman (1977a) [3,4]
4. Neo-Marxian	Baran & Sweezy (1966) [4,1] Harris (1978) [4,2] Nell (1980) [4,3,2,1] Bhaduri (1986) [4, 2] Skott (1989) [4,1] Graziani (1997) [4,2] Halevi (1998) [4,2] Duménil & Lévy (1999) [4,1]
5. Cumulative Causation[a]	Dixon & Thirlwall (1975) [5,2] Cornwall (1977) [5,2] McCombie (1982) [5,2] de Ridder (1986) [5,2] McCombie & Thirlwall (1994) [5,2] Argyrous (1996) [5,2]
6. New Institutionalist[b]	Brazelton (1981) [6,2] Hodgson (1988) [6,2] Harvey (1994) [6,2] Faber & Proops (1998) [6,7] Verspagen (2002) [6,2]
7. Neo-Austrian	Bohm (1999) [7,2] Burczak (2001) [7,2] Koppl (2002) [7,1]
8. Post-Walrasian[c]	Various Contributors to Colander (1996, 2000) [8,1,6,7]

a Phillip Toner (1998, p 1) argues that this is a separate school of economic thought
b Includes most Evolutionary Economics and Neo-Schumpeterian analysts
c Includes most Complexity Economics analysts

Table 1.1 suggests that all eight heterodox schools are developing linkages with each other and, in particular, with Post-Keynesian economics. This development is not universally welcomed, e.g. Steedman (1977), Roncaglia (1995) and Dunn (2000) have called for an end to the *rapprochement* between Neo-Ricardians and Post-Keynesians. Nonetheless, much of the inspiration for this thesis has been drawn from the continuing convergence among these schools of thought.

In the "Coda" to his *Big Players and the Economic Theory of Expectations*, Roger Koppl (2002) also identifies a convergence among schools, one that could see the emergence of a "new orthodoxy" to replace the "old orthodoxy" based on the GE paradigm. Koppl believes this will be forged by schools [6] through [8], plus Constitutional Political Economy and Complexity Economics. These were not included in Table 1.1 because the first is a mainstream/orthodox/Neoclassical school of thought, associated with the University of Virginia, and the second is not a school at all.

Complexity Economics, associated with the Santa Fe Institute, is consistent with all eight schools that comprise the heterodoxy. (The dynamic recursive nonlinear models of this thesis qualify as complex systems under the so-called "broad tent definition".) Complex systems theory possesses *pan-disciplinary* relevance, having been applied to issues in bio-diversity, energy, climate, demography, epidemiology, technological change, economic development, governance, computation, communication, physiology, sociology, cognitive psychology, and most other fields of scholarship. The complexity approach (built upon earlier theories of cybernetics, catastrophe and chaos) has enriched conventional static closed models in many disciplines and could lead the orthodox GE paradigm to evolve towards the complexity vision promoted by the Post-Walrasian school. It is too early to predict whether the "strange attractor" of complex systems theory will be strong enough to forge some sort of grand alliance between the Post-Keynesians (many of whom also practice complexity economics), the Post-Walrasians and some other heterodox schools.

This thesis may be viewed as a contribution to an emerging Post-Keynesian paradigm, the core of which may turn out to be some kind of cycles, distribution and growth (CDG) model – at a fairly high level of abstraction. As with the static GE paradigm, the dynamic CDG paradigm could consist of (i) the simplest feasible multisectoral model of a closed economy with no government, (ii) the minimum number of axioms needed to establish this model and (iii) a set of theorems, lemmas and corollaries derived from these axioms. It will *not* have to cover all bases. For instance, the GE paradigm permits neither money nor firms to exist, offering just a *numéraire* "commodity" and numerous "markets" for exchange between individual economic agents. Yet this limited *quaesitum* has not stopped Neoclassical economists developing canonical satellite theories to account for the behaviour of bank deposits and corporations, both important real-world institutions.

In the realm of economic policy, it is significant that Neoclassicals often refer to the Pareto-optimal solution of a well-specified set of GE equations as "the bliss point". They tend to see the world as (potentially) an harmonious set of interacting individual agents, all earning the respective MRPs of those factors of production in their ownership. Since the real world demonstrably is *not* in harmony, Neoclassicals search for reasons, usually finding them in "market failures" caused by departures from the axioms of the GE paradigm. Many of their policy prescriptions can best be understood as attempts to mould the real world into conformity with the abstract GE axiom-set, so as to more closely approach the economy's bliss point at some future time. The examples are legion and include anti-trust and anti-union legislation, privatisation of public utilities, "welfare reform", balanced budgets, "microeconomic reform", deregulation of telecommunications, "contracting out" of government functions, auctioning of "rights to pollute", tariff reductions, globalised capital markets, and floating exchange rates.

By contrast, Post-Classicals accept the real world's messy absence of harmony as given and are likely to prescribe economic policies designed to simultaneously tone down the inherent antagonism (equity) while striving to solve (effectiveness) at minimum cost/maximum benefit (efficiency) whatever economic problems happen to be troubling society at the time and threatening its cohesion most. Often, their prescription will be some new set of *institutional* arrangements, such as a "wages accord", an indicative planning authority, commodity stabilisation funds, debt cancellation for bankrupt Fourth World countries, a new Bretton Woods agreement, or a tax on the "blur money" that can destabilise entire economies *via* their balances of payments on capital account.

1.9 Post-Keynesian Economics

Much of the inspiration for this thesis has been drawn from one of the most prominent schools within the heterodoxy: the Post-Keynesians. Their elder statesman, Geoff Harcourt (1987, p 924), calls Post-Keynesianism "a portmanteau term which is used to contain the work of a heterogeneous group of economists who nevertheless are united not only by their dislike of mainstream Neoclassical theory ... but also by their attempts to provide coherent alternative approaches to economic analysis".

Sheila Dow (1991, p 176) identifies the Post-Keynesians as one among seven orthodox and heterodox schools of economic thought, although "its boundaries are not yet settled". Marc Lavoie (1992) goes further, claiming that a fully-articulated Post-Keynesian paradigm is emerging to challenge Neoclassical dominance of the mainstream, a thesis first advanced by Eichner and Kregel (1975). As explained above, it is expected that the Post-Keynesians – along with several other schools of heterodox economic thought – will continue with their project of forging a far broader, more fully-inclusive, *Post-Classical* synthesis. The complexity economics approach is likely to figure prominently in many of the key contributions.

The hallmark of any dynamic Post-Keynesian analysis is that it utilises an investment-driven model of a monetary production economy developing through irreversible historical time, with the question of how the potential commodity surplus is realised in money terms having central significance. The future returns to be expected from real investment are inherently uncertain (not merely risky), so cannot be reduced to a time-stream of certainty-equivalents. This leaves no room for Neoclassical optimisation procedures based on perfect foresight, nor for the highly data- and computation-intensive procedures that are claimed to lead the economy onto some dynamic long-run equilibrium growth path that exists and, furthermore, is unique, stable and Pareto-optimal.

Rather, there is an *infinity* of possible equilibrium growth paths, one for each value that the required rate of return or normal profit rate (i.e. the opportunity cost of capital) can take on for investors. Furthermore, dynamic equilibrium will last only for so long as the realised profit rate remains precisely equal to this normal profit rate. This is because lower (higher) than normal realised rates of return can depress (boost) the confidence of investors – hence also the level of investment – dragging the entire economy down (up) with it.

Such instability in this key investment aggregate will be accelerated by *positive* feedback from these profitability gaps, sending the economy into a downswing (upswing) until certain *negative* feedback mechanisms kick in and the system moves towards a cyclical trough (peak). In the CDG corn-credit models of this thesis, recessions (recoveries) eventually reach their cyclical turning points and reverse themselves because of deficient (excessive) supplies of foodcorn reaching the post-harvest markets. These demand-supply imbalances raise (lower) the equilibrium corn price and push the realised profit rate above (below) the normal profit rate. In Models D and E, the nominal interest rate, upon which this normal profit rate is calculated, is itself an endogenous cyclical variable. However, its volatility is far less than that of the realised profit rate, which is a "stylised fact" of the real world as well.

As noted above, the conventional pair of stable growth paths comprise the *stationary state* of zero growth and the *steady state* of constant positive exponential growth. In the CDG corn-credit models of this thesis, converging, diverging or regular (limit) cycles often are observed to trace out a stationary or a steady state growth *trendline.* This trendline is the principal object of study for Neoclassical economists – and for most early Post-Keynesians, including Kaldor (1957), Robinson (1956, 1962) and Pasinetti (1962). Neoclassicals believe the trajectory of this trendline is determined by supply-side factors, such as the growth in stocks of land, labour and capital – plus the rate of technical progress, however determined. Cycles are thought by most Neoclassicals to be the result of random *exogenous* shocks which temporarily knock the economy off its unique, stable, optimal, and pre-ordained long-run equilibrium path.

But to most Post-Classical economists, this trendline is merely a statistical artefact. "In fact", says Kalecki (1971, p 165), "the long-run trend is but a slowly changing component of a chain of short-period situations; it has no independent identity." In the corn-credit models of this thesis, all the supply-side factors are mixed in with all the demand-side influences to determine the investors' realised profit rate, which becomes their expected profit rate under a convenient "static expectations" assumption. Investor reactions to the profitability gaps that open up between the expected and normal profit rates will result in systematic *endogenous* cyclical, distributional and growth behaviour for the economy as a whole. Only such random exogenous shocks as affect "the bottom line" for investors (i.e. their expected *versus* their normal profit rate) will have any influence on cyclical behaviour. And even then, such shocks merely will *modify* (not determine) the shape and duration of the economy's inherent, investor-driven, non-optimal, endogenously-generated cycles.

1.10 Research Objectives

1. To utilise the minimum axiom-set of Post-Keynesian economics to construct, in five stages, a theoretical cycles, distribution and growth (CDG) model of a simple closed monetary production economy with no government;

2. To perform simple numerical analysis experiments by perturbing one key parameter at each stage of constructing this corn-credit model, starting from (a) its stationary-state dynamic equilibrium solution and (b) a steady-state fully-adjusted dynamic path generated by the model;

3. To use an expanded set of these "observed traverses" to elucidate the disequilibrium behaviour of the final Model E, an abstract *laisser faire* capitalist economy that grows, cycles and develops through simulated historical time;

4. To add a "government sector" and use the modified testbed (Model E*) to determine experimentally an effective, efficient and equitable fiscal policy package that automatically stabilises any disruptive traverse paths generated by the theoretical model; and

5. To so design Model E* that it may subsequently be expanded into a multisectoral complex economic systems model that potentially can explain the stylised facts of the typical open mixed capitalist economy, be calibrated against statistical data for specific regions and prove useful for short- and medium-term prediction based on realistic scenarios.

The research methodology adopted is known variously as “sequence analysis”, “dynamic process analysis”, "process dynamics", or “the method of shifting equilibrium”.

1.11 Organisation of the Thesis

In Chapter 2, the relatively sparse traverse literature is analytically surveyed. This survey is based on the history of economic thought up until 1973 and on Appendix A, which uses a coding system to classify the majority of traverse models developed since that pivotal year and accessible in the English-language literature.

In Chapter 3, the chosen research methodology is explained. Appendix B justifies the use of profitability gaps to determine levels of real investment at all stages of model construction.

In Chapters 4, 5 and 6 the CDG corn-credit model is constructed in five stages: A, B/C/D and E, respectively. Also in Chapter 6, numerous traverses of Model E are generated and analysed, starting from the primary basecase only, viz. the stationary-state dynamic path. (The secondary basecase could not be generated because Model E is not only the most flexible model, but the most sensitive to such perturbations as switching from zero to constant positive workforce growth.) A sensitivity matrix of long-period cause-effects (i.e. parameter-variables) elasticities is computed, then used to confirm some results from conventional comparative static analysis and to call others into question. Time-series plots reveal violent traverse behaviour, however and short- to medium-term results quite at variance with the long-term elasticities.

In Chapter 7, Model E mutates into Model E* *via* the addition of a public sector, which includes taxation, expenditure and budget deficits/surpluses implying growth/decline, respectively, of government debt. This model is the testbed for experimentally determining an effective, efficient and equitable “policy package” to regularise the unstable traverses and disruptive cyclical growth that Model E is found to exhibit. It features a [0, 1] “policy switch” to instantly toggle the model between regimes and compare the *laisser faire* results [0] with the policy-constrained [1] outcome.

In Chapter 8, the principal findings are listed, some important limitations are noted and certain directions are recommended for subsequent theoretical and applied research. Appendix C lists all parameters and variables used in the models and Appendix D discusses the accompanying CD-ROM, containing .htm and .xls computer files for Models A through E and E* plus the full text of this dissertation. A Bibliography that includes all references cited herein completes the thesis.

CHAPTER TWO

ANALYTICAL SURVEY OF THE TRAVERSE LITERATURE

2.1 Introduction

In this chapter, there are three objectives related to the traverse. The first is to define the "observed traverse" concept used during all five construction stages of the corn-credit model presented in this thesis. The second is to justify the originality of this thesis, given that so many traverse models are available in the literature. The final objective is to situate this thesis within the broad stream of economic thought concerning analysis of disequilibrium traverse phenomena. However, the objectives are not pursued in this particular order, e.g. before stating the operative traverse definition it is necessary to discuss the "early" (1821-1973) history of economic thought concerning this dynamic path of adjustment.

The traverse was named when J R Hicks published his First Traverse Analysis (based on Neoclassical theory) in 1965. However, only after John Hicks[2] presented his full Second Traverse Analysis (based on Neo-Austrian theory) in 1973 did the traverse literature take off into self-sustaining growth. Most post-1973 contributions to the traverse literature that are accessible to English-language readers are listed and classified in Appendix A.

Ricardo and Marx are first identified as the original progenitors of the traverse concept. Next, the work of the five pioneers who did most to convince the economics discipline of its importance for dynamic analysis is discussed, viz. the contributions of Kalecki, Lowe, Robinson, J R Hicks, and John Hicks. The analytical framework of this survey then is presented, before fitting into it these seven traverse models, spanning the 1821-1973 period. There also exists, it is conjectured, a vast trove of unrecognised traverse analyses, a "hidden literature" treating dynamic disequilibrium adjustment paths in a translucent, implied or even opaque, fashion.

The analytical framework is used again in Appendix A, this time to classify the post-1973 contributions mentioned above. This survey brings to the surface the frequencies with which certain elements of traverse modelling have recurred in the literature. In this way, the thesis can be positioned within the broad stream of economic thought concerning the way dynamic disequilibrium adjustment paths have been treated in the discipline's existing stock of traverse models. These findings also testify to the originality of the traverse model

[2] According to John Hicks (1975, p 365), he disowned his original identity (as the Neoclassical "J R Hicks") by publishing Hicks (1969), *A Theory of Economic History*. See also Hicks (1979).

constructed for this thesis, since its particular set of characteristics is shared with no other contribution identified by the analytical literature.survey.

2.2 Two Progenitors

Ricardo (1821) analysed the dynamic process of machine-labour substitution in Chapter 31 – On Machinery, which he added to the third edition of his 1817 *Principles of Political Economy and Taxation*. This was a recantation of his previous belief that the process benefited all classes of society by reducing commodity prices: "I am convinced", he wrote in 1821, "that the substitution of machinery for human labour, is often very injurious to the interests of the class of labourers" (p 388). Ricardo's traverse analysis showed that his earlier belief remained true, but *only* as a long-term proposition. In the short to medium term, he demonstrated, the working class would have to endure *transitional* unemployment and/or falling real wages as machinery displaced labour.

Marx (1885) presented an "embryonic consideration" of traverse phenomena in Volume II of *Capital*. Halevi & Kriesler (1998) note that Marx analysed "... the nature of the flows between the capital goods producing sectors and the consumption goods producing sectors ... In order to avoid disproportionality crises, Marx showed that certain conditions must be fulfilled by these flows. However, he also concluded that the satisfaction of these conditions was extremely unlikely in a capitalist economy ... this prepared the space for the analysis by Lowe and Hicks of the structural traverse ..." (p 1). With their neo-Marxian credentials and mutual respect, it can be said that Kalecki and Robinson also occupied this prepared space. Andrew Kliman (2001) goes further, arguing that Marx's two equilibrium reproduction schema constitute the initial and final states of an *unbalanced* growth model.

2.2.1 David Ricardo

Ricardo's historical-time traverse analysis pivots on two *fulcra*. The first is his distinction between a £15,000 "gross produce" and a £2,000 "net produce" of a business enterprise, the £13,000 difference being the capitalist's "wage fund" of corn to sustain his workforce. The second is the power of saving ≡ investment from an increasing net produce (i.e. "surplus" or profit) to add to the circulating capital of the enterprise (p 388).

> A capitalist we will suppose employs a capital of the value of 20,000*l*. and that he carries on the joint business of a farmer, and a manufacturer of necessaries. We will further suppose, that 7000*l*. of this capital is invested in fixed capital, viz. in buildings, implements, &c. &c. and that the remaining 13,000*l*. is employed as circulating capital in the support of labour. Let us suppose, too, that profits are 10 per cent., and

> consequently that the capitalist's capital is every year put into its original state of efficiency, and yields a profit of 2000*l*.

Then, in the *opening* year of Ricardo's traverse, the capitalist diverts half his workforce to construct a £7,500 machine, so that only £7,500 worth of circulating capital is reproduced that year, of which £2,000 flows to him as his usual surplus. In the *following* year, therefore, his wage fund will be only £5,500 worth of wage-goods with which to offer employment. So he fires all the machine-builders and more than one-quarter of those who lately were producing "corn" and (other) "necessaries", i.e. wage-goods.

By utilising the highly-productive new machine, this drastically-reduced workforce thereafter annually turns out a gross produce of £7,500-plus, and the capitalist is content to receive at least his usual profit of £2,000, after replacing the £5,500-plus wage fund of circulating capital. In addition, the unit costs of production of these wage-goods – hence also their prices – will have fallen due to mechanisation of the production process for corn and necessaries.

Ricardo then generalises this labour-saving impact to the entire economy (p 390).

> In this case, then, although the net produce will not be diminished in value, although its power of purchasing commodities may be greatly increased, the gross produce will have fallen from a value of 15,000*l*. to a value of 7500*l*., and as the power of supporting a population, and employing labour, depends always on the gross produce of a nation, and not on its net produce, there will necessarily be a diminution in the demand for labour, population will become redundant, and the situation of the labouring classes will be that of distress and poverty.

After a few more years of transitional adjustment along the traverse path, however, this situation will be ameliorated (and perhaps even corrected) because

> ... with the same wants he [the capitalist] would have increased means of saving, – increased facility of transferring revenue into capital. But with every increase of capital he would employ more labourers; and, therefore, a portion of the people thrown out of work in the first instance, would be subsequently employed; and if the increased production, in consequence of the employment of the machine, was so great as to afford, in the shape of net produce, as great a quantity of food and necessaries as existed before in the form of gross produce, there would be the same

ability to employ the whole population, and, therefore, there would not necessarily be any redundancy of people.

As for the *duration* of this traverse, Ricardo said "These savings, it must be remembered are annual, and must soon create a fund, much greater than the gross revenue, originally lost by the discovery of the machine, when the demand for labour will be as great as before" (p 396). Thus, the economy successfully negotiates the traverse and "soon" returns to its former stationary state.

This historic first-ever traverse analysis has all the right hallmarks. An initial dynamic equilibrium (a stationary state, in this case) is shattered by a parameter-change. This sparks off a transitional disequilibrium adjustment path which lasts for some years before a final state of dynamic equilibrium (also a stationary state) is successfully attained.

"On Machinery" was written less than five years after Britain's parliament had despatched 12,000 soldiers to quell the machine-breaking activities of Luddite textile workers opposing the introduction of mechanical weaving looms. And in France, the original *saboteurs* had been casting their wooden clogs into the mechanisms of the new Jacquard looms. Ricardo's analysis concluded that, for the duration of the traverse, "... the opinion entertained by the labouring class, that the employment of machinery is frequently detrimental to their interests, is not founded on prejudice and error, but is conformable to the correct principles of political economy." (p 392). Nonetheless, he supported the continued introduction of machinery because in the longer term it staved off diminishing returns in agriculture, improved the terms of trade and discouraged capital exports. No subsequent traverse study has overturned Ricardo's conclusion that labour-saving technical progress visits short-term pain upon a workforce that will, however, experience longer term gains.

The circulating capital and monetary production models of this thesis are not compatible with Ricardo's fixed capital traverse analysis. However, the same parameter perturbation methods are used, the adjustment paths are "observed traverses" and all models are capable of starting and finishing in a stationary state. In addition, their traverses can either continue indefinitely or move the economy into a limit cycle regime, depending on the nature of the farmers' seedcorn investment function. Ingrid Rima (1986, p 134) says that Ricardo's numerical example "... is analytically important because it introduces the technique of *sequence analysis* to examine the transition process from one equilibrium situation to another. The outcome of a change ... is traced out sequentially." In Chapter 3 below, it is noted that this thesis employs the same methodology, also known as "dynamic process analysis" or "process dynamics".

2.2.2 Karl Marx

Volume II of *Capital* introduced the "simple" and "extended" (or expanded) reproduction schemas of Marx (1885), which correspond to the classical stationary and steady states, respectively. According to Halevi & Kriesler (1998), Marx's Part III – The Reproduction and Circulation of the Aggregate Social Capital – addresses the problem of a lack of effective demand.

Marx's two schemas analyse real product flows between a capital-goods sector (Department I) and a consumption-goods sector (Department II), each using "constant capital" (means of production) and "variable capital" (paid labour) which generates "surplus value" (unpaid labour). He considers the conditions necessary for each sector to absorb its accumulation requirements (replacement or expansion of constant capital), both from its own production and from that of the other sector, without any co-ordination imposed *except* that derived from the market.

The basic aim of his schemas is to examine the conditions under which a capitalist economy can *grow* (extended reproduction), without enduring crises of overproduction in either sector. In Part III, Marx shows that the conditions necessary for such "balanced growth" are extremely restrictive and improbable, with the result being that overproduction within sectors is likely to generate structural imbalances in the flows between sectors.

The problem stems from the *dual* role of workers, as consumers of the output of the wage-goods sector *and* as a cost of production to both sectors, so that wages and profits are inversely related. This relationship, Marx believes, lies at the heart of capitalism and it provides an important obstacle to balanced growth, as it necessitates an increase in workers' powers of consumption that is antagonistic to capitalist class interests. As a result, the problem of unbalanced intersectoral flows will spread to the whole economy, the result being in a fall in investment, leading to an increase in unemployment.

When the initial crises caused by a skewed sectoral structure spread to become a general "underconsumption" problem, the link between disproportionalities and effective demand comes into its own. In other words, the reproduction schemas do not show the actual conditions of capitalist economies. Rather, Marx uses them to investigate the conditions under which such economies could grow without crises, in much the same way as in the Harrod-Domar growth model.

Having done this, the next stage might appropriately have been an investigation of what happens *outside* the stationary and steady states. Instead, say Halevi and Kreisler, Marx truncated his analysis precisely where Hicks, Robinson, Lowe, and Kalecki started theirs. Given the difficulty of growth without structural problems, the next step could have been a traverse analysis, to see how the capitalist economy will *respond* to these crises of disproportionality and underconsumption. Instead, Volume III (compiled by Friedrich Engels) abandons the sectoral approach of the reproduction schemas and so the opportunity was lost, according to Halevi & Kriesler.

However, Kliman (2001) credits Marx with taking the next step and performing a traverse analysis. He suggests that Marx's reproduction schemas constitute the first treatment of what Walter Rostow and later development theorists have called the (unbalanced) process of "take-off into self-sustaining growth".

"When regarded as two *distinct* models", says Kliman (p 1), "the schema of simple and expanded reproduction seem to depict balanced growth. In both cases, Departments I and II grow at the same (zero or positive) rate." However, "... the schema also demonstrate that growth – i.e., the *transition* from simple to expanded reproduction – requires that Department I grow faster than Department II." In more general terms, "... given the schemes' assumption of technological stasis, a long-run increase in the economy's growth rate always requires that Department I grow faster than Department II."

Kliman (p 2) goes on to say that Marx's schemas "... depict a process of transition ... If this transition is to take place, growth must be unbalanced." Kliman uses a principle of hermeneutics, a review of the secondary literature and textual evidence concerning Marx's own intentions to argue his case.

In support of Kliman, the models of this thesis do not *solve for* steady states, but *generate* them from initial stationary states. This is achieved by igniting traverses which attain a final positive constant exponential growth path. One also can quote Marx on the fact that his schema of simple reproduction contains the seeds of its own instability. Marx (pp 398-9) writes that

> Simple reproduction, reproduction on the same scale, appears as an abstraction, inasmuch as on the one hand the absence of all accumulation or reproduction on an extended scale is a strange assumption in capitalist conditions, and on the other hand conditions of production do not remain exactly the same in different years (and this is assumed).

As with Marx, this thesis also utilises the stationary state as a pure "abstraction". It is both a *basecase* from which traverses can be initiated and a *comparator* against which traverses can be assessed.

One can reduce Marx's prolix arguments and tortuous arithmetical examples of Part III to the following essentials. Divide the commodity output flows of Departments I and II into their constant capital (c), variable capital (v) and surplus value (s) components:

Means of Production	I = Ic + Iv + Is	socially-necessary labour hours
Articles of Consumption	II = IIc + IIv + IIs	socially-necessary labour hours

Workers consume all they earn, i.e. the variable capital equivalent of the wage bill. In Marx's general system, the social role of capitalists is to accumulate or invest, i.e. to *capitalise* most of the surplus value they extract from the working class. But under simple reproduction, their task is simply to keep the economy ticking over in a stationary state of zero growth, hence all surplus value must be *consumed* by the capitalist class.

As with Marx, in the abstract stationary states of Models A through D of this thesis, workers consume all they earn and capitalists do the same – after replacing their initial stocks of circulating capital.

The necessary articles of consumption can come only from Department II, whose workers consume IIv and whose capitalists consume IIs, leaving only IIc of consumption-goods available to sustain the workers and capitalists of Department I. *Prima facie* the only condition needed to maintain simple reproduction is that Iv + Is = IIc. The difficulties inherent in preserving such equality lead to the *inter-sectoral* "disproportionality crises" highlighted by Halevi & Kreisler.

However, there is an important *intra-sectoral* contradiction as well, centred on the Ic component of value, which as yet is unaccounted for. These capital-goods are produced internally by the capitalists of Department I. So, while the above equality is a necessary condition, it is far from being sufficient. Inside Sector Ic, where capital-goods firms exchange means of production for money, a traverse time-bomb is ticking away.

The constant capital of Department I is the sum of two separate flows: Ic = Id + Im, where d is a "fixed capital" flow (say, depreciation of machine-tools) and m is a "circulating capital" flow (say, inputs of raw material). The Im flow presents no real problem, since all intra-sectoral money-capital and commodity-capital exchanges balance each other. But the same is not true of the remaining portion of constant capital, the Id flow.

Department I's *stock* of fixed capital yields up its value to newly-produced means of production in increments as it depreciates, usually over many years. Unlike raw materials, the reproduction of machine-tools is "lumpy" rather than continuous. All Sector Ic capitalists are accumulating money in sinking funds, against the day when their depreciating machine-tools will have to be replaced. And some are, in fact, replacing them right now. Marx correctly points out that only by merest chance will the flow of newly-produced machine-tools precisely balance off against the Id flow of real depreciation. His is a kind of "impossibility theorem" for the stability and continuity of simple reproduction.

Marx (p 473) states that

> This illustration of fixed capital, on the basis of an unchanged scale of reproduction, is striking. A disproportion of the production of fixed and circulating capital is one of the favourite arguments of the economists in explaining crises. That such a disproportion can and must arise even when the fixed capital is merely *preserved*, that it can and must do so on the assumption of ideal normal production on the basis of simple reproduction of the already functioning social capital[,] is something new to them.

The contradictions inevitably must spark off a *traverse* in the direction of either more poverty (and possible collapse) or greater prosperity (and possible extended reproduction). Stasis is impossible under the capitalist mode of production.

In this thesis, however, simple reproduction can proceed indefinitely. Due to the absence of fixed capital, there are no endogenous forces working for destruction of the stationary state. The assumption that the only constant capital is circulating capital makes simple reproduction an effective basecase, against which traverse experiments can be run by perturbing the parameters of Models A through E. The other basecase from which traverse experiments are initiated is the steady state of extended reproduction.

2.3 Five Pioneers

The 1929-34 Great Depression inspired, *inter alia*, John Maynard Keynes's *Treatise on Money* (1930a, 1930b), Robinson's *Economics of Imperfect Competition* (1933) and Keynes's *General Theory* (1936), plus comparable demonstrations of the effective demand principle by Myrdal (1931) and Kalecki (1933). This massive "shock" or "disturbance" also was the "impulse" or "perturbation" that stimulated a fresh approach to the problem of structural traverse by Kalecki (early 1930s), who was followed by Lowe (1950s decade), Robinson (mid-1950s), J R Hicks (mid-1960s), and John Hicks (early 1970s). Their pioneering contributions are outlined below.

2.3.1 Michal Kalecki

Early in the 1930s decade, Kalecki published the first of a long series of macroeconomic models of endogenous cycles, distribution and growth, in which all three outcomes are driven by one strategic variable: the investment aggregate. Positive net investment has four separate effects within uncontrolled capitalist economies, viz. it increases productive capacity, raises aggregate demand, incorporates technical progress, and its trend-plus-fluctuations drive cycles, distribution and growth.

Steindl (1981, p 125) has rationalised Kalecki's impressive *oeuvre* into three underlying "versions". Courvisanos (1996, p 14) explains that

> Version I dates from an original Polish monograph (Kalecki, 1933) with two abbreviated English journal articles (Kalecki, 1935; 1937b). This version has an undamped endogenous business cycle, criticised mathematically by Frisch and Holme (1935). Version II dates from Kalecki (1943, 1954) ... This version maintains a linear equation. It has a damped business cycle which eventually requires a random shock to oscillate ... in the manner of a pendulum (Goodwin, 1964, p 421). Version III from Kalecki (1968) also has a linear damped business cycle, but allows for greater fluctuations as it concentrates on the profitability of new capital stock. In this way, it incorporates technical progress with a trend above cyclical oscillations.

Courvisanos (p 51) goes on to explain how "Kalecki never formally embraced the concept of the traverse in any of his work, but in three respects it is implicit in Kalecki's ... investment theories." The first aspect of his implicit traverse is the *production lag* which, in all versions, is a parameter of the investment function representing time elapsed between the decision to invest and the consequent deliveries of newly-produced capital goods. It makes its first appearance in Kalecki (1933) and figures in both subsequent versions

The second aspect of the implicit traverse concerns the long-period *path of economic growth*, which Kalecki (also Lowe and Robinson, see below) always viewed as no more than a slowly-changing component of the chain or sequence of short-period situations. Kalecki uses a short-period or sequential approach to permit specificity of capital goods, making the new vintages more productive than the old, thus capturing higher profit rates for the former.

The third aspect of the implicit traverse is the *shiftability of capital goods*, which Kalecki (1963) introduced to permit investment to increase without having to expand the capital-goods sector more rapidly than the economy's total productive capacity. Two shiftability factors can ameliorate this situation: changing the way some equipment is used, e.g. turning plant used for manufacturing consumer durables towards the production of machinery; and raising machinery imports, by cutting back on consumer-goods imports and/or increasing exports of such goods.

Kalecki had no need for an *explicit* traverse concept because, for him, the economy effectively was *always* in traverse. He knew that the fully-adjusted stationary state, steady state and regularly cycling time paths, although important theoretical constructs, are almost never observed in any real-world economy. This Kaleckian viewpoint (fresh traverses continually modifying earlier uncompleted traverses) also characterises the CDG models developed in Chapters 4 through 6 – and the traverse analyses of this thesis. A formal definition of Kalecki's implicit "observed traverse" is required and this is provided below, after the work of his four fellow pioneers has been outlined.

2.3.2 Adolph Lowe

Throughout the 1950s decade, Lowe developed his analysis of traverse processes by building upon the structural economics and business cycles work of the Kiel Institute of World Economics, which he had directed from 1926 to 1931. The relevant papers are Lowe (1951, 1952, 1954, 1955, 1959). This line of development culminated in *The Path of Economic Growth* (Lowe, 1976), which contains the fullest statement of his unique and highly original "instrumental traverse" concept, a melding of positive with normative economics that has significant policy-relevance and is utilised in Chapter 7 of this thesis.

Lowe's positive view of the *observed* path of economic growth (p 10) is identical with that of Kalecki and Robinson.

> ... what in retrospect appears as a secular process is, in fact, an abstraction derived from a sequence of short-term movements, the latter being the only "real" processes. We have long been accustomed to this kind of reasoning in statistical trend analysis. It is time to realize that it applies with equal force to the theoretical treatment of growth.

He expands on this in a footnote (p 10, n11):

> I have stated this position originally in the Introduction to ... [Lowe (1955)]. In the meantime, I received the valuable support of ... [Kalecki (1971, p 165)]: "In fact, the long-run trend is but a slowly changing component of a chain of short-period situations; it has no independent entity ..." The significance of these short-term processes for a theory of economic growth has been implicitly recognized in ... [Robinson (1956)] and in ... [Hicks (1965)], in which he discusses the intermediate processes required to achieve a change in the rate of growth under the heading of "traverse" – a suggestive term which we shall adopt.

Lowe accepts the "circular and horizontal" schemas of simple and extended reproduction, but he divides Marx's Department I (capital-goods) into Department Ia (machine-tools) and Department Ib (tractors). Department II (consumption-goods) produces corn with the aid of tractors. The machine-tools sector is of strategic importance, having the singular ability to initiate and sustain a circular production process of its own. His preferred analogy, which likens the machine-tools of an industrial economy to the "seedcorn" sustaining circular reproduction within an agricultural economy, sits well with this thesis.

In Lowe (1965), he reminisces that a study of bread production had shown "... seed-wheat as an input is capable of producing two types of outputs: bread-wheat as a potential consumer good and seed-wheat as its own replacement good". There follows an intuitive leap which solves the seeming paradox of infinite regress in the replacement of fixed capital, one which Austrian economists had papered over by positing "original inputs" of labour and land only. Searching for a special capital good that was capable of producing other capital goods, as well as reproducing itself, Lowe finds "... not one such mechanical instrument, but a comprehensive group which is defined as machine tools ... They play the same strategic role as seed-wheat plays in agriculture." (pp 269-70).

Following the lead of his former Kiel Institute student, Fritz Burchardt (1931-2), Lowe also incorporates the "linear and vertical" schema of Böhm-Bawerk's Austrian "stages of production" into his treatment of industrial structure. Thus his theoretical models also keep track of (a) the absorption within Department I of raw materials plus semi-finished capital goods and (b) the flows of finished capital-goods from Department Ib to Department II.

Sectoring of the industrial structure along its Marxian horizontally-integrated "width" dimension is complemented by adding an Austrian vertically-integrated "depth" dimension, to highlight the significance of working capital embodied in work-in-progress. Furthermore, while the Marxian analysis relates essentially to flows, Lowe explicitly includes their associated stock variables, which become especially significant when capital accumulation and other changes are to be analysed. As with Marx, the starting point of his analysis is the stationary state of zero growth. His model makes explicit the relevant stock-flow interactions involved in these changes and the adjustment processes (i.e. traverses) which they generate in restoring the stationary conditions.

It was in Lowe (1952), that his viewpoint began to move towards *normative* economics. Society should not necessarily accept a given structure of production and endure the inevitable traverses, but might democratically decide to control them. The division of net investment between the three sectors affects the overall growth rate. Then the "productivity of investment" (reciprocal of the marginal capital-output ratio) determines employment and output by interacting with the volume of investment allocated to both parts of Department I. Lowe's structural analysis inspired the similar corn-tractors models of economic planning by Mahalanobis (1953), Dobb (1955, 1960), Sen (1960), Raj & Sen (1961), Naqvi (1963), Mathur (1965), and Ishikawa (1967).

Too heavy a concentration of investment in the strategic machine-tool sector, instead of raising corn production after a short time-lag, can actually delay the increase in consumption unnecessarily. Making fewer tractors will certainly release resources for building extra machine-tools, but corn production cannot be increased without more tractors. A balance must be struck, so that Department Ia does not squeeze out Department Ib. Lowe also points out that the machine-tool sector is dependent on a balanced supply of intermediate inputs, which could slow its rate of growth for some time. Imports to supplement the domestic output of intermediate goods could, however, relieve such working-capital bottlenecks, at least for a time.

Eventually, in Lowe (1959), he gave precise expression to the possibility of an *instrumental analysis* of the traverse. By contrast with the deductive procedure of positive economic analysis – arguing "forward" from behavioural premises to terminal states – instrumental analysis resembles induction by searching "backward" for the determinants of given states. Except that the terminal states and processes are given by stipulation, rather than observation. He argues that the "invisible hand" of unfettered individualism cannot generate socially-optimal states unaided. Even if it could, it is only the *homo oeconomicus* assumption that decrees maximisation of the flow of consumer goods, or of the terminal capital stock (turnpike theory), to be the optimum that society always should aim for.

Therefore, Lowe calls for the deliberate "social engineering" of successful traverses to goal-states which have been selected in a democratic manner. A democracy might prefer the goal of preserving the people's traditional standard of living, for instance. But, whatever society decides, it is the task of instrumental controls to achieve these aims with the help of deliberate measures of economic policy – a contrived system taking the place of a self-regulating one. Lowe recommends that "carrot and stick" control measures should be designed to complement, not override, the behavioural-motivational patterns of economic agents.

In this thesis, the instrumental traverses of Lowe are employed to attain societal goals that the model *laisser faire* economy is incapable of reaching unaided. Model E* is created by adding a government sector to Model E and fiscal policies are developed to carry the economy (a) through a century of instability in its stationary state and (b) into a steady state of near full employment growth.

2.3.3 Joan Robinson

In the mid-1950s, Robinson (1956, pp 61-176) analysed "Accumulation in the Long Run" in Book II of *The Accumulation of Capital.* Therein, she describes the traverse as "The Process of Transition" (pp 140, 153, 168) between two "golden ages", i.e. steady states of constant full-employment growth. She distinguishes between a *change* in historical time within the same economy and a *comparison* in logical time between different economies. The transition process belongs to the former category, and comparative dynamics to the latter.

Her first traverse analysis initially *compares* two economies (Alaph and Beth) offering the same money wage. Beth has higher profit margins and money prices (hence a lower real wage) due to, say, "monopolistic rings". Alaph has the smaller investment (and the larger consumption) sector because "... we have caught them at a moment in their respective histories when the amount of employment is the same in both" (p 77). Hence its rates of capital accumulation, output growth and profit are all lower than in Beth. Two key assumptions are that both economies can draw on "unlimited supplies of labour", as in the Lewis (1955) development model, and that there is a single fixed technique of production.

Robinson next supposes a *change* within the more competitive of these two economies: "... the Alaph entrepreneurs begin to form themselves into rings and raise prices" (p 77). She shows that this must lower demand (hence also employment) in the C-sector, then eventually in the I-sector, which supplies it with capital goods. A likely outcome is "... the ratio of accumulation to the stock of capital is now the same in Alaph as it was in Beth, but this ratio has become established by a reduction in the stock of capital (and in the labour force) so that it now bears the low, Beth, ratio to the low, Alaph, level of accumulation" (p 78). On a brighter note, Robinson also refers to the possibility that "... if competition broke out in Beth, and gradually raised real wages there to the original Alaph level, a burst of extra accumulation (and immigration of labour) would establish the stock of capital at the high, Alaph, ratio to the high, Beth, rate of accumulation" (p 78).

Thereafter, she progressively drops the two key assumptions and analyses *changes* (within the same economy) that spark off traverses, starting from situations of labour surplus, which lowers the money wage (p 78); labour scarcity, which raises the money wage (p 80); and also "... (1) the rate of technical progress alters unexpectedly; (2) the competitive mechanism becomes clogged; (3) accumulation tends to vary relatively to the rate of increase of productivity; (4) technical progress fails to be spread evenly throughout the system" (p 89).

Following an interlude (pp 101-38) of *comparisons* (between different economies) concerning choice of technique, degree of mechanisation and measurement of capital[3], Robinson (p 139) returns to traverse analysis with this observation: "A change in the position of the mechanisation frontier in one economy is quite another story. It is an event taking place in time. It involves a change in the rate of profit and a revision of expectations."

This time she examines the effects of a labour shortage raising (p 140 *et seq*) – and a labour surplus lowering (p 153 *et seq*) – the "real-capital ratio", which is "The ratio of capital measured in terms of labour time to the amount of labour currently employed when it is working at normal capacity ... for this corresponds most closely to the conception of capital as a technical factor of production" (pp 122-3).

As a preliminary, Robinson (pp 139-40) makes several "Special Assumptions" designed

> ... to make it possible to analyse the transition from one technique to another as though it took place without any disturbance to tranquillity. The argument, for this reason, is somewhat fanciful, but setting it out in this way enables us to see the workings of the mechanism, which are hard to follow in the hurly-burly of short-period disequilibrium in which it actually operates.[4]

[3] Together with Robinson's (1953-4) article on the neoclassical production function, this interlude – which introduced the "Ruth Cohen Curiosum" (p 109) – ignited the long-running Cambridge controversies over capital theory, fully documented by Geoff Harcourt (1972, 1975, 1998).

[4] "We may speak of an economy in a state of *tranquillity* when it develops in a smooth regular manner without internal contradictions or external shocks, so that expectations based upon past experience are very confidently held, and are in fact constantly fulfilled and therefore renewed as time goes by." (Robinson, 1956, p 59).

After discussing shifts in the mechanisation frontier, she completes Book II by analysing those traverses set off by accumulation with neutral (pp 159-163) and biased (pp 164-72) technical progress – innovations that are both "capital-saving" and "capital-using".

Although Robinson views Book II as "… the central part of the work" (p ix), the remainder of her *magnum opus* contains many more traverse analyses, in Book III – The Short Period through Book VIII – International Trade. She describes the long-period golden age as representing "… a mythical state of affairs not likely to obtain in any actual economy" (p 99) and goes on (p 181) to state that

> In reality to-day is a break in time. Yesterday lies in the past, and has ceased to be relevant to what happens to-day, except in so far as experience of it colours expectations about what will happen next. To-morrow lies in the future and cannot be known. The short-period situation in existence to-day is like a geological fault; past and future developments are out of alignment. Only in the imagined conditions of a golden age do the strata run horizontally from yesterday to to-morrow without a break at to-day.

So, like Keynes, Kalecki and Lowe before her, Robinson reinforces the primacy of the Marshallian short period. Though not usually thought of in these terms, *The Accumulation of Capital* appears to be the most concentrated collection of traverse analyses in the history of economic thought. To Joan Robinson also, the uncontrolled capitalist economy was "always in traverse".

Although there is no choice of techniques for producing corn in the models of this thesis, Robinson's insistence that traverse analysis must be performed in historical (not logical) time is respected throughout. The traverses that are observed in Models A through E result from *changes*, as in her analyses. However, methods unavailable in 1956 allow *comparisons* to be made within the same economy, so that alternative dynamic paths (with and without an experimental parameter perturbation) can be compared.

2.3.4 J R Hicks

In the mid-1960s, Hicks (1965, pp 183-197) introduced a "new" method of dynamic economic analysis in Part 2 – Growth Equilibrium of his seminal work, *Capital and Growth*. The final chapter of Part 2 is Chapter 16 – Traverse, which presents what has become known as his First Traverse Analysis (FTA).

Hicks (p 184) introduces his Neoclassical concept of the dynamic disequilibrium adjustment path into mainstream economics and bestows the name "Traverse" upon it:

> Suppose that we have an economy which has in the past been in equilibrium in one set of conditions; and then ... a new set of conditions is imposed; is it possible (or how is it possible) for the economy to get into the new equilibrium, which is appropriate to the new conditions? We do not greatly diminish the generality of our study of disequilibrium if we regard it in this way, as a Traverse from one path to another. And there is some advantage to be gained from greater specification of the initial position from which the Traverse takes off ... Chiefly, it enables us to split up the kinds of adjustment that have to be made, so as to take different kinds separately.

Hicks (pp 184-185) first disposes of the Harrodian "knife-edge" problem. "If an economy has been in growth equilibrium at a particular growth rate, and is required to adjust to a different growth rate (maintaining full employment of labour), it cannot do so unless the propensity to save is varied, or the capital-output ratio is varied" – in response to an appropriate change in the rate of profit. After acknowledging the primacy of Kaldor's (1957) flexible-saving solution, Hicks admits Solow's (1956) flexible-technology solution as a secondary influence. "Indeed, if *anything* emerges to change the overall propensity to save out of income, along any channel, the Harrod difficulty can be got over. And (of course) if the change in the growth rate affects the capital-output ratio in the right direction, that also will help." So, a new equilibrium growth path *can* be defined. Hicks proceeds to derive a property that enables an economy to "traverse from" its initial steady state and "converge upon" its final steady state of growth.

Throughout Part 2 – Growth Equilibrium, Hicks develops his two-sector model of a farm-factory economy which employs labour and "tractors" to produce "corn" and tractors, with technical coefficients of production that are *fixed* for each *sector*. However, the *overall* capital-labour ratio (measured as tractors/worker) is *variable* and obviously the variation will depend on the economy's sectoral output composition.

> In that model ... the equilibrium ratio of 'tractors' to labour depends upon the rate of growth; with given technique it depends upon the rate of growth only. Thus if, at time 0, the economy is in equilibrium with a growth rate g_0, it will have to have a capital-labour ratio that corresponds. Now if the rate of growth is changed (either upwards or downwards) and the technique is unchanged, the equilibrium tractor-labour ratio will

> be changed; so that the actual tractor-labour ratio, at time 0, will not be that which is appropriate to the new equilibrium.

Hicks then performs his FTA, which examines "... four cases: according as g rises or falls, and according as m > or < 1" (p 186). Now g is the economy's growth rate and m the *ratio* of its two fixed sectoral capital-labour ratios. This variable is the economy's overall "relative degree of mechanisation"; m is a pure number that is formed by taking the tractor-labour ratio of the farm sector and dividing it by the tractor-labour ratio of the factory sector. So, if m > 1 the corn sector must be more "mechanised" than the tractor sector.

Hicks next proves algebraically "... that whether g rises or falls, there is a full-employment path to equilibrium, *provided that* m > 1. But if m < 1 (if the factory is more mechanized than the farm) such a full-employment path does not exist" (p 186). This is his famous Capital Intensity Theorem – although it was not Hicks who discovered it. Ronald Findlay (1963, p 6) first proved that the full-employment "state of bliss", analysed by Robinson (1956) in *The Accumulation of Capital*, could be attained only upon satisfaction of the Capital Intensity Theorem.

Like Ricardo before him, Hicks offers no opinion on how many years the economy might have to remain in traverse. But, in his logical-time model, that is not the point. J R Hicks simply is following the standard neoclassical methodology of deriving the "conditions" under which an equilibrium point or path will exhibit certain "properties". Necessary and/or sufficient conditions for the *four* classic properties of existence, uniqueness, stability, and optimality already had been proved by others.[5] What Hicks successfully derives is a new condition (m > 1) for a *fifth* property, one which might be termed the "traversibility" or "convergence" of an intertemporal Walrasian general equilibrium system that has experienced some exogenous shock.

This new (specifically dynamic) property of convergence or traversibility could be defined as: The ability of an economy to traverse from one equilibrium growth path to any other, while preserving equilibrium in the labour market – though not necessarily in the capital market. It is related to the stability property, which guarantees that temporary or accidental departures from the equilibrium path will not persist. Traversibility or convergence, however, guarantees that a structural change (such as faster workforce growth) will not prevent the economy from attaining the fresh dynamic equilibrium path which it entails.

[5] Most notably Walras (1874), Malinvaud (1953), Arrow & Debreu (1954), and Debreu (1959).

Hicks's Neoclassical traverse differs greatly from those of his fellow pioneers and progenitors. For instance, it runs its course in logical time and the world is assumed to be "ergodic".[6] This means an equilibrium must exist "out there", the economy's only task being to locate a fresh dynamic path and "manage" its way most efficiently towards this terminal steady state of growth.

By contrast, in this thesis, the corn-credit economy operates in historical time and the world is assumed to be "nonergodic".[7] Far from finding an equilibrium that already *exists*, the model endogenously *creates* a new dynamic path for itself, one that may or may not be an equilibrium trajectory. Whether the economy converges onto a fresh stationary or steady state, cycles endlessly or diverges is dependent on circumstances. In particular, its future evolution is governed by the nature of the farmers' seedcorn investment function.

2.3.5 John Hicks

Early in the 1970s decade, John Hicks (1970, 1973) introduced his Second Traverse Analysis (STA), a Neo-Austrian method based on the vertically-integrated linear production metaphor. It is "... descended from the 'Austrian' theory of Böhm-Bawerk and Hayek – a theory which had gone out of fashion, because of an obstacle which appeared to confine it to particular and practically unimportant applications. By developing an idea that was already present in *Value and Capital*, I was able to show that this obstacle could be overcome. There was something to be made of an 'Austrian' theory after all." (Hicks, 1973, p vi).

The "obstacle" he refers to is the Austrians' inability to handle *fixed* capital. They analysed production using a dated linear sequence of "point inputs" to yield a single dated "point output" of consumer goods. By contrast, the Neo-Austrian approach (p 8) would "... use an elementary process that converts a sequence (or stream) of inputs into a sequence of outputs ... The former difficulty of dealing with fixed capital is wholly overcome."

In his STA, Hicks defines two alternative dynamic paths, viz. Full Employment and Fixwage. In both cases, the "reference path" has a certain growth rate of "starts" of unit "elementary processes", each yielding a standard basket of consumption goods by using labour for a short *construction*, then for a longer *utilisation*, period. With vertical integration, labour is the only factor of production. Fixed capital equipment is classified as a process-specific intermediate good that is produced as part of the process and cannot exist outside it. An

[6] The Neoclassical "ergodic" world view is discussed by Samuelson (1968).

[7] The Post-Keynesian "nonergodic" world view is discussed by Davidson (1996).

elementary process is defined by its "profile", comprising information about service life, length of construction period and absorption of labour inputs.

Steady-state growth proceeds according to a demographic metaphor. The population of identical processes expands due to births exceeding deaths, i.e. process starts outweigh the number of processes reaching the end of their optimal service lives. Thus, in the steady state, a balanced age-distribution of live processes is maintained. But suddenly, at time t = 0, a new technique with its profile "strongly forward-biased" (using more construction but less utilisation labour) is invented and these "more mechanised" new processes begin to replace the less mechanised old processes throughout the model economy. Replacement can be *via* natural attrition and/or "truncation", i.e. the premature scrapping of old processes because their internal rates of return (IRRs) or "yields" are less than those of the new processes.

Along the Full Employment path, the real wage rate gradually rises with growth, while along the Fixwage path, the workforce poses no constraint and unemployment is possible. Investment, which determines the rate of new process starts, depends on the saving behaviour of capitalists, supplemented by resources released *via* the truncation of old processes. Investment consists in supplying consumption goods (during the construction periods) to workers engaged in starting new processes. Saving is governed by Hicks's Full Performance assumption: "extrawage consumption" (principally by capitalists) is a simple function of time, while workers consume all their wage income. With neither expected nor realised capital gains and losses along the traverse path affecting extrawage consumption, the labour market is likely to struggle towards its new equilibrium in a series of "jerks".

Traverse begins with an Early Phase of old processes being wound down (or truncated) and new, more productive and profitable, ones entering the population of live processes. The Late Phase opens once all the old processes have been killed off, thanks to their relatively lower IRRs. The Late Phase *may* end once a final steady state (compatible with the innovation) has been attained, yet ... "It cannot be taken for granted that the sequence, generated in this manner, will tend to a new equilibrium. It may or it may not." So there is no guarantee "... that our sequence can properly be considered as a *Traverse* from one steady state to another ... There are other possibilities." (p 82).

As in Ricardo's "On Machinery" chapter, although the Fixwage traverse initially is unfavourable to labour, its longer-term effects are likely to go the other way because the new steady-state path has greater expansionary potential. However, such steady states "... will be no more than a means to an end – to the study of an economy which is *not* expanding uniformly, an economy in which things actually happen." (p 47). With these words, Hicks signalled that his interest had switched from the Neoclassical traverse of *Capital and Growth* to the Neo-Austrian traverse of *Capital and Time.*

Although there is no population of elementary processes in the corn-credit models, there are some points of comparison between the observed traverse of this thesis and the Neo-Austrian traverse of Hicks's STA. Both are dynamic paths through historical time and both may attain final growth regimes that are far removed from the classic stationary and steady states. Also, the extrawage consumption of capitalists has a counterpart in Models A through D, where foodcorn is retained from each end-of-year harvest and earmarked for consumption within farming households during the following year.

2.4 Defining the Observed Traverse

To John Hicks, the traverse is "Neo-Austrian". It comprises an historical-time disequilibrium adjustment path that is sparked off by some technical process innovation which promises higher yields to those entrepreneurs who choose to adopt it. The traverse *may* converge on a final steady-state growth path, but it is quite likely that the Late Phase never actually terminates. Thus, the population of live processes continues to feature some mix of the old and new technologies. Hicks's non-smooth Neo-Austrian traverse closely resembles the "observed traverse" defined below.

To J R Hicks, the traverse is "Neoclassical". It comprises the conceptual logical-time adjustment path that connects an initial to a final steady-state equilibrium growth path, after some exogenous shock or impulse has hit the economy. His traverse *is* a genuine disequilibrium phenomenon but, as a true Neoclassical, J R Hicks permits smooth adjustment towards the new equilibrium path, provided there is sufficient flexibility in quantities, prices, saving behaviour, or techniques of production along the way.

To Lowe, the traverse is "instrumental". It comprises the optimum historical-time adjustment path that a democratic society would choose to implement – *via* the deliberate exercise of economic policy – in order to absorb growth in its stock of labour. For Lowe, the ultimate economic goal-state has been stipulated to be optimal *via* democratic processes, not *via* an untestable axiom-set that specifies *in vacuo* the nature of *homo oeconomicus*.

To Ricardo, Marx, Kalecki and Robinson, the traverse is "observed". It comprises the *actual* historical-time adjustment path traced out by an uncontrolled capitalist economy, as plotted by its statisticians. Dynamic disequilibrium states are possible in the Ricardian system and likely in the Neo-Austrian model of Hicks. For Marx, Kalecki and Robinson, dynamic disequilibrium is all-pervasive, so that one could even say "Life is a Traverse", the phrase coined by Peter Kreisler (1989, p 10). Lowe developed his instrumental traverse – which is subject to democratic social control – specifically to "tame" the disruptive observed traverse that he viewed as the alternative outcome.

Marx, Kalecki, Robinson, and Lowe share the same opinion concerning the effects of uncontrolled capitalism. Their particular traverse concepts differ only in that the first three economists use positive economics to *describe*, whereas the fourth uses normative economics to *prescribe*. J R Hicks, on the other hand, had not yet renounced his Neoclassical past, so his traverse adjusts smoothly to a final growth equilibrium.

As noted above, *Capital and Time* presents the Second Traverse Analysis of Hicks. When he published *Methods of Dynamic Economics*, John Hicks (1985, pp 144-5) finally disowned his First Traverse Analysis.

> We had to suppose, when analysing a Traverse, that capital (tractors) could be transferred, in various quantities, from one 'industry' to another, between one period and the next. If the end of the one and the beginning of the other are simultaneous, the transfer must take place instantaneously. But that is quite hard to accept. (Marshall, assuredly, would not have let us have it.)
>
> One gets no help … from the celebrated device of von Neumann, according to which the whole of the terminal capital (of the period) is treated as output of the period, while the initial capital is treated as input. For it remains the case that it must be possible, at the join, for the capital to be reallocated.

Having jettisoned the Neoclassical traverse, Hicks (1985, p 145) states that "Instantaneous reallocation may well be practicable if it is planned reallocation. But this will not do for Traverse analysis, for the study of what happens when an equilibrium is disturbed." The rejection by Hicks of his FTA Neoclassical traverse (together with the "observed" flavour of his STA Neo-Austrian traverse) makes unanimous the opinion in favour of the observed traverse (at least for *positive* economic analysis) among all pioneers and progenitors of the concept.

Courvisanos (1996, p 67, fn 29) credits Allen Oakley (private communication) with coining the apposite term "observed traverse" which, broadly defined, is Kalecki's "… dynamic (out of equilibrium) adjustment path in historical time", according to Kriesler (1989, pp 1-2). As for what, precisely, the economy is always adjusting to, Kalecki's voluminous writings most frequently mention changes in the powerful investment aggregate, which (on the supply side) embodies technical progress and determines productive capacity and (on the demand side) determines aggregate demand and drives cycles, distribution and growth.

In narrowing down Kriesler's broad definition, Courvisanos (p 50) writes that

> The traverse examines a sequence of irreversible events within the structure of production. When a change occurs (or is induced by policy) to alter the level of demand or supply in the economy at a macro level, there is a sequence of slowly evolving production decisions made by industries and firms in response to such changes. This production sequence is the [observed] traverse, or path of economic growth.

This thesis adopts both the broad and narrow definitions of Kalecki's observed traverse. For clarification, the term "structure of production" encompasses *all* parameters of the structural forms of Models A through E, and the term "production sequence" covers the simulated historical time paths of *all* endogenous variables in these models. In common with Lowe, this thesis regards all economies as "engines of provision" for the populations they must sustain, hence "production" is paramount and all-encompassing.

Throughout Chapters 3 through 6, the observed traverse concept is used exclusively. For purposes of this thesis, its broad and narrow definitions are simplified rigorously into:

> That set of dynamic disequilibrium adjustment paths traced out over historical time by all endogenous variables, following perturbation of one or more structural parameters of an economy initially evolving in a fully-adjusted stationary or steady state of long-period growth.

The observed traverse, therefore, tracks the path-dependent evolution of an initial state of growth into some final state, which may or may not possess the same long-period fully-adjusted character. In Chapter 7, Lowe's instrumental traverse concept is used to develop a suite of rational economic policies which the abstract corn-credit economy could adopt to attain (with minimum disruption) a steady state of growth that approximates Robinson's "Golden Age".

The above definition of the observed traverse avoids any mention of long-period growth *equilibrium* – even though it is accepted that the stationary state is an equilibrium growth path. In this thesis, it is demonstrated – *contra* Hicks (1965) and many other authors, both

Neoclassical and Post-Keynesian – that the steady state is *not* one of long-period equilibrium, but an historical time path of *disequilibrium* growth that happens to be smooth. The classic steady state is, in fact, a special case of *traverse*. Thus, all traverses which commence from the steady state are actually *second-order* dynamic processes. A first-order traverse always starts from the stationary-state equilibrium solution of a dynamic model.

In the real world, of course, steady states are rare events of short duration and stationary states non-existent. As Robinson (1971, p 3) says

> To find a stationary economy in real life we should look for some corner of the world untouched by war and trade where tradition rules and the cycle of production and distribution repeats itself from year to year, from generation to generation, without changes in population, technical innovations, or concentration of wealth ... The stationary state in economic theory was not supposed to describe any actual society. It was an analytical device intended to throw light upon relationships in the changing world in which the economists were living.

To many Post-Classical and Neo-Austrian economists, "Dynamic Disequilibrium Rules" and "Life is a Traverse". This makes it extremely difficult to "unravel" the separate contributions of exogenous (including policy) variables and structural changes to the behaviour of those historical time series which record what has been happening within any real-world economy.[8]

In the abstract model economies of pure theory, however, the experimenter can reverse direction and begin to "ravel", so to speak. As simulated historical time passes, the researcher can generate fresh traverses – to be stacked upon the cumulating aggregate of existing ones – thereby building up a "layered" set of time series which completely describes the history of the model economy. This geological "stratigraphic" metaphor is used in one section of Chapter 6, in which a fresh traverse is sparked off every 25 years by changing, in its turn, each structural-form parameter of Model E.

2.5 The Analytical Framework

Given the contemporary coexistence of at least two broad approaches to economics, encompassing competing paradigms that underpin more than a dozen different schools of

[8] Nonetheless, an attempt has been made by the "disequilibrium econometrics of cyclical growth" (DECG) researchers, e.g. Claude Hillinger (1992, Preface) notes that "... explanations of economic fluctuations ... focus on technology and on adjustment lags in investment and production."

thought, it is not surprising to discern a wide spectrum of opinions concerning the real-world *relevance* of traverse processes.

At the far right of this spectrum of Traverse Relevance are those who claim the economy is *never* in traverse (**NT**), while at the far left are those who maintain that the economy is *always* in traverse (**AT**). The first group comprises Monetarists, New Classicals (including Real Business Cycle theorists) and Radical Libertarians, all of whom embrace the rational expectations hypothesis. This group is the polar opposite of the second, which comprises Post-Walrasians (and other Complexity Economics theorists) plus Marx, Kalecki, Robinson, and their Post-Classical successors.

2.5.1 Traverse Relevance: AT, CP, MP, IP, UP, NP, or NT

There exists a spectrum of views concerning the relevance of traverse phenomena:

AT ⇐ CP ⇐ MP ⇐ **IP** ⇐ UP ⇐ NP ⇐ **NT**

As the traverse is a dynamic disequilibrium adjustment path, it is not surprising to find those at the rightmost *extremum* (**NT**) believing a free market economy to be effectively always at an equilibrium point (along its long-run fully-adjusted path) exhibiting the stringent Walras-Arrow-Debreu properties of existence, uniqueness, stability, and optimality, plus the Hicks FTA property of convergence.

As one ranges leftwards across this spectrum, other schools of thought (and individual economists) are encountered, who view the traverse as

NP – a Non-Problem	Stability and convergence properties are so strong that even large departures from equilibrium are soon corrected; there is no such thing as "path-dependence". [Neo-Keynesians, New Keynesians]
UP – an Unimportant Problem	Classical "normal prices" (also Neoclassical multiple equilibria) exist, so an unstable economy may gravitate (or be attracted) to some *adjacent* dynamic equilibrium trajectory; a weak form of path-dependence. [Most Classical economists, some GE theorists]

IP – an Important Problem	The bulk of ("traverse-aware") economists would occupy this centre ground: the dynamic path of disequilibrium adjustment *is* problematic, due to its long time-span and/or path-dependence. [David Ricardo, J R Hicks]
MP – a Major Problem	Time lags and/or hysteresis make the economy strongly path-dependent, so disequilibrium states change the underlying data on which the final fully-adjusted target state is based. [John Hicks, Nicholas Kaldor, Richard Goodwin]
CP – a Crucial Problem	Traverse phenomena are all-pervasive, so carefully-designed economic policies and plans are required to attain any generally-accepted target state. [Adolph Lowe, Janos Kornai, Oskar Lange]

Finally, one arrives at the leftmost *extremum* (**AT**), whose adherents believe that free market economies effectively *always* operate in disequilibrium states. Any long-period growth path is merely a statistically-smoothed version of the underlying succession of linked Marshallian short periods over a span of historical time.[9] As noted above, Kreisler describes this world view as "Life is a Traverse".

The Traverse Relevance spectrum is one important tool for classifying contributions to the traverse models literature that is accessible in English-language publications. However, many other key *differentia* apply to this literature as well.

As this literature covers such a wide range of traverse models, several additional dimensions are needed to classify them accurately, prior to any firm conclusions being drawn.

[9] This does *not* imply that these economists shun comparative statics and dynamics. On the contrary, they view such analyses as valuable for "sorting out" causal relationships between endogenous variables and for use as theoretical benchmarks.

2.5.2 World View: ERG or NER

The world is viewed as either ergodic (ERG) or nonergodic (NER). These terms are defined and discussed in two key contributions. The Neoclassical "ergodic" world view is presented by Samuelson (1968) and the Post-Keynesian "nonergodic" world view is given by Davidson (1996).

2.5.3 Time: HT or LT

The dynamic paths evolve in either irreversible historical time (HT) or reversible logical time (LT), as discussed in Chapter 1.

2.5.4 Initial State: STAT, STED, ATED, or DSEQ

The analysis begins with the model economy in a stationary (STAT), steady (STED), either stationary or steady (ATED), or some other disequilibrium (DSEQ) state.

2.5.5 Traverse Impulse: LAB, CAP, INN, WAG, R/Y, SAV, CRC, SUP, DEM, or PAR

The impulse/trigger/disturbance/perturbation that initiates/ignites/sparks off a traverse is a change in the workforce (LAB), capital stock (CAP), technological innovation (INN), real wage rate (WAG), profits share (R/Y), saving behaviour (SAV), credit-creation (CRC), consumer goods supply (SUP) or demand (DEM), or one of several parameters (PAR).

2.5.6 Human Agency: REP, SIC or AIB

The model economy's behavioural parameters reflect the behaviour of a representative agent (REP) or social/income classes (SIC). Otherwise they simply measure an average of individual behaviours (AIB).

2.5.7 Value: REAL or MONY

The economy is represented by a barter (REAL) or monetary (MONY) model. To be classified as "monetary", the model must include a stock of financial capital, not merely a stock of "real balances".

2.5.8 Production: CLRE, LNPR or CLLN

The model features Classical circular reproduction (CLRE), Austrian linear production (LNPR) or both (CLLN). Horizontal or circular reproduction is also known as the "interindustry", "input-output" or "sector" model, while vertical or linear production has also been termed the "stage" model. Circular reproduction was introduced by Quesnay and developed by Marx, Lowe, Leontief, Sraffa, Pasinetti, and others. Linear production was introduced by Böhm-Bawerk and developed by the Austrian economists who followed him.

2.5.9 Integration: VIN or NVI

If all intermediate inputs are represented by the dated quantities of labour they directly and indirectly contain, then production is said to be "vertically-integrated" (VIN). Otherwise, intermediate inputs are expressed in physical or real terms and production is "non-vertically-integrated" (NVI).

2.5.10 Capital Stocks: FK, WK or FW

The model contains stocks of fixed capital (FK), working capital (WK) or both (FW). In this thesis, working capital and "circulating capital" are synonymous.

2.5.11 Technology: FIX or VAR

The model assumes fixed (FIX) or variable (VAR) technical coefficients of production. The assumption of variable technology is rarely made in traverse models.

2.5.12 Sectors: CORN, COTR, CTLA, CJET, or CSEC

The economy is represented by a one-sector (CORN), two-sector (COTR), three-sector capital (CTLA), or three-sector consumption (CJET) model. These acronyms stand for corn, corn/tractors, corn/tractors/lathes, and corn/shoes/tractors models, respectively. These products merely indicate some broad commodity categories: necessary consumption goods (corn), luxury consumption goods (jewels), capital equipment (tractors), and machine-tools for making capital equipment (lathes). Multisectoral (CSEC) models also have been subjected to traverse analysis.

2.5.13 Traverse Propulsion: UR, CU, CR, SD,RN, PI, or EP

The dynamic model seeks uniform profit rates (UR), normal capacity utilisation rates (CU), both of the foregoing (CR), supply-demand equilibrium (SD)[10], or equality between two earning rates (RN).[11] Otherwise, it is driven by the propagation of innovations (PI), or the implementation of economic policies and plans (EP).

2.5.14 Inspiration

["Original"] and/or [Name] of economist or school of thought, whose writings inspired this traverse model.

2.5.15 Main Finding

Short summary of the principal conclusion(s) from this particular traverse analysis.

2.6 The Early Literature

Using the analytical framework developed above, the earliest models to be found in the traverse literature are classified as follows, beginning with the Traverse Relevance characteristic (in bold).

Ricardo (1821)

IP, NER, HT, STAT, INN, SIC, REAL, CLRE, NVI, FW, FIX, CORN, UR

[Original, Smith] The machine-labour substitution process generates transitional structural unemployment during the traverse towards a bigger aggregate surplus, until higher saving ≡ investment out of profits replenishes the depleted corn wage fund to a degree compatible with some final stationary state.

Marx (1885)

AT, NER, HT, ATED, CAP, SIC, MONY, CLRE, NVI, FW, FIX, CJET, UR

[Original, Ricardo] Free markets are incapable of co-ordinating intersectoral flows along a balanced growth path, so the unpaid unemployed victims of a *sectoral* "crisis of overproduction"

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[10] Supply-demand equality in a macroeconomic model implies saving-investment equality.
[11] Examples include realised *vs* required and expected *vs* realised profit rates.

	may, *via* their dearth of spending power, amplify this into an *economy-wide* "crisis of realisation", due to lack of effective demand.
Kalecki (1933+)	**AT**, NER, HT, DSEQ, CAP, SIC, REAL, CLRE, NVI, FW, FIX, CJET, SD [Original, Marx] Traverses are driven by the production/delivery time-lags on investment and the fact that newer vintages of capital equipment are more productive (have higher profit rates) than older ones. Dynamic disequilibrium can be ameliorated by making plant flexible between the tractor and corn sectors and by importing more tractors (at the cost of importing less/exporting more corn).
Lowe (1955+)	**CP**, NER, HT, ATED, LAB/INN, AIB, REAL, CLRE, NVI, FW, FIX, CTLA, EP [Original, Classical, Marx, Austrian] To absorb faster workforce growth, the lathe sector must be made to expand its capacity at the expense of the tractor sector, thus temporarily denying tractors to the corn sector and reducing its output. This lowers the real wage (*via* forced saving) for the duration of the traverse. This is paradoxical: In order ultimately to *increase* the output of corn, such output must, to begin with, be *reduced*.
Robinson (1956, 1962)	**AT**, NER, HT, STED, PAR, SIC, MONY, CLRE, NVI, FW, VAR, COTR, RN [Original, Classical, Marx, Keynes] Attaining a new steady state appropriate to a faster growth rate requires a rise in the proportion of productive capacity in the tractor sector. Corn sector output must fall for a time, to permit the higher rate of accumulation to get under way. Conversely, adjustment to a lower growth rate entails a period either of unemployment or of increased consumption. In the real world, frequent and erratic changes in the growth rate or in the bias of technical progress destroy tranquillity and steady-state conditions along with it.

J R Hicks (1965)	**IP**, ERG, LT, STED, LAB, SIC, REAL, CLRE, NVI, FK, FIX, COTR, SD [Original, Neoclassical] In this, his First Traverse Analysis (FTA), Hicks proves the Capital Intensity Theorem: a full-employment adjustment path to faster (or slower) workforce growth is impossible unless the corn sector is more mechanised than the tractor sector.
John Hicks (1970, 1973)	**MP**, NER, HT, STED, INN, SIC, REAL, LNPR, VIN, FW, FIX, CORN, SD [Original, Austrian, Classical] In this, his Second Traverse Analysis (STA), Hicks studies what happens when a new "strongly forward-biased" technique (using more construction but less utilisation labour) is invented. Traverse from the Fixwage path begins with old processes being wound down (or truncated) and the new, more productive and profitable, ones entering the population of live processes. Although initially unfavourable to labour, the new steady state has greater expansionary potential. However, there is no guarantee that the economy will attain this dynamic path.

2.7 The Hidden Literature

It is no accident that all seven progenitors and pioneers of traverse analysis also developed theories of economic growth and/or income distribution and/or cyclical output fluctuations. Such theories are to be found, *inter alia*, in Ricardo (1817), Marx (1885), Kalecki (1933), Lowe (1926), J R Hicks (1950), Robinson (1956, 1962), and John Hicks (1974). In terms of dynamic analysis, there exist close connections between the analysis of CDG models and the disequilibrium adjustment path that constitutes the traverse.

This connection is drawn out further in the conjecture of this thesis concerning the existence of a "hidden literature" on traverse phenomena. There also exists, it is conjectured, a vast trove of *unrecognised traverse analyses*, an esoteric or hidden literature which treats dynamic disequilibrium adjustment phenomena (possibly misrecognised) in a translucent, implied or even opaque, fashion.

Perhaps economic dynamicists have been "speaking traverse all their lives", to coin a phrase. The possibilities are legion: the "cumulative causation" processes of Knut Wicksell, Johan Akerman, Erik Lindahl, and Gunnar Myrdal; the increasing returns growth spiral of Adam Smith, Allyn Young and Nicholas Kaldor; the "kaleido-static process" of George Shackle; the Cobweb Theorem explanation of the hog cycle; most early trade cycle theorists, as well as the European inter-war business cycle theorists; Austrians like Friedrich von Hayek; Thorstein Veblen and other Institutionalists; process analysts like Dennis Robertson, Knut Wicksell, Erik Lindahl, Bertil Ohlin, Gunnar Myrdal, and Mabel Timlin; the development theorists of "poles of growth" (François Perroux), "balanced growth" (Paul Rosenstein-Rodan and Ragnar Nurkse), "unbalanced growth" (Albert Hirschman and Paul Streeten) and "self-sustaining growth" (Walter Rostow); and, of course, Joseph Schumpeter and Alfred Marshall.

To survey this putative *corpus* is beyond the scope of this thesis. If professional historians of economic thought were to trawl the works of the great dynamicists for implicit traverse analyses, there might well be a large catch. Certainly some fascinating specimens will be hauled up, including perhaps those "Sheaves" of Liquidity Functions in the "Field" of the Shifting Equilibrium, as manipulated by Timlin (1942) at Saskatchewan.

2.8 Results of Analytical Survey

Just as Classical corn models came back into fashion following Sraffa (1960), explicit traverse models began appearing after J R Hicks (1965) named the concept and John Hicks (1973) demonstrated, for a second time, that the analysis of dynamic disequilibrium paths could be a respectable object of mainstream economic research. There are seven traverse models in Section 2.6 above and 32 models in Appendix A. All 39 traverse models have been classified and coded, using the analytical framework developed in Section 2.5 above.

In Table 2.1 below, all but two row frequencies sum to 39, the number of traverse models included in the analytical survey. The "Traverse Impulse" and "Inspiration" row-sums exceed 39 because some models have two or more characteristic codes. For instance, Lowe (1976) uses both labour supply changes and innovations to initiate traverses and Robinson (1956) was inspired by the Classical economists, by Marx and by Keynes.

John Hicks's Second Traverse Analysis (STA) has inspired most subsequent models. The first substantive column (i.e. the codes appearing most frequently) contains all the Hicks STA codes except those for "Production" and "Integration". Traverse modellers seem to like all the Neo-Austrian characteristics *except* the two that most define this school's approach, viz. linear production and vertical integration. The First Traverse Analysis (FTA) of J R Hicks inspired seven models, which accounts for there being only seven contributions viewing the world as ergodic and time as logical.

Table 2.1 – Frequencies of Characteristics in 39 Traverse Models

Traverse Relevance	17 MP	9 IP	8 AT	5 CP			
World View	32 NER	7 ERG					
Time	32 HT	7 LT					
Initial State	29 STED	4 DSEQ	3 ATED	3 STAT			
Traverse Impulse	19 INN	10 LAB	5 CAP	5 PAR	2 SAV	2 WAG	4 Others
Human Agency	29 SIC	6 AIB	4 REP				
Value	30 REAL	9 MONY					
Production	19 CLRE	16 LNPR	4 CLLN				
Integration	26 NVI	13 VIN					
Capital Stocks	25 FW	12 FK	2 WK				
Technology	37 FIX	2 VAR					
Sectors	19 CORN	11 COTR	4 CTLA	3 CSEC	2 CJET		
Traverse Propulsion	25 SD	6 EP	3 UR	2 CR	1 CU	1 PI	1 RN
Inspiration	15 Hicks STA	7 Hicks FTA	6 Lowe	5 Kalecki	4 Marx	3 Keynes	10 Others

Source: "The Early Literature" (Section 2.6) and "Traverse Models After 1973" (Appendix A)

For most analysts, the Neoclassical traverse is a dead end. Hicks himself disowned his FTA, largely because dynamic disequilibrium traverses *necessarily* are path-determined, evolving through irreversible historical time in a nonergodic world. Mathematical tractability seems to be the reason for the predominance of fixed technologies and models having a single sector producing consumption goods.

Table 2.1 can be of considerable assistance in designing an original traverse model to be specially constructed for this thesis. From the first substantive column, it is advisable to select the codes NER, HT, CLRE, NVI, FIX, and CORN, for reasons given above. To be a good testbed for running traverse experiments, the model should contain *many* parameters and disequilibrium would *not* be an appropriate initial state or reference time path. So, PAR is accepted and DSEQ rejected. Instead, ATED is chosen so that experimental traverses can be initiated from stationary and/or steady state basecases, which will serve as traverse comparators. Originality often means taking "the road less travelled", so the remaining codes are selected from those with low frequencies of mention: MONY, WK, AT, AIB, and RN.

The corn-credit model of this thesis, therefore, has the characteristics: **AT**, NER, HT, ATED, PAR, AIB, MONY, CLRE, NVI, WK, FIX, CORN, RN. Inspiration is provided by Robinson, Kalecki and Lowe, whose own influences were the Classical economists, Marx, Böhm-Bawerk, and Keynes.

2.9 Conclusion

This chapter defined the "observed traverse" concept used for the corn-credit model of this thesis and located it within the broad stream of economic thought concerning analysis of traverse phenomena. The "early" (1821-1973) history of economic thought on this important dynamic adjustment path was discussed, with Ricardo and Marx being identified as progenitors, followed by five pioneers, viz. Kalecki, Lowe, Robinson, J R Hicks, and John Hicks. A "hidden literature" treating dynamic disequilibrium adjustment paths in a translucent, implied or even opaque, fashion was conjectured to exist.

An analytical framework was developed to classify the early literature and post-1973 contributions, helping position the thesis within the broad stream of economic thought. The findings also testify to the originality of the corn-credit model, whose construction commences in Chapter 4, once the research methodology employed for this thesis has been outlined and discussed in Chapter 3.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

The principal aim of this chapter is to present and justify the methodology employed for this theoretical traverse research. First, this chapter shows how the five model construction stages, comprising Models A through E, are "nested" to form a sequence. There follow discussions of the "shifting equilibrium" concept, dynamic disequilibrium traverse adjustment paths, stationary and steady growth paths, and ways of analysing observed traverses. The common features of the five models are next listed, the most important being the *investment function* determining annual retentions of seedcorn by farmers. This equation, a member of the Keynes/Kalecki *marginal-efficiency* group of investment theories, is discussed in the chapter, with a fuller treatment in Appendix B. Its principal explanator – the "profitability gap" – is demonstrated to be a kind of "genome" that is present in most investment functions.

Features specific to each model then are listed, followed by an explanation of how the models are *solved for* the stationary-state basecase and, finally, how this solution is used to *generate* a steady-state basecase. Experimentation proceeds by using *ceteris paribus* changes in parameters or variables as "triggers" for traverses that depart from (and, under certain conditions, return to) these classic growth states, which are utilised as reference dynamic paths. The evolution of each traverse path over a century of simulated historical time is observed and analysed for what it reveals about the dynamic behaviour of these recursive systems.

The disequilibrium dynamics are especially rich in the fifth and final model of pure *laisser faire* in Chapter 6, viz. Model E. The addition of a "government sector" to this model introduces *no* new theoretical content. In Chapter 7, Model E* shows that there exists at least one "fiscal policy package" capable of ameliorating traverse-generated instability in the unregulated capitalist economy of Model E.

Appendix B demonstrates that all Classical *uniform-profitability*; Keynesian *marginal-efficiency*; Neo-Keynesian *multiplier-accelerator*; and Neoclassical *q-ratio* and *user-cost* investment functions are individual "ontogenic" expressions of a general "phylogenic" investment function, *viz.* the one used in Models A through E. By analogy with an organism's genes, the profitability gap is characterised as the common "genome" of all specimens within the investment equation species. An alternative analogy would characterise the profitability gap as the Greek-language key to translating the heiroglyphic, demotic, etc. "Rosetta Stone" scripts in which several (essentially equivalent) investment functions have been expressed by different schools of economic thought.

3.2 A Sequence of Nested Models

This thesis involves the staged construction, solution, manipulation, and analysis of a sequence of five minimal, but progressively more flexible, recursive dynamic CDG models of a monetary production (or corn-credit) economy. Its money-of-account is "dollars" and its only output is sacks of "corn". At the end of each crop year, farmers store part of the harvest in their barns (investment in "seedcorn") but the bulk is held in their granaries and progressively released for sale at markets held weekly during the new year (consumption of "foodcorn"). These are abstract closed economies of unregulated agrarian capitalism, having no government sector and no constraints on the stock of arable land.

For analytical purposes, the starting point is always the same "primary basecase" or reference growth path, representing a stationary state of long-period equilibrium. This is the state of Marxian "simple reproduction", in which the "baseyear" (year-zero) situation is replicated *ad infinitum* or, in these CDG models, for 100 years of simulated historical time. Such stationary states of zero growth are the simplest of all long-period growth paths – and were the first to be developed by the Classical economists. In the primary basecase, precisely 40,000 sacks of seedcorn are invested annually, the realised profit rate is always five per cent per annum and the graph of every economic variable, when plotted against time, forms a perfect horizontal flatline.

These corn-credit models also can achieve classic steady states of Marxian "extended reproduction", in which all relevant variables (production, investment, employment, consumption, etc.) exhibit growth at a constant, positive and uniform exponential rate.[12] For analytical purposes, a fully-adjusted long-period steady state is utilised as a "secondary basecase". This particular reference growth path can be generated, *inter alia*, by making the corn economy's (formerly stationary) workforce grow annually at some constant positive exponential rate. After an initial period of adjustment – a traverse, in fact – all relevant growth rates come to match that of the workforce.

[12] Analysis of steady-state growth paths in logical time is favoured by the Neoclassical schools. Robinson (1956, 1962) and other Post-Keynesian economists such as Harrod (1939), Kaldor (1957), Kahn (1959), and Pasinetti (1974, 1981) analyse steady-state growth paths in historical time.

Once the structural form of each model has been specified and its reduced form derived, one or more of its parameters (i.e. its initial values and constants) are perturbed to generate the resultant traverses from the opening basecase of stationary- or steady-state growth. In all such experiments, the parameter changes are made during year 30 of the 100-year span of simulated historical time. There is no presumption that the system will converge on a new fully-adjusted state, these being observed traverses as defined in Chapter 2.

Table 3.1 below shows how the fixprice Model A is extended and transformed into the flexprice Model E by progressively relaxing its four most stringent assumptions. These comprise a fixed corn price, money wage and interest rate, together with the absence of a conventional demand function for foodcorn in Models A through D.

Table 3.1 – Five Stages in Constructing a Flexprice Corn-Credit Economy

Model A	–	Fixprice Corn Economy	See Chapter 4
Model B	–	Model A with Flexible Corn Price	See Chapter 5
Model C	–	Model B with Flexible Money Wage	See Chapter 5
Model D	–	Model C with Flexible Interest Rate	See Chapter 5
Model E	–	Model D with Foodcorn Demand Function	See Chapter 6

This sequence of monetised corn models is "nested" in the sense that A forms the core of B, B lies at the heart of C and so on up to Model E. As noted in Chapter 1, these are really the five *stages of construction* for Model E, which is the one most intensively analysed in Chapters 6 and 7. Nonetheless, each is self-contained and can operate independently.

In fact, Model B could be used as a simple undergraduate teaching model for imparting the rudiments of economic dynamics, including the basics of Post-Keynesian cycles, growth and distribution theory. In Chapter 8, it is argued that the recursive dynamic system of Model E* holds potential for almost limitless future development into a realistic applied multisectoral open-economy CDG model having maximum flexibility and real-world policy relevance – subject to parameter identification and statistical validation using empirical data from a contemporary capitalist mixed economy.

3.3 The Concept of Shifting Equilibrium

Peter Skott (1989, p 8) states that "In general, any non-vacuous and internally consistent theory will describe a number of regularities and define a non-empty set of outcomes satisfying the regularities. This set of consistent outcomes constitutes the equilibria of the theory. A proof of the existence of equilibrium therefore is simply a check on the logical consistency of the theory. A theory without equilibrium ... is logically false." All five CDG corn-credit models *do* have unique long-period equilibrium stationary-state solutions, hence must be logically true and consistent.

Skott's argument does not imply that a system need always (or ever) *operate* in equilibrium, merely that an equilibrium notionally must *exist*. That is why, at each of its five construction stages, the model first is solved for the stationary state consistent with its parameter-set.

There are differing conceptions of "equilibrium" within schools of economic thought. Victoria Chick (1983, p 21) defines it as a situation of rest, in which *the forces leading to change* are either absent or countervailing. Hers is the broad Post-Keynesian view, which contrasts with the highly-specific Neoclassical definition, viz. zero excess demand in all commodity (including labour) markets. This difference in terminology accounts for the difficulty many mainstream economists face in accepting that Keynes proved the existence and persistence of an "unemployment equilibrium", which to them sounds like an oxymoron.[13] For Keynes and the Post-Keynesians, equilibrium is a state in which expectations are being fulfilled, so that economic agents are content to keep on doing whatever it is they have been doing in the recent past.

Jan Kregel (1976, pp 214-17) distinguishes between three "equilibrium" models used by Keynes (1936). Each has different characteristics, viz.

- i long-period expectations are stable and short-period expectations are fulfilled;
- ii long-period expectations are stable but short-period expectations are unfulfilled; and
- iii long-period expectations are unstable because
  - iiia they are affected by unfulfilled short-period expectations, or
  - iiib the autonomous influences on them are changing

Kregel is answering criticisms by Ludwig Lachmann (1973) and Mark Blaug (1974), who claims that the "Cambridge School" of early Post-Keynesian economists (Robinson, Kalecki, Kaldor, et al.) had no Keynesian *imprimatur* for their favoured methodology. This comprises *comparative dynamics*, i.e. comparing and contrasting long-period steady-state time paths, such as Robinson's "Golden Age" of full-employment growth *versus* other trajectories.

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[13] In Keynesian unemployment equilibrium, the involuntarily unemployed can bid down the money wage, but the real wage does not change because the price level falls *pari passu* with the money wage. The "sticky" real wages which Neo-Keynesians and New Keynesians claim as a *cause* of involuntary unemployment are actually an *effect*.

Kregel (1976, p 220) demonstrates that the Cambridge School economists are using Keynes' Model ii, choosing it largely because of Robinson's "... often repeated *caveat* that the analysis of steady growth has to be fully worked out and understood before moves can be made to the analysis of dynamic change over time" (i.e. before any traverse analysis can be undertaken). Kregel shows that Keynes uses Model i for most of his *General Theory*, but occasionally switches to Model ii, e.g. in Chapter 17 – The Essential Properties of Interest and Money. Significantly, he also uses Model iii in those parts which so often puzzle exegetists and commentators, e.g. in Chapter 5 – Expectation as Determining Output and Employment, in Chapter 12 – The State of Long-Term Expectation and in Chapter 22 – Notes on the Trade Cycle.

Model iii is the model of "shifting equilibrium" and the Model iiib "autonomous influences" are parameter changes. Keynes's shifting *equilibrium* trajectory is identical with the dynamic *disequilibrium* observed traverse of this thesis. As Kregel (1976, p 217) notes, "If ... realisation of error alters the state of expectations and shifts the independent behavioural functions, Keynes's model of shifting equilibrium will describe an *actual path* of an economy over time chasing an ever changing equilibrium – it need never catch it." (Italics added). Kregel's "actual path" is what Kriesler terms the "observed traverse", which *might* converge upon some fully-adjusted regime: a final stationary or steady state, or even a regularly recurring limit cycle. However, there is no guarantee that this *must* occur ... in which case the entire 100 years of simulated historical time represents a *continuation* of the actual path or observed traverse.[14]

Keynes (1936, pp 48-50) clearly describes the shifting equilibrium time path. He commences with "Let us consider, first of all, the process of transition to a long-period position due to a change in expectation ..." and, after describing over- and under-shooting of the target, says

> Thus a mere change in expectation is capable of producing an oscillation of the same kind of shape as a cyclical movement, in the course of working itself out. It was movements of this kind which I discussed in my *Treatise on Money* in connection with the building up or the depletion of stocks of working and liquid capital consequent on change.

He concludes that such an uninterrupted process of transition to a new long-period position can be complicated. "But the actual course of events is more complicated still. For the state of expectation is liable to constant change, a new expectation being super-imposed long before the previous change has fully worked itself out ..." In Chapter 6 of this thesis, six

[14] One of the metrics reported in Chapters 5, 6 and 7 is "traverse duration" measured in years.

separate traverses are sequentially layered or stacked or "super-imposed" in this Keynesian manner.

3.4 Stationary and Steady States of Growth

Keynes, Kregel and Chick agree that there are two types of expectations which need to be satisfied for equilibrium to prevail, viz. long-period (investment) and short-period (sales) expectations. Significantly, in the corn-credit models of this thesis, these two species of expectations *coincide*, because both the long and the short periods are of one crop year's duration.

The allocation of *last* year's grain harvest between investment goods (seedcorn) and consumption goods (foodcorn) was determined by farmer-entrepreneurs as that year closed. Once the new year opens, that earlier *flow* of seedcorn invested represents an opening *stock* of circulating capital, which, by being sown, determines *this* year's production and employment levels. However, *this* year's sales receipts (hence also dollar profits) are realised at weekly markets where *last* year's supply of foodcorn (Qso sacks)[15] is sold by farmer-traders to meet *this* year's consumer demand.

When farmers make their investment decision, they cannot forecast reliably what corn price (P dollars/sack) they will receive. They are ignorant of what profit (R dollars) they will make, what the capital value of their proposed investment in seedcorn (Ka = P Qio dollars) will turn out to be and, most significantly, what profit rate (r = R / K % pa) they will realise. Keynes (1937, p 114) asked "How do we manage in such circumstances to behave in a manner which saves our faces as rational, economic men?" The investing farmers, in fact, are forced to adopt some plausible *convention* for developing their expectations of what profit rate they will realise fully twelve months into the unknown (and inherently unknowable) future.[16]

It is "expectation-fulfilment" (no surprises) that keeps the farmers investing such that the primary basecase stationary states of these corn-credit models persist as simulated historical time passes. Keynes (1937, p 114) says that investing entrepreneurs adopt a three-part convention: (1) "assume that the present is a ... serviceable guide to the future", (2) "assume that the *existing* state of opinion as expressed in prices ... is based on a *correct* summing up of future prospects" and (3) "endeavour to conform with the behaviour of the majority or the average."

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[15] Variables with a lower-case "o" are lagged by one year.
[16] Paul Davidson (1996) contrasts the Keynesian view of reality ("nonergodic, unknowable, and transmutable") with the Neoclassical view ("predetermined, immutable, and ergodically knowable").

Keynes's convention is consistent with a working assumption of naïve or myopic or static expectations.[17] In these CDG models, it is assumed that *this* year's realised profit rate is what each farmer confidently expects to earn *next* year (re = r% pa). So, if r% pa is higher (lower) than n% pa – the normal profit rate or opportunity cost of capital that each desires to earn – the farmers will react to this "profitability gap" (a = [r – n]% pa) by retaining more (less) seedcorn at the end of this year (Qi sacks pa) than they did after last year's harvest (Qio sacks pa). The average strength of all their individual reactions to the average of all their individual profitability gaps is measured by the parameter ϕ, a constant called the economy's "reaction coefficient".

Like all other behavioural parameters of these corn-credit models, ϕ is not the same for all individuals. The farmers, workers and bankers (who also are consumers) that populate this model economy *cannot* be construed as Neoclassical "representative agents". Rather, each relevant parameter of the model comprises an "average of individual behaviours". This "AIB assumption" is the same implicit assumption that economists habitually use to horizontally sum numerous individual demand functions into a single market demand function. Each consumer has a different set of demand elasticities, so "the" own-price, cross-price and income elasticities governing total consumption of (say) corn must represent an average of individual behaviours.

In the *stationary state*, farmers have been realising their expectations by earning a rate of profit (re = r) = n% pa on the dollar value of their capital stock of seedcorn. This keeps them content to continue sowing Qi = Qio sacks pa indefinitely. In the *steady state*, farmers have been earning (re = r) > n% pa, their positive profitability gap remaining constant. They react by annually retaining a stock of circulating capital that keeps on growing (Qi > Qio sacks pa) at some constant exponential rate (gQi% pa). Thus Qi = (1 + ϕ a) Qio sacks pa is the profitability gap investment function governing farmers' retentions of seedcorn in all five corn-credit models. Obviously gQi = ϕ a% pa is the rate of capital accumulation along a steady-state growth path exhibiting constancy of the profitability gap.

As a crude analogy, in each of these Post-Keynesian models it is the "tail" (profitability gap) which wags the "rump" (seedcorn investment) which moves the "dog" (corn production, employment, incomes, prices, ...). Subsequently, the profits realised from sales of last year's foodcorn feed back in the form of a *fresh* profitability gap, thus driving this circular and cumulative process inexorably on through simulated historical time. Note that all the

[17] Alternative nonergodic assumptions would include "adaptive" and "least squares" expectations, but never "rational" expectations, which require the world to be ergodic.

foodcorn stored in farmers' granaries after any particular year's 31st December harvest is sold to households at weekly markets held during the *following* year.

A constant profitability gap of a > 0% pa implies that farmers are *never* in long-period dynamic equilibrium. In such steady states they are forever *dissatisfied*, believing the volume of this year's opening stock of circulating capital to be "too low". The farmers' experience of continually realising an abnormally high profit rate (r > n% pa and r > ro% pa) is what prompts them to keep *net* investment positive by retaining Qi > Qio sacks of seedcorn year after year, and this makes the economy's capital stock grow over time at the rate gQi = ϕ a% pa.

In this thesis, the *only* long-period equilibrium time path is the stationary-state *solution* of each model. Traverses are dynamic disequilibrium phenomena but, surprisingly, so too is the steady-state growth path. This trajectory is, in fact, a species of perpetual traverse – the only one along which the profitability gap remains both positive *and constant*. This gap is what *generates* the constant positive exponential rate of growth in investment that characterises a steady-state time path. As Victoria Chick (1983, pp 22-3) notes, "While net investment is positive – or negative – the capital stock is changing; therefore the economy cannot be in equilibrium in the sense that the magnitudes of all the variables, stocks as well as flows, are stationary." It is more appropriate to describe both classic long-period growth paths as "fully-adjusted", with only the stationary state having the *additional* property of being an "equilibrium" dynamic path.

Joan Robinson (1971, pp 75) says of her "Golden Age" – a long-period steady-state growth path generated using a comparable monetary production model – that "This is not a system in equilibrium; there is no mechanism to keep it on its path. The only point of setting it up is to see where it is liable to go wrong." In other words, even a smooth exponential growth path can be diverted onto a disruptive traverse path at a moment's notice, typically by a parameter perturbation.

Most modern heterodox economists are aware of the impressive *corpus* of theoretical insights these early Post-Keynesians achieved using Model ii – including Robinson's taxonomy of "metallic ages" of economic growth; her concept of an "inflation barrier" being erected by workers against further investment, whenever too-rapid accumulation is causing unacceptable real wage reductions; the "Cambridge" and "Golden" Rules relating the economic growth rate to the profit rate and saving propensity; the Keynes/Kalecki/Kaldor macrodistribution theory of investment creating profits; the "Pasinetti paradox" that worker saving has no effect on the growth rate; the logical impossibility of any "real capital" aggregate being independent of the profit rate; and the associated "reswitching" and "capital reversal" debates.[18]

Considering the massive analytical resources devoted to Model ii by the Cambridge School and others, any further theoretical advances most likely will flow from deployment of Keynes' far less tractable shifting equilibrium Model iii by contemporary Post-Classical economists.

3.5 The Methodology of Dynamic Process Analysis

The intrinsic recursive dynamics of the five corn-credit models developed for this thesis *are* of Keynes's Model iii (shifting equilibrium) variety. Yet they invariably *start* from fully-adjusted dynamic paths, either the stationary-state primary basecase or the steady-state secondary basecase. The model economies track these classic growth paths, unless and until disrupted by a perturbation to one or more of their variables, initial values or constants. Such perturbations are the "triggers" that spark off the dynamically shifting Model iii behaviour which underlies all disequilibrium traverse processes that are caused by any sequence of non-zero profitability gaps.

As noted in Chapter 1, Kriesler believes that "Life is a Traverse". His implication is that such dynamic adjustment paths are commonplace – in fact, that they constitute a *permanent* feature of the economic landscape. It is clear that economic cycles constitute shifting equilibrium (equivalently, dynamic disequilibrium) phenomena,[19] as do the panics, depressions, manias, bubbles, and booms that afflict capitalist economies from time to time. So, given the multitude of exogenous influences incessantly impinging upon modern real-world economies, the time series published by their statistical bureaux could be viewed as the *resultant* of numerous separate traverses, layered or stacked on top of each other over some underlying endogenous dynamic pattern.[20] Chapter 6 displays two time series graphs, generated from Model E by perturbing each of its parameters at 25-year intervals. A new traverse is initiated before one or more of the previous traverses have terminated. By analogy with the earth's layered crust, one could say that "Life is a Series of Stacked Traverses".

In these abstract corn-credit models, the economy always *opens* in a fully-adjusted long-period stationary- or steady-state basecase, but something happens in year 30 which upsets the farmers' short-period expectations of profit from selling that portion of their harvest

[18] *Vide* Sraffa (1960) and Harcourt (1972, 1975, 1998).
[19] This is clear to all economists except Neoclassical "real business cycle" (RBC) theorists.
[20] This is in complete contrast to the Frisch-Slutsky hypothesis that the business cycle results from a series of exogenous shocks to a linear Neoclassical model. *Vide* Mullineaux (1990, pp 19-29).

allocated to the foodcorn market. Profit (R dollars pa) is the difference between sales proceeds and costs of production, so this highly-significant *residual* is impacted by any and all changes in the economy – as is the closely-related realised profit rate (r% pa). Under the static expectations assumption, any change in r% pa will alter the profitability gap[21] and react upon the aggregate of seedcorn invested.

By changing a variable, initial value or constant, a traverse experiment drives a wedge between r% pa and n% pa, which gives the profitability gap (a% pa) its new non-zero value and "surprises" the farmer-entrepreneurs, making them react by retaining more or less seedcorn in year 31 than the 40,000+ sacks annually stored in the previous 30 years. This parameter-change triggers the Marx/Kalecki/Robinson *observed traverse process*, which is fuelled by the capitalist farmers' reactions to their short-period expectations being unfulfilled.

Typically an endogenously-generated disequilibrium economic *cycle* will ensue and these computed traverses can be observed as numerical time series and/or their associated graph plots. The cycles are driven by farmers reacting to a time-stream of changing profitability gaps, a% pa being the measure of *disequilibrium*, i.e. of "unfulfilled expectations" that r = n% pa should have been the profit rate that actually was realised.

The value of "a" may be decreasing, increasing, cycling, or even remaining constant. If the profitability gap is decreasing during the traverse, the economy's cycles will converge and an equilibrium stationary or fully-adjusted steady state eventually will become re-established. If "a" is increasing, the economy's cycles will diverge indefinitely. If "a" is cycling regularly, the economy will follow, describing a sequence of fully-adjusted limit cycles. Likewise, if "a" is cycling irregularly, the economy will do so too. Finally, if "a" settles down to a constant gap above (below) zero, the traverse process will have led the economy into a long-period steady state of constant growth (decline).

Accurate deployment of Model iii was not feasible in Keynes's day, although Dennis Robertson (1933) and Bertil Ohlin (1937) attempted it, using tedious manual calculation procedures. Mabel Timlin (1942) employed Keynes's shifting equilibrium model to extend his liquidity preference theory of the nominal interest rate. As Chick (1983, p 16) states, "The *General Theory* can hardly be said to be concerned with a static economy. Yet paradoxically the method used was statics" (*ie* Keynes' Model i). She notes that "Mrs Robinson (1952) resolved the paradox thus: 'Past history is put into the initial conditions, so that the analysis is static in itself, and yet is part of a dynamic theory.'" Chick (p 24) calls Keynes's preferred

[21] It is a "stylised fact" of the real world that r% pa is more volatile than n% pa and the same is true of these CDG models.

Model iii "process dynamics" and points out that "It is in the 'shifting model' (Model iii) that investment is allowed the volatile behaviour which some would say was the essence of Keynes's theory."

Abba Lerner (1940) called Keynes's (then-unattainable) *desideratum* "dynamic process analysis", noting that a version of Model iii also had been favoured by two Swedish contemporaries (Gunnar Myrdal and Erik Lindahl), with their *ex ante* "Dynamic Approach". Lerner (1940, pp 238-9) was scornful: "... this is nothing but another prayer for LaPlace's equation of the universe", he wrote, pointing to

> the impossible complexities to which the process method [Model iii] leads before it can be applied to any practical problem ... The assumption of short-period equilibrium is basic in Mr. Keynes's work ... If in the real world short-period equilibrium is approximately reached, then analysis like that of Mr. Keynes [Model i] can usefully be applied and there is no need for the complexities of non-equilibrium process analysis.

“If short-period equilibrium is not approximated”, Lerner continued, “economics will be fruitless until someone has invented a way of using non-equilibrium *ex ante* process analysis. Meanwhile, premature attempts to apply the scheme before the invention has come, are tempting but dangerous.”

A lot has happened since 1940. The invention Lerner alluded to *has* now come. With the advent of numerical analysis and computer simulation using spreadsheet software with graph-plotting capabilities, modern-day economists have been granted the power to perform dynamic process analysis *via* experimentation on models that are highly nonlinear and recursive.

3.6 Analysis of Observed Traverses

From the discussion thus far, it will be apparent that the research methodology employed in connection with these corn-credit models may be termed "process dynamics" or "shifting equilibrium analysis". It is ideal for analysing Kalecki-Robinson “observed traverse” processes. Attempting to capture the essence of an economy’s development through simulated historical time is certainly difficult, but what this particular methodology allows the economist to do is separate out the various causes of dynamic disequilibrium traverse behaviour, one by one, so as to study their effects. In each traverse experiment, therefore, *ceteris paribus* is enforced for the entire economy, not merely assumed.

Each of the five corn-credit models is solved for its 100-year equilibrium stationary state by numerical simulation using a computer spreadsheet. Each model is systematically perturbed during year 30 by varying one or more parameters, which comprise its set of initial values and constants. In this way, the duration of the traverse may be timed and the model's dynamic behaviour closely observed, documented, investigated, and explained.

Various sensitivity matrices, tables, graphs, and diagrams are presented to show the economy-wide responses to these stimuli along the observed traverse to a new, fully-adjusted growth path (if one exists), consistent with the changed parameter-set. The typical result is a pattern in which endogenous forces interact to generate time paths having constant trend rates of growth (zero in the stationary state), around which most economic variables cycle in a series of peaks and troughs that may be convergent, regular or divergent. Analysis of reduced-form models shows that this depends mainly on the value of the "reaction coefficient" (ϕ), which operates on the profitability gap (a% pa) and on lagged investment (Qio sacks pa) to determine the current volume of seedcorn invested (Qi sacks pa). From the steady-state basecase, the dynamic path displays the familiar "saw-tooth" pattern that characterises real-world business cycles around an economic growth trend.

Historically, mainstream economics has privileged mathematically tractable equilibrium models, with their convenient assumptions, smoothly differentiable functions and precise theorems, corollaries, lemmas, and results. However, with a personal computer, the modern experimental economist can trace out and study the entire observed traverse process using numerical simulation and spreadsheet software.

David Colander (2000, pp 3-4) uses an economic argument to suggest that computational solutions eventually will prevail over analytical ones: "Computers do not provide analytic solutions to equations, but instead provide numerical solutions using brute force. [This] is not as elegant as deductively showing it from assumed first principles, and standard science prizes elegance. The bias against computer solutions runs deep, but as computational technology continues to advance, the relative cost of elegance inherent in deductive solutions will rise, swaying more and more scientists toward computational solutions."

Tõnu Puu (1997, p 4) points out that analytical solutions may not even exist: "In a science, such as economics, that, as far as formal models are concerned, for so long has resorted [to] general existence proofs, simulation may make the impression of an inconclusive, heuristic, and hence inferior scientific procedure. Such an attitude would, however, declare all experimental science as inferior ... we now know that dynamical systems that can be solved in closed form are, not only a sparse subset, but a nontypical subset. Accordingly, experiment, *ie* simulation ... is our only way of getting ahead with the systems."

The spreadsheet analysis chosen as part of this research methodology is an appropriate technique because (a) a built-in "hill-climbing" search algorithm can be utilised to solve for the opening stationary state; (b) every time-step along the observed traverse in each model run appears as a fresh column of data describing the state of the economy during that particular crop year; (c) it is easy to pose "what if?" questions concerning parameter changes; and (d) spreadsheets are quickly transferable to other researchers wishing to replicate the analyses and check the results obtained. In accordance with (d), an important component of this thesis is the CD-ROM of Appendix D, containing a spreadsheet data file for each model, which may be opened in any Web browser.

3.7 Features Common to All Models

All five recursive dynamic systems are "cycles, distribution and growth" (CDG) models. The classical corn model tradition of having only circulating capital is upheld in these corn-credit models: Qio sacks is the *total* stock of seedcorn to be sown as each year opens and Qso/κ sacks is the *average* stock of foodcorn farmers hold during that year as their sales inventory. Although this inventory is run down weekly, rather than continuously, the capital turnover ratio ($\kappa \approx 2$) indicates that, on average, about half the initial stock of foodcorn is on hand throughout the year. This inventory is the "consumption fund" that sustains the population until the new crop is harvested and, in Models A through D, it resembles the Classical "wage fund". In all five model economies, it is assumed that *per capita* foodcorn consumption exceeds the subsistence mininimum.

Wherever possible, simple *linear* functional forms have been used to specify the technology and behaviour underlying Models A through E, although not all functions are "linear in the variables". Also, apart from the given initial value(s), all variables have their base-period or year-zero values *computed*, using the equations and identities of the model, rather than being *specified*. (This is in accordance with an important principle, viz. the best economic models always "explain much by little".) Even the most complex system, Model E, needs only *three* initial values to start it computing a simulated historical time path of indefinite duration.

Lagged variables are not "distributed", i.e. they never extend beyond a single time-lag of one year. This simplifies notation, as no time subscripts are required in the algebra describing the models. Almost all variables are unsubscripted, i.e. they carry an implicit *current-year* time subscript. There are only two exceptions.

First, if a variable's name is followed by a lower-case "o", this denotes its "one-year lagged" value, i.e. its value in the immediately preceding crop year. Thus, for instance, Qi sacks pa is the quantity of seedcorn stored *this* year, while Qio sacks pa is *last* year's investment in seedcorn – which, of course, determines this year's total production of corn, Q sacks pa. (From this, the Qi sacks are withheld for planting next year and Qs sacks are stored, ready for release onto next year's weekly foodcorn markets.) Secondly, for each initial value parameter, its year-zero value is denoted by a lower-case "z". For instance, Qiz = 40,000 sacks is the base-year volume of seedcorn, which is an initial value common to all models. In any *particular* year, however, seedcorn invested is Qi sacks pa and its lagged value is Qio. (These variables only keep repeating the Qiz = 40,000 sacks initial value in the stationary-state or equilibrium solution.) The initial values inserted into each model sum up the entire history of that corn-credit economy prior to year zero.

Some variables represent *physical* quantities, e.g. sacks of corn and numbers of workers. Others, such as consumption and investment expenditure at constant prices, represent *real* magnitudes, i.e. current dollar money values deflated by the price level. Yet others are expressed in terms of *ratios*, such as shares and growth rates. Most variables, however, represent *nominal* magnitudes (i.e. current dollar money values), such as the aggregate of investment expenditure or the value of the economy's capital stock. A complete list of parameters and variables used in this thesis is provided in Appendix C. Most variables are classified as "aggregates", i.e. pure accounting identities that have no part to play in determining the solution time paths.

In the literature, most corn models are meant to represent barter production economies. In such economies, workers are paid in sacks of foodcorn from the wage fund and consumers' real saving behaviour (weighing the utility of future, as against present, consumption of foodcorn) determines how much the farmers are left with to invest as a stock of circulating capital. These five corn-credit models are quite different, however. The dollar is not the medium of exchange, simply the unit of account and standard of value. All business is transacted *via* book entries. Workers are credited their wages in dollars and the aggregate of farmers' decisions to invest in seedcorn (weighing expected profitability against the opportunity cost of capital) is what determines the *real* saving of all consumers in terms of foodcorn consumption foregone. Consumers can save as much *money* as they choose, yet not one extra sack of *corn* will be invested by farmers to swell the opening stock of seedcorn and make the next harvest bigger than the last.

Once the crop is harvested, every sack of grain that farmers earmark as seedcorn ensures that one less sack will be available for consumption as foodcorn. The number of dollars annually outlayed as consumption expenditure – for purchasing Qso sacks of foodcorn at a series of weekly markets – merely determines the dollar price of the average sack of grain in Models B, C and D. Less money outlay due to extra saving by *some* simply lowers the money price of foodcorn that *all* must pay. Savers are radiating benefits to non-savers now, in the form of price reductions. Whether they actually manage to raise their consumption of foodcorn in the future will depend on the price of corn at the time today's money savings are spent.

Neoclassicals would counter this by arguing that extra money saving should depress the interest rate (i% pa, the price of money loans), hence also the normal profit rate sought by farmers (n = [i + φ]% pa), where φ% pa is the risk premium. Because they assume the realised profit rate (r% pa) – which is supposed to be "the" marginal productivity of "capital" – remains unchanged, a lower interest rate ought to *widen* the profitability gap and increase real investment in seedcorn (Qi sacks pa). However, this assumption is untenable; it is an illegitimate resident of Marshall's "*ceteris paribus* pound".

In fact, the profitability gap will *narrow* due to an even bigger fall in realised than in normal profitability. Recall that farmers base their profit expectations on their most recent profit realisations, as summed up in the static expectation function: re = r% pa. If consumers increase their money saving flow by spending less from their money incomes, this will lower the corn price (P dollars/sack) and reduce the flow of profit (R dollars pa) to farmers, who consequently experience a fall in r% pa. So, in accordance with the seedcorn invested equation Qi = (1 + ϕ a) Qio, the profitability gap (a% pa) would be *lower* and the number of sacks of grain held back by farmers to form their new opening stock of circulating capital would *diminish*, rather than increase.

In these corn-credit models it is true that more consumer saving cannot result in one extra sack of *corn* being invested by farmers to swell the opening stock of seedcorn and make the next harvest bigger than the last. In fact, it has been shown above that the very opposite will occur. This "barter illusion" makes consumers *think* they exert power over the quantum of future *versus* present consumption of foodcorn. Barter illusion carries over into Model E, despite that economy's consumers having a demand function that permits continuous choice over allocating any extra money income between raising their bank deposits and purchasing more foodcorn at the weekly markets.

In all five CDG models, the production period is one year, the technical coefficients are fixed and there are constant returns to scale. The level of corn production is determined by last year's stock of seedcorn and its yield: Q = θ Qio sacks pa. Employment is given by this year's output and productivity: L = Q / λ workers. There is no government (anarchic) and these are all closed economies (autarkic).

Farmers own unlimited land of uniform fertility (hence rent is zero) while workers own labour of uniform productivity (hence the money wage rate is uniform). Even if they have accumulated savings, workers cannot acquire land and set up on their own account as farmers. Farmers also *compete* to sell foodcorn from their granaries, hence both the corn price and their realised profit rate are uniform. As each year opens, the money wage is set and labour is hired on 12-month contracts, then the seedcorn so recently stored by farmers – immediately after last year's harvest – is sown. During each year, the crops are tended by these hired workers, money wages accumulate fortnightly in their ledger accounts and last year's flow (i.e. this year's stock) of foodcorn is purchased and consumed weekly.

As each year closes, the new crop (Q sacks pa) is harvested and farmers move fresh seedcorn (Qi sacks pa) into their barns, in the volume decreed by their profitability gap investment equation. In Models A through D they also retain some foodcorn (Qf sacks pa) for their own household consumption during the coming year, the balance (Qs = Q – Qi – Qr sacks pa) being stored in their granaries as sales inventory for the coming year's weekly markets. Only in the final Model E do workers *and farmers* both purchase all their consumption requirements on the open market, so that Qf vanishes and Qs = Q – Qi sacks pa are stored in the granaries.

Analytical clarity is the sole reason for this procedure. It demonstrates the theoretical importance of the difference between *contractual* incomes (money wages) and *residual* incomes (realised profits). Premature introduction of a conventional demand function in own-price, prices of related goods and all income (whether contractual or residual) would have rendered "invisible" the underlying reality of Kalecki's dictum that "workers spend what they earn, while capitalists earn what they spend", the implication to be drawn from Kalecki (1937b). Independently, the same principle was discovered by Keynes (1930a, p 125), who stated that "profits ... are a widow's cruse which remains undepleted however much of them may be devoted to riotous living."

As the annual harvest time approaches, after the final weekly market has been held, farmers record the average price (P dollars/sack) their opening stock of foodcorn (Qso) fetched. They use this price to assign values to their new crop (Q sacks pa), once it is harvested, and to the opening stock of seedcorn that was sown to produce that crop. This allows them to compute their gross surplus (Rg = P Q – w L dollars), current value of opening capital stock (K = Ka + Kb = P [Qio + Qso/κ] dollars), realised profit (R = Rg – Ka dollars), and realised profit rate (r = R / K % pa).

After each end-of-year harvest, farmers decide what portion of their crop to allocate as seedcorn in accordance with the profitability gap investment function. They know the ruling normal profit rate (n% pa), but have only this year's realised profit rate (r% pa) as a guide to what profitability they might expect to realise from selling this year's foodcorn on next year's weekly markets. Their key decision problem is how to split the newly-harvested crop between seedcorn for sowing and foodcorn for marketing in the most rational manner. On the basis of the quotations from Keynes (1937, p 114) discussed above, farmers' long-period (investment) expectations are likely to be formed on the basis of their most recently achieved profit rate, which they then use to define what profitability gap (a = [r – n] % pa) they expect to occur.

All farmers desire to earn *at least* the normal profit rate (n% pa) on that fraction of the economy's total capital stock (K dollars) owned by them. The starting point for their profitability expectations is this year's realised profit rate (r% pa), so first they consider the profitability gap (a% pa), as defined above. The aggregate of their individual decisions regarding investment in seedcorn (Qi sacks pa) will rise above last year's figure (Qio sacks pa) if a > 0, and fall below it if a < 0. The investment function which captures this behaviour is specified as Qi = (1 + ϕ a) Qio sacks pa at all five stages in the construction of Model E.

3.8 The Investment Function

The right-hand sides of the reduced-form model equations in Chapters 4 and 5 will show just how significant and pervasive is the influence on the corn-credit economy of the *lagged* volume of seedcorn invested (Qio sacks pa). Therefore, the independent equation determining the *current* year's value of that variable (Qi sacks pa) must be crucial. In every model's structural form, the second equation is always the following investment function

$$Qi = (1 + \phi\, a)\, Qio \qquad \text{sacks pa}$$

where a = [r – n] % pa is termed the "profitability gap".

This equation for farmers' annual gross physical investment in stocks of seedcorn to be sown next year (i.e. that portion of their crop *not* destined for sale next year as foodcorn) has two determinants. Qi depends on its own lagged value *and* it flexes directly with the size of any gap between the realised profit rate (r% pa) and the normal profit rate (n% pa), a.k.a. the farmers' opportunity cost of capital or target/hurdle/required rate of return.[22] Should this profitability gap always remain at zero, then Qi = Qio sacks pa will persist and a classic stationary state will be maintained indefinitely.

Now Qio sacks, the economy's opening stock of circulating capital in the form of seedcorn, is *completely consumed* in producing this year's crop of corn (Q sacks pa) – from which Qi sacks pa subsequently will be re-invested as seedcorn and the remainder (Qs sacks pa) consumed as foodcorn. Thus the accumulation of seedcorn represents a *flow* of gross investment (Qi sacks pa) in any one year, but also a *stock* of circulating capital (Qio sacks) as the next year opens. With *circulating* capital consumption being total, Qio may be likened to a stock of *fixed* capital that undergoes a 100% pa rate of depreciation during the year it enters into production.

In Models A through E, the realised profit rate (r% pa) is the principal determinant of investment behaviour. The reason is that this endogenous variable contains within itself *every* particle of economic information that may influence the investment decisions of farmers. It has realised net surplus or "profit" as the numerator of r = R / K % pa. Significantly, this monetary value (R dollars pa) is a *residual* – not a contractual – income stream, which flows to the capitalist-farmers. Any and every change in the corn economy must have *some* effect on this information-rich residual, and be reflected in the dollar value it finally takes on.

This year, farmers considering what portion of the coming harvest they should hold back as seedcorn have only to consult their latest-available realised profit rate – which happens to be r% pa. Recall that they are assumed to hold static long-period expectations of what the market is likely to deliver (re = r% pa). Naturally, farmers would dearly love to *know* what next year's realised profit rate will turn out to be, but that figure is not (and can never be) available to them until they have finalised their accounts at the end of next year. They live in the same nonergodic world as we all do.

In a nonergodic world, there are gaps in the knowledge of decision-makers, gaps in profitability and gaps in time between expectations and realisations. Even net investment itself is defined to be a gap variable – in both types of worlds, nonergodic and ergodic.

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[22] Recall that this rate is determined by n = [i + φ] % pa, where φ% pa is the risk premium farmers apply to the resources they allocate to agriculture, rather than liquidating them and lending out the proceeds at i% pa in the market for money loans.

3.8.1 Net Investment as a Gap Concept

In any economy, each year's opening capital *stock* is either completely or partially absorbed in the current year's production process. This loss of capital value is the *flow* of depreciation. Thus, a stock of *circulating* capital is equivalent to a *fixed* capital stock that suffers a δ = 100% pa depreciation rate.

In the absence of price inflation, an opening stock of fixed capital worth Ko dollars will grow indefinitely over historical time (Δt = 1 year) so long as *net* investment

$$I = K - Ko = \Delta K = \Delta K/\Delta t \qquad \text{dollars pa}$$

is positive, i.e. if *gross* investment each year (Ig dollars pa) exceeds the loss of opening capital value (δ Ko dollars pa) due to depreciation

$$I = Ig - \delta\, Ko > 0 \qquad \text{dollars pa.}$$

In a cyclical trough, net investment also may be negative (Ig < δ Ko) and, in the long-period equilibrium of a classic stationary state, it would be zero (Ig = δ Ko).

Now in circulating capital models having δ = 100% pa, economic growth occurs when net investment is positive, just as in fixed capital models. The only difference is that no visible evidence of the previous year's opening capital stock remains; Ko has been entirely absorbed in (i.e. depreciated by) the process of production during the current year. An example of *positive net* investment in circulating capital is a stock of seedcorn used to produce a flow of corn output, from which the opening stock is *more than* replaced one year later, after the harvest. (If the opening stock is *less than* replaced one year later, there has been *negative* net investment.)

Already it can be seen that "gaps" are important in capital and investment theory, e.g. the positive, zero or negative "differences" between K and Ko are identical with those between Ig and δ Ko. These gaps or differences may also be expressed as "ratios", i.e. (K / Ko) = 1 is mathematically equivalent to (K – Ko) = 0, a situation of zero net investment. Thus "gaps" may be present in gap theories, difference theories and/or ratio theories.

If net investment (I dollars pa) is itself a gap, it may come as no surprise that economists of all schools have theorised that it is *determined by* some kind of gap, difference or ratio. In Appendix B, it is shown that many different species of investment functions are driven by entrepreneurial reaction to gaps of one kind or another. Furthermore, it is demonstrated that all such gaps *ultimately* reduce to a *single* difference, viz. the profitability gap: a = [r – n] % pa. That is why this particular gap has been likened to a "genome" and to a "Rosetta Stone" key.

In this thesis, each farmer-investor is assumed to *react* to any positive (negative) gap in expected profitability by increasing (decreasing) this year's retention of seedcorn relative to the volume held back from last year's harvest. The corn economy's overall reaction-coefficient ($\phi > 0$) is the mean of all farmers' *subjective* responses. It cannot be construed as a degree of "gap closure" or as a "speed-of-adjustment" towards some objectively-determined optimal capital stock. If the profitability gap is zero, the positive reaction-coefficient can have *no* effect, farmers remaining content to keep aggregate physical investment at last year's level (Qi = Qio sacks pa).

3.8.2 Gap Theories of Investment

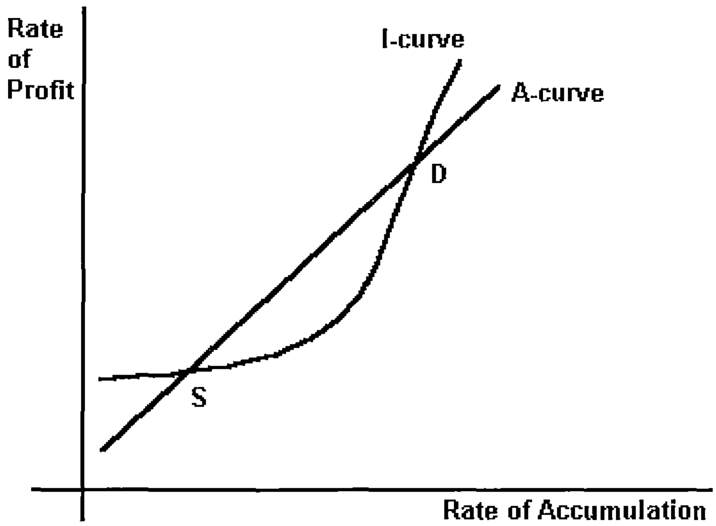
In the Harrod/Samuelson/Hicks multiplier-accelerator theories, the relevant gaps lie between the current and lagged values of output or consumption. The Jorgenson user-cost theories feature differences between the actual and optimal capital stocks or production capacities. The Tobin "q" theories are based on a ratio between the Marshallian demand and supply prices of capital equipment.

Only in the Smith/Ricardo/Marx uniform-profitability and the Keynes/Kalecki marginal-efficiency theories is the profitability gap mechanism present in almost pristine form. For instance, Keynes (1936, pp 315-7) insists that gaps between the subjective marginal efficiency of investment (MEI)[23] and the objective long-term rate of interest are responsible for fluctuations in the investment aggregate. These, he claims, get amplified (by the multiplier) into instability, the trade cycle and the infrequent crises that afflict capitalist economies.

Robinson (1962, pp 47-48) includes the mutual positive feedbacks of (a) expected profitability onto investment, (b) investment onto realised profitability and (c) realised profitability back onto expected profitability. She calls this "... the double-sided relationship between the rate of profit and the rate of accumulation" which "... involves the whole question of the mechanism of fluctuations in a private-enterprise economy."

[23] Keynes (1936, pp 135-46) called it the marginal efficiency of capital (MEC), but the context makes clear that he really meant the marginal efficiency of investment (MEI). This was pointed out by Lerner (1944, 1953) and reinforced by Pasinetti (1974, pp 60-4).

Figure 3.1 – The Robinsonian Investment Function



Source: Robinson (1962, p 48)

Robinson states that "The A curve represents the expected rate of profit on investment as a function of the rate of accumulation that generates it. The I curve represents the rate of accumulation as a function of the rate of profit that induces it." Desired rates of capital accumulation are shown at points S (unstable) and D (stable). Stability is assured at point D because the I-curve cuts the A-curve from below. To the right (left) of D, investment is higher (lower) than that which would be justified by the rate of profit that it generates, hence investment will be reduced (increased) once the lower (higher) rate of profit is realised in the accounts.

Significantly, Robinson refers to "desired" rather than "equilibrium" rates of capital accumulation. This is because the very fact that gross investment in excess of depreciation is going on indicates that the opening capital stock was *not* in equilibrium – given the *expected* future course of revenue, expense and associated profitability. And being out of equilibrium must mean that some kind of "gap" has developed, one which prevents capitalist investors from continuing to feel comfortable about the size and/or structure of their opening capital stock.

3.8.3 The Gap Zoo

Nuclear physicists were embarrassed by their "particle zoo" until, beginning in 1964, Murray Gell-Mann and others demonstrated that these 200-plus different sub-atomic particles detected in their accelerator rings were built out of only six quarks, six leptons and three bosons. Economists today should feel similarly uncomfortable about the "gap zoo" inhabiting their own accelerator models. Among the resident "animals" is the profitability gap of this thesis, appearing in the Smith/Ricardo/Marx and Keynes/Kalecki families of investment functions. The profitability gap fascinates economists from other schools of thought as well.

With respect to capital accumulation and growth, Edmond Malinvaud (1986, p 382) agrees with economic historians "... that an essential element ... is the course of business profitability ... this latter is precisely a deviation from the flexprice equilibrium".[24] Malinvaud notes that the existence of non-zero pure profit rates is inconsistent with existing Neoclassical flexprice growth theories, basically because the Walrasian barter model of perfect competition and price flexibility cannot sustain any $r \neq n$% pa gaps in supply-demand equilibrium. He says this is not consistent with the empirical data of capitalist economies and concludes that "... the observed differences, with at some times and places negative, at others high pure profit rates ... truly reveal what is best interpreted as disequilibria of the price system."

With respect to the business cycle, Alan Freeman (1999, p 4) proposes a Neo-Marxian nonlinear, continuous-time, two-equation investment model which is driven by a gap between realised and normal profitability and proves that even such a simple abstract system will generate stable, persistent business cycles. He believes that neither Neoclassical nor Marxist thinkers have "... constructed formal models in which the rate of profit itself exercises the predominant influence on investment behaviour, notwithstanding ... [its theoretical importance and] ... the significant empirical evidence uncovered, by authors of both schools, of profit rate variations during the course of the cycle".

Model E of this thesis is a nonlinear, discrete-time, six-equation economic model which also is driven by a profitability gap and also can generate stable, persistent business cycles. In addition, this Post-Keynesian model can engender traverse adjustment paths of diverse shape and duration.

In Appendix B, it is argued that practically *all* the various flow and stock gaps, differences and ratios appearing in investment functions are merely imperfect proxies for something

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[24] Malinvaud's profitability "deviation" is the the profitability "gap" of this thesis: $a = [r - n]$% pa.

deeper and more fundamental: the profitability gap of this thesis. The difference between the profitability of capital (r% pa) and the opportunity cost of capital (n% pa) is the "genome" that is shared by all animals in the economists' Gap Zoo.

3.9 Features Specific to Each Model

In addition to the general features discussed in section 3.7 above, each of the five systems has its own set of *specific* assumptions. All assumptions of earlier models continue to apply unless and until they are altered in a later model. In all models, an extra equation is needed before the stationary-state solution can be obtained. This additional equation is the equilibrium condition that r = n% pa, or its equivalent a = [r – n] = 0% pa. It specifies that zero pure profits are earned by farmers only in the tranquil long-period equilibrium cocoon of a fully-adjusted stationary state.

3.9.1 Model A

This model has a fixed corn price (P dollars/sack), money wage (w dollars/worker pa) and interest rate (i% pa). The workforce is unlimited, so there is no constraint on the level of employment (L workers). Excess demand for labour is assumed to be satisfied by the immigration of *gastarbeiters* from some neighbouring less developed economy. These "guest workers" are returned to their native land when their services are no longer required.

The model economy's wage bill (W = w L dollars pa) is a form of contractual income, but profit is a pure residual (R = P Q – W – Ka dollars pa), where Ka is the value of the opening stock of seedcorn (Ka = P Qio dollars). The price level is defined as p = P / Pz, so p = 1.000 implies no change. The inflation rate is defined as gp = (p / po) – 1, therefore gp = 0% pa implies no inflation. There are 3 equations, 1 equilibrium condition, 10 identities, 4 lagged variables, 9 constants, and 1 initial value.

3.9.2 Model B

This is Model A with a flexible corn price. Following each harvest, a certain volume of foodcorn retained (Qf sacks pa) is held back for consumption by farmers' households. Foodcorn supplied is defined as Qs = Q – Qi – Qf sacks pa. Under the Classical saving assumption, workers spend all their money income (i.e. the economy's wage bill) on foodcorn. All of last year's volume of foodcorn supplied is purchased by the workers only. So, P = W / Qso dollars/sack necessarily determines this year's dollar price of foodcorn.

There are 4 equations, 1 equilibrium condition, 10 identities, 4 lagged variables, 8 constants, and 1 initial value.

3.9.3 Model C

This is Model B with a flexible money wage. Migration of "guest workers" is no longer permitted. The workforce is constant at η = 16,000 workers, so that L < η indicates unemployment, while L > η implies overtime working. The economy's employment rate is defined as e = L / η, so that e = 1 implies full employment of the workforce and e < 1 shows that unemployment exists. A situation of "overfull employment" is indicated by e > 1, so that the wage bill includes overtime payments equal to w (L – η) dollars pa, the overtime hours being spread across the workforce in some equitable fashion. In wage bargaining, the money wage flexes directly with e and with gpo, the lagged rate of price inflation. There are 5 equations, 1 equilibrium condition, 11 identities, 6 lagged variables, 10 constants, and 2 initial values.

3.9.4 Model D

This is Model C with a flexible interest rate. The minority of richest farmers are trusted to be bankers, as they own the largest stocks of seedcorn and foodcorn available for liquidation, should their banks fail. Over a year, all farmers carry average debt of D = W / μ dollars to meet their *fortnightly* payrolls, implying a wage bill turnover ratio of μ = 52 = 2 x 26 fortnights. The farmers' debt:assets ratio is d = D / Ko and its growth rate is gd = (d / do) – 1. Ko dollars is the lagged value of the total capital stock (K = Ka + Kb dollars), where Kb = P Qso / κ dollars, implying a capital turnover ratio of κ = 2 = 2 x 1 year. The figure 2 appears in these turnover ratios because the average holding of an opening stock run down to zero over any period necessarily is *half* that opening stock.

The interest rate flexes directly with gd, which is a proxy for lender's risk. The economy's interest bill (J = i D dollars pa) is a form of contractual income. Income of J dollars pa is shared between the bankers and their depositors. All foodcorn supplied is bought by all consumers, using their contractual income streams only. So now, P = (W + J) / Qso dollars/sack necessarily determines this year's dollar price of foodcorn. Profit remains a pure residual, but now is reduced by the interest bill (R = P Q – W – J – Ka dollars pa). There are 6 equations, 1 equilibrium condition, 15 identities, 9 lagged variables, 11 constants, and 3 initial values.

3.9.5 Model E

This is Model D with a flexible demand function for foodcorn. Farmers no longer retain Qf sacks pa of foodcorn – they purchase their household supplies on the open market instead. So, foodcorn supplied is now defined as Qs = Q – Qi sacks pa and its lagged value (Qso sacks pa) is the volume offered for sale in any given year. Household income includes this year's (contractual) wage and interest bills, as well as last year's (residual) profit earnings by farmers. So, household income is defined as Yh = W + J + Ro dollars pa. Any money income not spent on consumption is saved, so foodcorn purchases compete with bank deposits. The price of bank deposits (1 / i) is the reciprocal of the return on bank deposits, i.e. the interest rate.

Foodcorn demand depends on own-price, price of bank deposits and household income, with the (log-linear) demand function for foodcorn being lnQd = f (lnP, ln[1 / i], lnYh). However, Qd = Qso is the condition for clearing the weekly foodcorn markets over an entire year, therefore P = exp[g (lnYh, ln[1 / i], lnQso)] now determines this year's dollar price of foodcorn. There are 6 equations, 1 equilibrium condition, 16 identities, 10 lagged variables, 14 constants, and 3 initial values.

3.9.6 Complexity of Models

Table 3.2 below shows how the five corn-credit models increase in complexity as they are sequentially nested by replacing Roman-letter price constants with Greek-letter constants from the new pricing equations. An indicative "index of complexity" is created by summing across all rows (excluding equilibrium conditions and Roman-letter constants), then relating all later models back to Model A, whose index is normalised to unity. These row entries record the numbers of structural-form relationships, parameters and lagged variables appearing in each recursive dynamic model. An alternative index of complexity, one based on the increasing length of the reduced-form corn price equation, appears in brackets in the final column of the table below.

Table 3.2 – The Corn-Credit Models at a Glance

Stage[1]	Structural Form Relationships			Parameters			Lagged	Indices of
	Independent Equations	Equilibrium Condition	Identities	Constants (Greek)	Constants (Roman)	Initial Values	Variables	Complexity[2]
A	3	1	10	5	4	1	4	1.00 (1)
B	4	1	10	5	3	1	4	1.04 (5)
C	5	1	11	8	2	2	6	1.39 (12)
D	6	1	15	10	1	3	9	1.87 (46)
E	6	1	16	14	0	3	10	2.13 (na)

Note 1 – Model E* is Model E with a government sector, including revenues, expenditures, government debt, and policy instruments, hence Model E* is not tabulated.
Note 2 – The first index is the row-sum of each model, relative to the row-sum of Model A (excluding equilibrium condition and Roman-letter constants). The bracketed index is based on the reduced-form corn price equation for each model. It is a count of the number of symbols appearing on the right-hand side, with some constants and variables included more than once, because of squared terms, etc. Model A has one constant in its corn price equation, whereas Model E is so complex that no reduced form can be obtained.

The Wolfram "Mathematica" computer program is used to derive the reduced form of each model. Reduced-form equations show how all unknown quantities are determined by other quantities (i.e. parameters and variables) whose values are known. This is in contrast with structural-form equations, which do have unknown quantities on their right-hand sides. In brief, structural forms define the behaviour of economic agents and the technologies they utilise, while reduced forms display the economic mechanisms which the structure implies. Table 3.2 shows how increasing structural-form complexity is amplified into an extremely rapid rise in reduced-form complexity, even to the point that no reduced form can be obtained for Model E.

3.10 Solving for the Stationary-State Basecase

At each of the five model construction stages, there are between $n = 4$ and $n = 7$ unknown variables to be determined. However, in every case there are only $n - 1$ independent equations available to do this. The nth relation is *not* independent. It is a mere identity, the definition of the realised profit rate: $r = [R / K]\%$ pa. Therefore, all five models are under-determined, each having one too many unknowns than independent equations to determine them.

This situation of under-determination is quite common in theoretical economics. For instance, the typical Neoclassical general equilibrium model specification *also* has one too many unknowns for its simultaneous system of excess demand equations to determine. Walras' Law states that if n – 1 commodity and factor markets exhibit zero excess demands, then the nth market *also* must be in a state of zero excess demand. General equilibrium theorists achieve a just-determined model by reducing the number of unknowns. Typically, they set the price of one commodity (say, peanuts) or one factor of production (say, labour) – it doesn't matter which – to unity. The price of this *numéraire* commodity or factor is set to Pn = 1 and the solution vector of relative (*not* absolute) prices implies that a "peanut theory of value" has precisely the same status as a "labour theory of value". (Some variant of the Quantity Theory of Money – "money" being a medium of exchange rather than a unit of account – then must be enlisted if absolute or "dollar" prices are required.)

In like fashion, the Post-Keynesian models of this thesis also are made just-determined by reducing the number of unknowns. In this case, however, the procedure is complicated by the equation systems being dynamic and recursive (historical time), rather than merely simultaneous (logical time). In principle, one simply sets the profitability gap identity to zero (a = [r – n] = 0% pa), then solves the model. But to enforce this stationary-state condition as simulated historical time passes, one needs to follow a certain procedure commonly used for numerical analysis. One must apply a powerful "hill-climbing" search algorithm ("Solver"), which keeps on making small alterations to one or more model parameters until the realised profit rate (r% pa) comes into strict equality with the normal profit rate (n% pa) during every year of the simulated century of historical time.

This Post-Keynesian solution procedure guarantees a stationary state that must persist, because each and every annual profitability gap (a = [ro – n]% pa) has been reduced to zero by the Solver algorithm. In other words, the farmers' profitability expectations for next year (re = r% pa) are always being realised (r = ro% pa) and, in addition, their opportunity cost of capital is always being covered (r = n% pa). Farmers, therefore, experience no surprises or dissatisfaction and so are content to keep on investing Qi = 40,000 sacks of seedcorn after each crop has been harvested.

The unreal, abstract nature of the stationary state time path is actually its greatest strength, for theoretical purposes. By changing a single variable or parameter, one is able to perfectly isolate, for close investigation, the particular *complete observed traverse* that is ignited by that change. One such change in year 30 will inject an initial dose of short-period disequilibrium, whose dynamic knock-on adjustment effects will propagate through simulated historical time and alter dozens of time series for up to the next 70 years. Their new positions and shapes then can be plotted, compared, contrasted, and explained. In Model E, for instance, there are 17 parameters (3 initial values plus 14 constants) available to be perturbed during year 30. As for the "dozens of time series" to be plotted and analysed, this model has 55 endogenous variables available, including 32 "aggregates" having no effect on the solution. These mainly comprise conventional national accounting identities.

3.11 Generating the Steady-State Basecase

Rather than solving for the steady-state basecase, one *generates* it. This can be done by continuous (rather than once-off) perturbations to a model constant, such as by making the workforce (η workers) grow at some constant exponential rate ($g\eta$ > 0% pa), starting in year zero rather than in year 30. Thus, a steady state can be established using the standard traverse-generation procedure described above. It is not an equilibrium time path, but a disequilibrium regime that is "fully-adjusted", in the sense that it will continue indefinitely if left alone. The same is true of the stationary state which, in addition, *is* an equilibrium dynamic path, as explained above. The only other fully-adjusted time path is the regularly-cycling economy, in which all endogenous variables fluctuate above and below their own stationary- or steady-state trendlines in an endless sequence of limit cycles. This sustainable regime – which is just as unlikely to occur in the real world as the stationary and steady states – can be termed "the cyclical state".

All other dynamic paths are observed traverse adjustment paths. The economy struggles to adjust to the impact of one or more parameter perturbations, hopefully "traversing towards" – and converging on – one of the only three sustainable paths, whether stationary, steady or cyclical. If this does not occur, the traverse path is divergent and the economy collapses after experiencing a first or a final cycle of immense amplitude. Such outcomes are due to unsustainable boom conditions. An extremely rapid increase in the realised rate of profit – in the presence of a far less violent interest rate rise – causes the profitability gap to widen and the associated volume of seedcorn investment to soar.

As before, after imposing a sudden exogenous change, the experimenter can perfectly isolate (then plot, compare, contrast, and explain) the particular *complete observed traverse* that is sparked off by that change. Superficially, the use of steady-state analysis resembles that of the Cambridge School economists, as discussed above. However, the differences are significant and they can be found in Robinson's distinction between historical-time "changes" and logical-time "comparisons".

The basic Cambridge School methodology is to make formal *comparisons* between smooth theoretical steady-state growth paths and derive their properties, then to informally discuss the real-world implications of *changes* (using the shifting equilibrium model). By contrast, this thesis formally adopts Keynes' model of shifting equilibrium from the outset, tracks only *changes* through historical time and draws out nothing but theoretical implications. Even the "policy recommendations" made in Chapter 8 apply only to such abstract worlds, in which farmers and workers exchange labour and foodcorn at prices expressed in a money of account, with current production determined by lagged investment in seedcorn and financial intermediation performed by farmer-bankers whose credit-worthiness is beyond question.

3.12 Conclusion

Dynamic process analysis is central to the overall research methodology of this thesis, as discussed in section 3.5 above. The availability of computer hardware (together with spreadsheet, hill-climbing, graph-plotting, and mathematical software – see section 3.6 above) allows modern Post-Keynesian economists to construct and test their model economies, then generate numerous observed traverses over long spans of simulated historical time. This opens up possibilities for going far beyond the theoretical insights and policy conclusions that the discipline already has achieved, using the conventional tools of comparative statics and comparative dynamic analysis of business cycles and steady-state growth paths.

It is significant that no reduced form could be derived from the apparently simple structural form of Model E, with only six independent equations and a single equilibrium condition. The even simpler Model D was found to have 46 instances of known quantities determining its unknown flexible corn price. (Chapter 5 shows that the right-hand side of this equation includes four squared parameters and one raised to the fourth power.) The methodological future of theoretical economic research, it seems, will be intertwined with the broad Complexity Economics approach, which is discussed in section 1.8 above.

In the next chapter, the fixprice Model A is constructed. It is the first of five stages leading up to the ultimate flexprice Model E and its Model E* instrumental traverse variant.

CHAPTER FOUR

A FIXPRICE CORN-CREDIT ECONOMY

4.1 Introduction

This chapter introduces Model A, the first of five construction stages in modelling a theoretical Post-Keynesian monetary production (or corn-credit) economy. It is a pure fixprice model, in that the corn price, money wage and interest rate are all given as constants. The model's structural form is specified first, followed by a discussion of its equations, identities and parameters. Then the corn-credit economy is described, using a flowchart to display the dynamic inter-relationships of all variables. Some 30 "aggregates" (i.e. variables having no dynamic feedback effects) are defined as accounting identities, built up from the endogenous variables of the structural form. The model's reduced form is obtained, in order to identify the ultimate "drivers" of its dynamic behaviour. Then the model is realised as a computer spreadsheet, in readiness for simulating the abstract economy under investigation.

Model A is first solved for a zero-growth stationary state lasting 100 "years". Then this particular solution is used as the starting point for generating a steady state of constant positive exponential economic growth. Finally, two specimen traverses are computed, one from each growth path, i.e. the stationary state and the steady state. Both traverses are initiated by perturbing the same variable, viz. the volume of seedcorn invested.

This alters the model's computed allocation of one particular end-of-year harvest (between seedcorn for start-of-year sowing and foodcorn for year-round selling), thus sparking off a dynamic disequilibrium traverse process. The stationary and steady states serve as experimental benchmarks. They are basecases or *reference* time paths, smooth trajectories to which the model's *actual* traverse paths are compared. This "Misallocation Scenario" is the standard perturbation applied at all stages, although in Chapter 6 the ultimate pure flexprice Model E also provides a testbed for numerous additional simulation experiments.

4.2 Structural Form

Model A is a simultaneous system of three equations, one equilibrium condition and ten identities. Its structural form is classified as "dynamic" and "recursive" because the model contains first-order difference equations having one-year time-lagged explanators. This can be seen in Table 4.1 below, i.e. three lagged endogenous variables (Qio, Qso and po) appear on the right-hand sides of five of the model's 14 structural-form relations, its equations and identities.

The identities simply make the behavioural and technical equations easier to understand. Equation (D), for instance, defines the realised profit rate simply as r = R / K. However, by using the identities for R, Ka, Kb, and K this definition expands into

$$r = (P\,Q - W - Ka) / P (Qio + Qso / \kappa)$$

making it far less readable with eight symbols cluttering up the right-hand side.

The numerical values chosen for Model A's ten parameters (viz. nine constants and one initial value) are shown in bold type. Note that there are conventional Greek-letter symbols for the first five constants, but the final four carry Roman-letter symbols. This is to foreshadow that P, w, i, and Qf lose their status as constants in Models B, C, D, and E, respectively. (All but Qf become endogenous variables determined by extra equations added to the structural forms of these nested extensions to Model A.) By the time the fifth and final construction stage (Model E) is introduced in Chapter 6, there are 14 constants in its structural form and *all* are represented by conventional Greek-letter symbols. (Model E also has three initial values, making a total of 17 parameters.)

After assigning near-arbitrary values to nine of the ten parameters, the unique long-period equilibrium stationary-state solution of Model A is found by using the "hill-climbing" search algorithm Solver to determine the one remaining parameter as P ≈ $27.80 per sack. Given the other nine parameters, this is the *only* corn price for which the economy's profitability gap remains on a = 0% pa over 100 years of simulated historical time. This equilibrium condition implies that farmers keep on experiencing equality between their expected, realised and normal profit rates (re = r = n = ne = ro = no = 5% pa) along the entire time path.

Table 4.1 – Structural Form of Model A

Equations			
Corn Produced	$Q = \theta\ Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = (1 + \phi\ a)\ Qio$	sacks pa	(B)
Employment	$L = Q / \lambda$	workers	(C)
Profit Rate	$r = R / K$	percent pa	(D)
Identities			
Wage Bill	$W = w\ L$	dollars pa	(1)
Seedcorn Capital	$Ka = P\ Qio$	dollars	(2)
Foodcorn Capital	$Kb = P\ Qso / \kappa$	dollars	(3)
Capital Stock	$K = Ka + Kb$	dollars	(4)
Profit	$R = P\ Q - W - Ka$	dollars pa	(5)
Normal Profit Rate	$n = i + \varphi$	percent pa	(6)
Profitability Gap	$a = r - n$	percent pa	(7)
Foodcorn Supplied	$Qs = Q - Qi - Qf$	sacks pa	(8)
Price Level	$p = P / Pz$	ratio	(9)
Inflation Rate	$gp = (p / po) - 1$	percent pa	(10)
Constants			
Reaction Coefficient	ϕ = **0.4388**	ratio	(a)
Seedcorn Yield	θ = **4**	sacks/sack pa	(b)
Labour Productivity	λ = **10**	sacks/worker pa	(c)
Risk Premium	φ = **1.0**	percent pa	(d)
Capital Turnover	κ = **2.0**	ratio	(e)
Corn Price	P = **27.80**	\$/sack	(w)
Money Wage	w = **200.00**	\$/worker pa	(x)
Interest Rate	i = **4.0**	percent pa	(y)
Foodcorn Retained	Qf = **4878**	sacks pa	(z)
Initial Value			
Seedcorn Invested	Qiz = **40,000**	sacks pa	(I)

Note – Corn flows are measured in sacks pa, value flows in dollars pa, labour stocks in workers, corn stocks in sacks, value stocks in dollars, and rates of return in percent pa.

Model A is path-dependent, in that each overlapping *pair* of years is self-contained, while being linked together by three endogenous variables carrying one-year time-lags, viz. Qio, Qso and po. As simulated historical time passes, the current year keeps on getting "frozen into history", by becoming the previous year of the *new* current year. Base-year variables are identified by "z" (indicating year **z**ero) and previous-year variables by "o" (indicating a **o**ne year time lag), all others being current-year variables, including Qi, Qs and p. While unusual, this form of notation dispenses with the forest of time-subscripts that otherwise would disfigure the algebra of a model in which, apart from a few year-zero initial values, the only years that matter are t (not identified) and t – 1 (identified by "o").

Equations (A), (B) and (C) are the only independent equations of Model A. But *three* technical/behavioural equations are not sufficient to determine *four* unknowns, viz. Q, Qi, L, and r. Equation (D) – which defines the realised profit rate – is <u>not</u> independent, and this means the system is under-determined. So, to select one from among the resulting infinite number of possible solutions, the model is closed by searching for that parameter-set which will ensure the profitability gap remains on a = 0% pa over 100 years of simulated historical time. After near-arbitrarily fixing nine of the ten parameters (*vide* Table 4.1 for their values), it was found that P ≈ \$27.80 per sack is the *only* corn price consistent with maintaining a long-period equilibrium stationary-state time path.

This particular parameter-set guarantees that a stationary state will endure indefinitely. The economy's farmers continue having their profitability expectations exactly fulfilled, while simultaneously realising the *only* profit rate that meets their rate of return target (r = n% pa), i.e. it covers their opportunity cost of capital or required profitability. These long-period equilibrium conditions keep farmers content to go on accumulating Qi = 40,000 sacks of seedcorn after every harvest. This procedure of setting a = 0% pa to close an under-determined dynamic Post-Keynesian monetary production model is comparable with choosing the nth commodity as *numéraire* and setting its price to unity (Pn = 1) in order to close an under-determined static Neoclassical barter exchange model of general equilibrium.

4.3 Components of the Model

As historical time passes, the future is transformed into the present and then into the past, i.e. "next year" becomes "this year" and then becomes "last year". Using the realised profit rate variable for illustrative purposes, re% pa is *next* year's expected value, r% pa is *this* year's actual value and ro% pa is *last* year's actual value.

Twelve months later, the old r% pa has become the new ro% pa and farmers now know whether their old re% pa expectation was in fact fulfilled by the new r% pa. Their new re% pa expectation of profitability twelve months' into the future will be based on this new r% pa, the one realised during the present year. At all five construction stages, the simplest possible assumption is adopted, viz. that re = r% pa. As discussed in Chapter 3, farmers' expectations of future profitability are thus assumed to be naïve, myopic or static.

4.3.1 Equations

Equation (A) is the corn production function. It shows that, on 1st January *this* year, farmers already possess a certain opening stock of seedcorn (Qio sacks), which they "yesterday" (viz. 31st December *last* year) decided to hold back from being supplied to any of *this* year's weekly foodcorn markets. Given that the average yield of seedcorn (θ sacks/sack pa) is known, Qio determines what volume of corn (Q sacks pa) will be produced during the coming twelve-month growing season and harvested at the end of this year – assuming sufficient labour is applied.

Equation (B) is the seedcorn investment function, which specifies the average behaviour of farmers on 31st December *this* year, with respect to how they choose to store the newly-harvested crops in their barns (as seedcorn), granaries (as foodcorn) and farmhouses (for household consumption). The average profit rate realised at *this* year's weekly foodcorn markets is expected to be realised at *next* year's markets (re = r% pa). Static expectations also apply to *this* year's known normal profit rate (ne = n% pa), so farmers compute their profitability gap as a = [r – n]% pa on 31st December and make their investment decisions accordingly.

If the profitability gap is positive (negative), farmers will retain Qi > Qio (Qi < Qio) sacks pa out of the newly-harvested crop in their barns as seedcorn. However, should the gap be zero, then farmers will retain Qi = Qio sacks pa from this year's harvest. For as many years as this equality continues, the economy will remain in a classic stationary state of zero growth in seedcorn invested. The reaction coefficient (ϕ ratio), multiplied by the profitability gap (a% pa), fixes the percentage by which Qi differs from Qio. There will be no reaction, hence no difference between Qi and Qio, whenever a = 0% pa. Naturally, the balance of *this* year's crop (Q – Qi sacks pa) ends up in the granaries and farmhouses, earmarked for sale as foodcorn at *next* year's weekly markets and for consumption by the farming families.

Equation (C) is the labour demand function. The overall level of employment (L workers) offered by farmers depends on the size of this year's crop (Q sacks pa) and on the average productivity (λ sacks/worker pa) of the labour hired to raise it. There is no risk or uncertainty about any of these values.

Equation (D), however, is purely definitional. It defines this year's realised profit rate (r% pa) as the profit (R dollars pa) earned by farmers on the crops they raised this year, divided by the current replacement value of the opening stock of circulating capital, viz. K dollars worth of last year's corn. This rate of return has the same algebraic sign as their realised profit –

which may be zero, of course (or even negative if the farmers' accounts show a loss on this year's corn harvest).

4.3.2 Identities

Identity (1) defines this year's aggregate wage bill (W dollars pa) as the fixed average money wage (w dollars/worker pa) multiplied by the overall level of employment (L workers).

Identity (2) assigns a value to the *smaller* portion of the economy's stock of circulating capital, viz. seedcorn capital. It shows that farmers value their opening stock of seedcorn at Ka dollars on 31st December. By then, the weekly foodcorn markets have revealed what average price (hence also what profit and profitability) they realised from this year's trading operations. In preparing their accounts, farmers value the Qio sacks of seedcorn they planted last January at its opportunity cost, viz. its current replacement value (P Qio dollars). Its historic cost (Po Qio dollars) is irrelevant to economists, though not to accountants.

Identity (3) assigns a value to the *larger* portion of the economy's stock of circulating capital, viz. foodcorn capital. It shows that farmers value their opening stock of foodcorn at Kb dollars on 31st December. They know that Qso sacks of corn from last year's harvest were in storage twelve months ago and that their granaries are now empty, following the series of weekly foodcorn markets. On average, about one half the opening stock (Qso / κ sacks, where $\kappa = 2$) was on hand all year round, at an opportunity cost of P dollars/sack.

Identity (4) sums both types of circulating capital to define the current replacement value of the farmers' capital stock (K dollars), which comprises the denominator of their realised profit rate (r% pa).

Identity (5) defines the economy's flow of realised profit or net surplus (R dollars pa) as the surd or residual of gross product (P Q dollars pa) after meeting all expenses. Profit is whatever remains of gross product after subtracting both the wage bill (W dollars pa) and the current replacement cost of seedcorn capital – effectively a 100 percent depreciation charge of Ka dollars pa because this capital is not fixed but circulating. This flow of money profit may be positive, zero or even negative – depending on what average corn price was achieved at this year's series of weekly foodcorn markets.

Identity (6) sums two constants to define the farmers' normal profit rate (n% pa). The first is the ruling rate of interest (i% pa) on risk-free loans of money capital. This rate is fixed by tradition until Model D, when banks are introduced and the interest rate begins to flex because of lender's risk. The second constant is the risk premium (φ% pa) that compensates farmers for continuing to bear the risks of adverse demand shifts by holding their wealth as stocks of seedcorn and foodcorn. These risks are that they may be left with unsold stocks in Model A or experience corn price reductions from Model B onwards.

Identity (7) defines this year's profitability gap (a% pa) as the realised profit rate (r% pa) minus the required profit rate (n% pa). This profitability gap will be negative (positive) if the realised profit rate is lower (higher) than farmers consider normal. Should r = n% pa, the profitability gap will be a = 0% pa, which is consistent with the maintenance of a stationary state. This is a state of equilibrium, in that farmers experience no "surprises", i.e. abnormal events that could lead them, on 31^{st} December this year, to revise the seedcorn investment decision they took twelve months earlier.

Identity (8) shows that farmers, on 31^{st} December this year, allocate Qs sacks pa as foodcorn supplied. They decide the level of Qs by subtracting two quantities from their harvest of Q sacks pa. These are the volume of seedcorn invested – Qi, given by equation (B) – and of foodcorn retained – Qf, a constant which is discussed below.

Identity (9) defines the economy's price level (p) by relating this year's corn price (P) to the corn price (Pz) that historically was realised in year zero, the model's base year. Of course, p = po = pz = 1 will hold for the entire century because P remains constant in this fixprice model.

Identity (10) defines this year's inflation rate (gp% pa) as the growth of the price level, i.e. this year's price level (p) is divided by last year's (po), before subtracting 1 from the result. Of course, gp = gpo = 0% pa will hold and the entire century remains free of price inflation in this fixprice model.

4.3.3 Constants

Constant (a) is the reaction coefficient (ϕ = 0.4388). This parameter controls the degree of "pass-through" of any non-zero profitability gap (r $\neq$ n% pa) in its effect upon the volume of seedcorn invested this year (Qi sacks pa), relative to last year (Qio sacks pa). The reduced form of Model A (see Table 4.3 below) shows that the previous year's investment of seedcorn "drives" this abstract corn economy, in that it "explains" all four "unknowns" on the left-hand sides of the structural-form equations.

As simulated historical time passes, every current Qi soon becomes a lagged (Qio) variable, so the question "What drives the drivers?" becomes relevant. In fact, the profitability gap (a% pa) is the *ultimate* driver, whose effect is *muted* by the farmer-investors' reaction coefficient being set at $\phi < 1$. In Models A, B and C, a profitability gap of +10 per cent would cause Qi to exceed Qio by 4.388 per cent. Were the farmers' reaction coefficient to average $\phi = 1$, then *all* the "excess" or "pure" profitability would be "passed through" to affect seedcorn investment, thus raising Qi the full 10 per cent above Qio. (In Models D and E, the reaction coefficient is set slightly higher than $\phi = 0.4388$, as explained in Chapter 5.)

Constant (b) is the seedcorn yield ($\theta = 4$ sacks/sack pa). This parameter reflects the inherent plant biology of corn propagation, in the given state of technology. On average, every sack of corn, planted at the start of any given cropping year, fructifies into four new sacks of corn within 12 months – provided it is properly tended by the application of labour.

Constant (c) is the labour productivity coefficient ($\lambda = 10$ sacks/worker pa). This parameter sums up the average annual output per worker employed in tilling the soil and planting, tending, then harvesting the crop of corn. It measures the volume of corn produced if we "Suppose a man could with his own hands plant a certain scope of Land with Corn, that is, could Digg, Plough, Harrow, Weed, Reap, Carry home, Thresh, and Winnow so much as the Husbandry of this Land requires; and had withal Seed wherewith to serve the same." (Petty, 1662, p 43).

Constant (d) is the risk premium ($\varphi = 1\%$ pa). This parameter comprises the return above the interest rate on money loans (i% pa) that farmers require before agreeing to lock up their wealth in a *specific* form of physical capital. Farmers must be recompensed for agreeing to hold illiquid stocks of corn instead of liquidating them at the weekly markets.

Constant (e) is the capital turnover ratio ($\kappa = 2$). It expresses the fact that an opening stock of anything that is run down to zero (in roughly equal decrements over the course of a certain period) implies that, on average, the owner has about *half* the opening stock on hand throughout that period – one year in the case of stored foodcorn.

Constant (w) is the fixed corn price ($P \approx \$27.80$ per sack). This is the average revenue realised on all of last year's foodcorn supplied (Qso sacks) when sold at this year's series of weekly markets. In Model A, this is the *only* parameter whose value is *not* specified near-arbitrarily. The search algorithm Solver identifies it as the *only* corn price to guarantee that r = n = 5% pa will be realised by farmers each year over a century of simulated historical time

– given the initial value parameter (Qiz = 40,000 sacks pa), the other eight constants and the functional forms of the model's three independent equations.

Constant (x) is the fixed money wage (w = $200 per worker pa). It is the average monetary remuneration offered by farmers and accepted by workers. The workers are paid in a money-of-account which serves as the standard-of-value in this simple economy. The fortnightly payroll debts of farmers (*qua* corn producers) to their employed workforce are debited to their own ledger accounts and credited to their workers' "paybooks". These dollar balances subsequently are transferred by all workers to those offering foodcorn for sale at the weekly markets, i.e. the very same group of farmers (*qua* corn retailers, this time).

Constant (y) is the fixed interest rate (i = 4% pa). In the absence of a banking sector, this is the average rate of return on risk-free loans of money capital traditionally agreed between farming families on the basis of trust.

Constant (z) is the volume of foodcorn retained on 31st December this year (Qf = 4,878 sacks pa) by farmers to sustain themselves and their families during the 12 months it will take for next year's crop of corn to be produced.

4.3.4 Initial Values

Initial Value (I) is the only one required to solve the model for its stationary-state dynamic path. This parameter is the base-year volume of seedcorn invested (Qiz = 40,000 sacks pa). It may be viewed as an artefact of the entire previous "history" of this abstract corn-credit economy. All other year-zero values are *computed* using the equations and identities programmed into column E (which represents the base-year) in the computer spreadsheet of Table 4.4 below.

4.4 Corn-Credit Economy Description

Model A's intra-column simultaneous ("circular") and inter-column recursive ("cumulative") dynamic system of structural-form equations is displayed in Figure 4.1 below. The arrows indicate how the model's equations and identities determine its endogenous variables. Lagged variables and Roman-letter constants are italicised and explicit (but Greek-letter constants are implicit) in the flowchart. Due to the corn price (*P*) being a constant, it has no connection with either the lagged volume of foodcorn supplied or the money wage in the fixprice Model A. These necessary links first appear in Model B, wherein P is the first Roman-letter constant to be transformed into a variable.

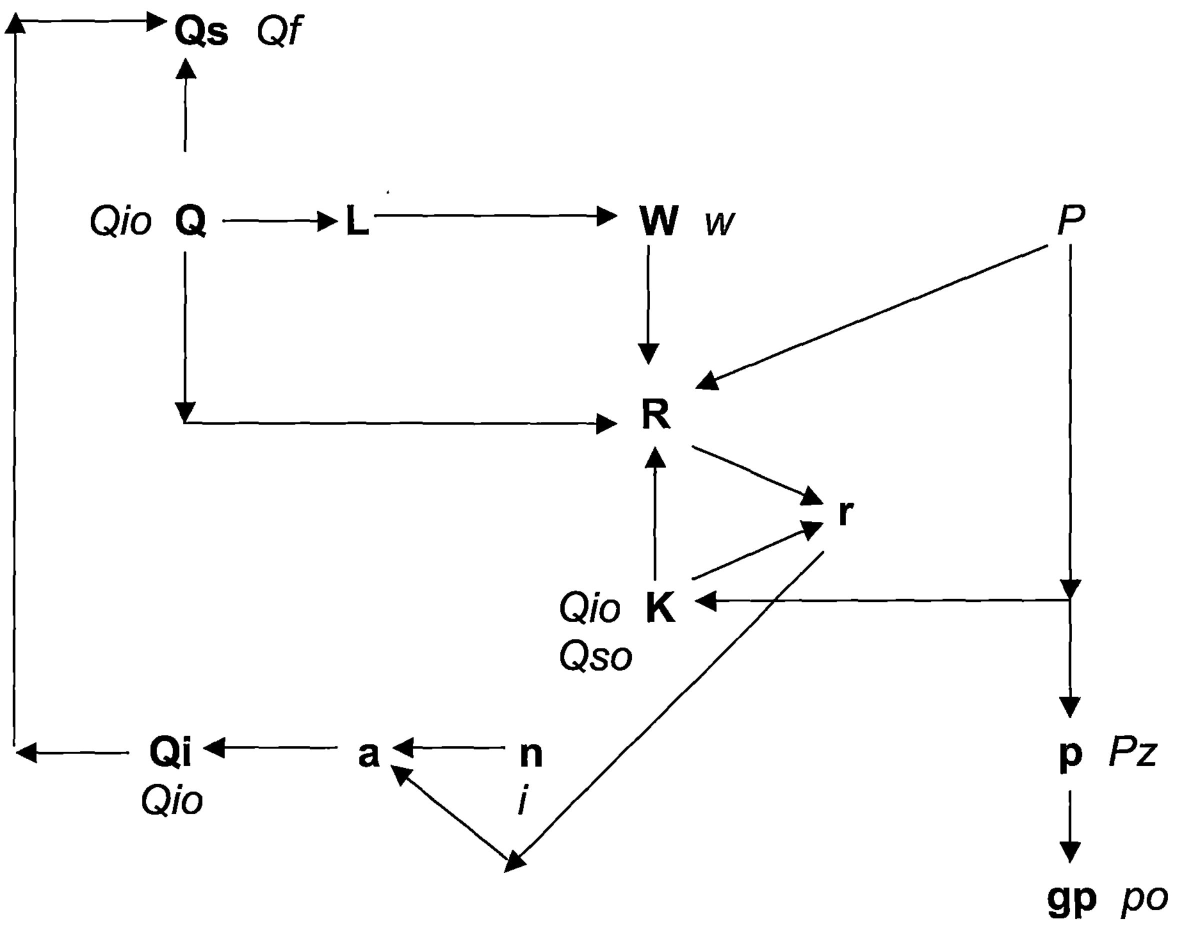
Beginning with the harvesting of Q sacks of corn on 31st December this year, one sees that this event must have required the sowing of *Qio* sacks of seedcorn, the economy's opening stock of circulating capital on 1st January this year. Throughout the twelve-month growing season, L workers tended the crop at a money wage of *w* dollars pa, so the farmers' wage bill was W dollars. Four arrows point towards profit (R dollars), indicating that farmers multiply Q by *P* to value their crop, then subtract W and the opportunity cost of the seedcorn needed to raise it, viz. Ka = *P Qio* dollars. The remaining portion of the farmers' K dollars worth of capital stock is Kb = *P Qso* / κ dollars, the average value of foodcorn stored in their granaries during this year. On 31st December, soon after this year's harvest, farmers are able to calculate the rate of profit (r% pa) that they realised this year on the capital allocated to their joint farming/trading businesses.

Under the static expectations assumption, this year's realised profit rate is identical with the profit rate farmers expect to earn next year: re = r% pa. In addition, this year's opportunity cost of capital is expected to prevail next year: ne = n% pa. All this explains why *two* arrows point to a% pa: this variable measures the gap between r% pa and n% pa. The arrow from a% pa shows that this profitability gap, together with the opening stock of seedcorn (*Qio*), comprises the economy's investment function. It determines how much of this year's crop (Qi) is stored in the barns as seedcorn invested on 31st December, after this year's harvest. Finally, the arrows from Qi and from Q show that the balance of the crop (Qs + *Qf*) is stored in the granaries and farmhouses for consumption next year.

Taking any adjacent pair of years, the chain of causation runs from farmers' expected profitability in year two (i.e. their realised profitability at the end of year one) to investment, to their realised profitability at the end of year two, very much as Robinson (1962, pp 48-51) theorised. The central importance to the corn-credit economy of realised profit (R dollars pa) is evidenced by the fact that the greatest number of arrows converge to determine *its* value.

This year's realised profit rate (i.e. next year's expected profit rate) is plugged into the identity a = r – n% pa so the investment function can determine this year's *flow* of seedcorn invested (Qi). As this also comprises next year's opening *stock* of seedcorn (Qio), the corn-credit model thereby is driven through successive years of simulated historical time. The motive force is supplied by the profitability gap, which maintains a stationary state indefinitely, while ever a = 0% pa (hence also Qi = Qio) remains true. If, on the other hand, a positive or negative profitability gap opens up, then this destroys the tranquillity of the stationary state and initiates some form of dynamic disequilibrium traverse behaviour as the system attempts to adjust its profitability gap back towards zero.

Figure 4.1 – Flowchart of Model A



4.5 Nature of the Corn-Credit Economy

Model A represents an abstract Post-Keynesian monetary production or corn-credit economy that is closed (autarkic) with no government (anarchic) and is history-bound and path-dependent. Labour of uniform productivity is in perfectly elastic supply to farmers at the fixed *money* wage. Keynes (1936, pp 276-7) noted that wage-bargains are always made in money terms; *real* wage rates are jointly determined in the markets for labour and consumer goods. They only emerge once the dollar price of wage-goods (here called Qso or "lagged foodcorn supplied") also becomes known. In this particular economy, however, the *money* price of foodcorn is fixed as well, so workers can be certain of earning a particular *real* wage. This volume of foodcorn is assumed to be above the subsistence minimum for feeding a family comprising (in the stationary state) a couple and two children to replace them.

The economy is inhabited by individual workers and farmers, together with their families. Farmer-traders are the owner-managers and entrepreneur-landlords of unregulated capitalist farm-firms. Although acreages vary, all farms operate at constant returns to scale. This year's opening *stock* of circulating capital is last year's *flow* of investment, i.e. seedcorn retained in barns after the latest end-of-year harvest. During this year, the households of workers (farmers) consume foodcorn retained in granaries (farmhouses) right after the latest end-of-year harvest was brought in.

Barns and granaries are not considered to be income-earning capital assets by farmers, who view them simply as a necessary adjunct to the farmhouses they live in. Land of uniform quality is in perfectly elastic supply – to the farmer-landlord class only – and hence attracts zero rent. The landless workers reside in huts, which they themselves erect on plots of wasteland, owned by whichever farmer they have contracted to work for each year. The workers' dwellings and any implements they feel they need are fashioned in their own time from wood freely taken from forestland owned by their employers.

The opening stock of seedcorn (Qio, retained from last year's harvest) must have labour applied before it can fructify into this year's crop of corn. All the seedcorn is sown, i.e. the circulating capital is absorbed in its entirety, and this is equivalent to a 100 per cent annual depreciation rate. Twelve months later, after the harvest, farmers store this year's fresh investment in seedcorn in their barns and the remaining foodcorn (destined for consumption) in their granaries and farmhouses. They look on *this* year's corn price as the opportunity cost of that volume of seedcorn historically retained from *last* year's harvest. Thus P (current value) rather than Po (historic cost) is used to value Qio, the circulating capital sacrificed to raise *this* year's crop of corn.

To determine this year's gross surplus – a pure accounting residual – farmers use the same corn price (P) to value their crop (Q), then they subtract the wage bill (W). After subtracting the value of their opening capital stock of seedcorn (Ka = P Qio dollars), this yields the farmers' net surplus – which may be positive, zero or even negative. So, the realised rate of profit can be defined consistently as dollars of net surplus divided by dollars of opening capital stock, all evaluated using this year's corn price as determined at the weekly foodcorn markets. The fact that P dollars/sack is the average revenue from selling *last* year's foodcorn (Qso) at *this* year's series of markets – markets held while this year's crop (Q) was still in the fields – does not prevent this price from being used to measure the current value of this year's crop of corn. Gross product and profit are being measured using the production and income methods, not the expenditure method. Expenditure helps determine this year's corn price, i.e. the opportunity cost of this year's output of corn.

The next points are crucial for the analysis of dynamic disequilibrium traverse processes. If, following the harvest, this year's realised profit rate is found to be higher (lower) than this year's opportunity cost of capital – the normal profit rate or target rate of return that farmers have come to expect and demand – there will be a positive (negative) profitability gap and they will decide to retain a higher (lower) volume of seedcorn from this year's than from last year's harvest.

Farmers naïvely or myopically expect to realise the same profitability next year as they did this year. The react, *qua* investors, to any difference between their realised and normal rates of profit, viewing any such profitability gap as an unusual departure from their opportunity cost of capital. So the farmers' profitability gap – which may be zero, of course – is what drives their decision to invest, i.e. to retain a certain volume of seedcorn after each harvest, so that a fresh crop may be sown the following year. In the tranquil conditions of a stationary state, the profitability gap always remains on zero, so expectations of future profitability are never under- or over-fulfilled and aggregate investment remains constant as year follows simulated year.

In this one-commodity world, although the decision to invest necessarily also is a decision to save, it is the investment decision that is paramount. Farmers do not refrain from consuming by making *primary saving decisions* based on (say) a process of inter-temporal maximisation of their lifetime utility, as in the Neoclassical Ramsay-Phelps-Solow corn models.[25] Rather, being classic profit-seeking entrepreneurs, they make *primary investment decisions* based on the expected future profitability of altering their most recently accumulated capital stock of seedcorn. Thus, saving is purely a secondary consideration for the farmers and no explicit saving assumptions (for either farmer or worker households) are required in Model A. For the workers, money saving/dissaving is simply the residue of their wage income (i.e. the economy's wage bill of W dollars) after purchasing Qso sacks of foodcorn at the fixed price of P dollars/sack.

Post-Keynesian economics emphasises the importance of one-way historical time (not two-way logical, mechanical, abstract, or analytical time) and of irreducible uncertainty (not calculable risk) as it affects profitability expectations. Therefore it is important to define both the "period of time" and the "basis of expectation" used in this suite of nested dynamic models.

[25] *Vide* Hicks (1965, pp 251-63,) for trenchant criticism of the Ramsay-Phelps-Solow optimal saving theory.

Neoclassical microeconomic concepts of the short and the long *runs* originate with Walras and are relevant to logical time only. Post-Keynesians tend to follow Marshall, Kalecki, Keynes, and Robinson in viewing any long period of historical time as comprising a sequence of short *periods*, linked together by last period's closing (identical with this period's opening) realised profit rate and stocks of factors and products. In order to define such short periods for economic analysis (and for computational convenience), the natural continuous one-way flow of historical time must be interrupted at discrete intervals – particularly in the agrarian setting of a corn-credit model. This periodisation mimics what real-world business firms do, viz. annually performing stocktakes, preparing accounting statements, determining profit and profitability, and reporting to their shareholders. Therefore, one year was chosen as the Kaleckian short period for Models A through E.

The Neoclassical paradigm assumes that reality is ergodic[26]. This particular *Weltanschauung* craves existence proofs for some grand market-clearing situation which, desirably, also is stable, unique and Pareto-optimal, and which the economy can attain – perhaps after undergoing some smooth and convergent long-run equilibrium-seeking traverse process. By contrast, Post-Keynesians accept that reality is nonergodic and that traverse processes intrinsically are open, history-bound, path-dependent, and subject to hysteresis. So, although the traverse is always a dynamic disequilibrium "adjustment path", the actual or observed traverse process may be seeking a new fully-adjusted growth trajectory – or not – depending on circumstances.

Keynes (1937, p 114) realised that the future is not only unknown, but also inherently unknowable, so that investors and consumers alike fall back on conventions and rules of thumb "... to save our faces as rational men". Keynes (1936, p 157) earlier spoke of "... the forces of time and our ignorance of the future", so it is obvious he believed the universe to be nonergodic. He did not subscribe to the ergodic axiom of Neoclassical economics, whereby economic agents "... draw samples from the past or present, assume that such samples are equivalent to drawing samples from the future, and then place them into an optimising algorithm", as Courvisanos (1996, p 164) puts it. Ergodic-universe rational expectations imply that the profit rate to be realised over the life of an investment project can be foreseen precisely, even in the planning stage.

Economic models of investment always should disclose which particular "expectation function" is being employed to generate the expected profit rate. Economists have used several nonergodic specifications to model real-world investor behaviour, including "static", "adaptive" and "least-squares" expectations. Keynes saw economic agents – particularly

[26] *Vide* Davidson (1996), pp 479-483

entrepreneurs and rentiers – as responding "rationally" to their expectations concerning the future, but theirs necessarily is a bounded or "procedural rationality". By contrast, the perfect "substantive rationality" of the Neoclassicals is quite unattainable in the nonergodic world of the Post-Keynesians. It is logically impossible to reduce uncertainties to risk-equivalents, no matter what volume of information and power of calculation is brought to bear on the problem.

It is assumed, therefore, that a procedurally rational response to irreducible uncertainty is for farmers to act on the expectation that this year's economic conditions (particularly the realised and normal profit rates) will continue into next year. One year is so short a period of historical time that these agrarian entrepreneurs feel comfortable developing their expectations on the basis of what their financial accounts reveal about this year's weekly foodcorn markets, in terms of the average corn price (P dollars/sack) they received and the average profit rate (r% pa) they realised.

4.6 Aggregates

Accounting definitions for 30 macroeconomic aggregates are set out below, together with the units in which they are measured. None of these values has *any* effect on the 14 endogenous variables (comprising four unknowns and 10 identities) of this fully fixprice model. In all these corn-credit models, conventional macroeconomic concepts are of *secondary* importance, including consumption, saving, income shares, the multiplier, the capital-output ratio, the inflation rate, and all variables defined in real terms.

Table 4.2 – Aggregates of Model A

Gross Product	Y = P Q	dollars pa
Gross Surplus	Rg = Y – W	dollars pa
Net Surplus	Rn = Rg – Ka	dollars pa
Real Gross Product	Yr = Y / p	constant dollars pa
Real Wage Bill	Wr = W / p	constant dollars pa
Real Gross Surplus	Rgr = Rg / p	constant dollars pa
Real Net Surplus	Rnr = Rn / p	constant dollars pa
Real Profit	Rr = R / p	constant dollars pa
Consumption	C = P (Q – Qi)	dollars pa
Consumption Ratio	c = C / Y	ratio
Real Consumption	Cr = C / p	constant dollars pa
Saving	S = Y – C	dollars pa
Saving Ratio	s = S / Y	ratio
Real Saving	Sr = S / p	constant dollars pa
Investment	I = P Qi	dollars pa
Real Investment	Ir = I / p	constant dollars pa
Real Interest Rate	ir = i / p	percent pa

Real Wage	$wr = w / p$	constant $/worker pa
Real Profit Rate	$rr = r / p$	percent pa
Wage Bill Share	$ws = W / Y$	ratio
Gross Surplus Share	$rs = R / Y$	ratio
Investment Multiplier	$k = Yr / Ir$	ratio
Real Capital Stock	$Kr = K / p$	constant dollars
Capital-Output Ratio	$v = Kr / Yr$	ratio
Capital-Labour Ratio	$x = Kr / L$	ratio
Money Wage Growth Rate	$gw = w / wo - 1$	percent pa
Real Normal Profit Rate	$nr = n / p$	percent pa
Prime Cost	$pc = w / \lambda$	dollars/sack
Margin	$mn = P - pc$	dollars/sack
Markup	$m = mn / pc$	ratio

4.7 Reduced Form

The Wolfram Mathematica computer program is used to eliminate all identities from the structural form before solving Model A. The resulting reduced form is shown in Table 4.3 below, with nothing on the right-hand sides of its equations but parameters and predetermined variables. Eight of the model's nine constants and two of its four lagged endogenous variables are present.

Table 4.3 shows that, apart from two lagged variables (Qio and Qso), reduced-form equations (A) through (D) have nothing but constants on their right-hand sides. The four unknowns of Model A (Q, Qi, L, and r) all depend on the predetermined variable Qio (lagged seedcorn invested), with two of these unknowns (Qi and r) also depending on Qso (lagged foodcorn supplied). Q, Qi, L, and r, therefore, are path-dependent. All four are driven through simulated historical time by the sequence of previous-year values for Qi, while two are affected also by the sequence of previous-year values for Qs.

Table 4.3 – Reduced Form of Model A

Corn Produced	$Q = \theta Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = \left[1 - \phi(i + \varphi) + \phi\dfrac{(\theta\lambda P - \lambda P - w\theta)Qio}{\lambda P(Qio + Qso/\kappa)}\right]Qio$	sacks pa	(B)
Employment	$L = \theta Qio/\lambda$	workers	(C)
Realised Profit Rate	$r = \dfrac{(\theta\lambda P - \lambda P - w\theta)Qio}{\lambda P(Qio + Qso/\kappa)}$	percent pa	(D)

The reduced-form equations for corn produced (A) and for employment (C) are easy to interpret, but not so the other two. Equation (D), multiplied by the reaction coefficient (ϕ), constitutes the third term of equation (B). Thus it is the path-dependent realised profit rate (r% pa) of equation (D) which drives the dynamic paths followed by Qi (including *disequilibrium* traverse paths), even while its own previous-year value (Qio) is supplying feedback to this key profitability variable.

As noted in section 3.3 of the previous chapter, there are differing conceptions of "equilibrium" within schools of economic thought. The broad Post-Keynesian view defines it as a situation of rest, in which *the forces leading to change* are either absent or countervailing. This contrasts with the narrow Neoclassical definition, viz. zero excess demand in all commodity (including labour) markets. In these corn-credit models, "the forces leading to change" are non-zero profitability gaps between the realised and normal rates of profit along the abstract economy's dynamic path. Should this time sequence of gaps turn out to be a time series of zeros, the *abnormal profitability* forces leading to change are "absent" and a tranquil equilibrium will prevail as the economy simply reproduces itself from one year to the next.

The tranquil *equilibrium* solution of Model A is a stationary-state time path created by forcing the realised profit rate (r% pa) into equality with the normal profit rate (n% pa), which is a constant. Writing the equilibrium condition $r = n = (i + \varphi)$% pa in place of equation (D) and plugging it into equation (B), one obtains $Qi = [1 - \phi(i + \varphi) + \phi(i + \varphi)]Qio = Qio$ sacks pa. This particular dynamic path, the one along which Qi = Qio for every pair of years, defines the stationary-state (or flatline) solution of Model A.

4.8 Spreadsheet Realisation

The structural form of Model A was programmed into a Microsoft "Excel" spreadsheet and given the computer filename Astat (see Appendix D, with enclosed CD-ROM). The spreadsheet formulae corresponding to the equations, identities and constants of Table 4.1 are set out in Table 4.4 below. Formulae for the aggregates of Table 4.2 are not shown.

Table 4.4 – Model A Spreadsheet Formulae

	A	B	C	D	E	F
1	**A - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**=+E1+1**
2	**Equations**					
3	Corn Produced	Q			=+E4*E20	=+E4*F20
4	Seedcorn Invested	Qi			**40000**	=+E4*(1+F19*F14)
5	Employment	L			=+E3/E21	=+F3/F21
6	Realised Profit Rate	r			=+E12/E11	=+F12/F11
7	**Identities**					
8	Wage Bill	W			=+E25*E5	=+F25*F5
9	Seedcorn Capital	Ka			=+E24*E4	=+F24*E4
10	Foodcorn Capital	Kb			=+E24*E15/E23	=+F24*E15/F23
11	Capital Stock	K			=+E9+E10	=+F9+F10
12	Profit	R			=+E24*E3-E8-E9	=+F24*F3-F8-F9
13	Normal Profit Rate	n			=+E26+E22	=+F26+F22
14	Profitability Gap	a			=+E6-E13	=+F6-F13
15	Foodcorn Supplied	Qs			=+E3-E4-E27	=+F3-F4-F27
16	Price Level	p			=+E24/$E24	=+F24/$E24
17	Inflation Rate	gp			=+E16/E16-1	=+F16/E16-1
18	**Constants**					
19	Reaction Coefficient	ϕ			**0.4388**	=+E19
20	Seedcorn Yield	θ			**4**	=+E20
21	Labour Productivity	λ			**10**	=+E21
22	Risk Premium	φ			**0.01**	=+E22
23	Capital Turnover	κ			**2**	=+E23
24	Corn Price	P			**27.7966104639471**	=+E24
25	Money Wage	w			**200**	=+E25
26	Interest Rate	ii			**0.04**	=+E26
27	Foodcorn Retained	Qf			**4878**	=+E27

The rows of Table 4.4 are numbered 1 through 27 and the columns are tagged A through F. Columns A and B list the long and short names, respectively, of all variables and constants in the model.[27] The purpose of columns C and D is discussed below, but in Table 4.4 the formulae they contain are suppressed. Column E (year zero) holds the single initial value, the nine constants and all formulae for the model's equations and identities. Column F contains all relevant formulae for year one. The (missing) columns for years two through 100 continue the pattern established in column F.

In the spreadsheets for the stationary-state (Table 4.5) and the steady-state (Table 4.6) basecases below, one observes that column C = column D. The two-letter codes stand for "reference difference" (rd) and "actual difference" (ad) in Table 4.5 and, in Table 4.6, for "reference growth rate" (rg) and "actual growth rate" (ag). Only when a specimen traverse spreadsheet is presented (e.g. in Table 4.7 below) is there potential for "actual" figures to depart from "reference" figures. With no parameter perturbation (hence no associated traverse) in the basecases, then necessarily rd = ad and rg = ag. The column C reference percentages are common across all runs of one particular model; only the column D actual percentages may change, and then due to traverse behaviour alone. Thus the distance of

[27] In row 26, the interest rate's short name is shown as ii because the Excel software capitalises all single letters i.

any ad/ag above or below its corresponding rd/rg is an indicator of the quantitative significance and direction of whatever traverse process is responsible for creating that divergence.

All differences (ad, rd) and growth rates (ag, rg) are calculated as 70-year averages from year 31 through year 100. This is because *all* traverse-inducing perturbation experiments begin in year 30, thus preserving a segment of the reference basecase behaviour for comparison against the actual traverse behaviour that follows. Visual inspection of time-series graphs complements numerical [ad ↔ rd] and [ag ↔ rg] comparisons.

In Table 4.4 the single initial value (**bold** type) in year zero is given by history as 40,000 sacks pa of seedcorn invested. Apart from the nine constants (also **bold** type), all other year-zero values are *computed* rather than specified. *Prima facie*, the whole of column E could have been filled with known historical base-period data. Yet, as this is not an empirical model, there are no historical data. So, reliance is placed on the assumption that, in any given year, history cannot be internally inconsistent. That is why the model's standard set of equations and identities is used to compute all remaining year-zero values.

This spreadsheet realisation of Model A is reminiscent (albeit not an implementation) of Gunnar Myrdal's (1957, p 30) "... circular causation in the cumulative processes of economic change". The *circularity* resides in the columns, where the 14 structural-form equations simultaneously determine the endogenous variables. The *cumulation* occurs along the rows, where the time path that each of these variables (plus the 30 aggregates, not shown) traces out is dependent on the recursive levels of Qio and Qso in each previous column and on the initial value Qiz in column E. This mix of "circular and cumulative" (or "simultaneous and recursive") causation will become clearer once the stationary state is discussed and Model A encounters steady-state economic growth.

4.9 Solving for the Stationary State

The first step is to enforce the stationary state condition a = r – n = 0% pa for all columns within the spreadsheet, in order to make Model A "just-determined". A set of near-arbitrary numerical values for all but one of this model's parameters is chosen, then its single remaining constant (the corn price, P dollars/sack) is manipulated until r = n = 5% pa characterises each year for a century of simulated historical time. In this way, the tranquil long-period equilibrium of a stationary state (which Marx termed "simple reproduction") is achieved. Given all other parameter-values, the *only* corn price consistent with stationarity is P ≈ $27.80 per sack, as reported in Table 4.4 above. This figure can be confirmed by treating P as the unknown (and r = 5% pa as a known quantity) in the reduced-form equation (D) of Table 4.3 above.

Table 4.5 below displays years 0, 31 and 100 of a century-long stationary state, as simulated in the Astat spreadsheet file. This reference solution (or basecase) constitutes the starting point for all subsequent Model A computer runs. The 23 numbers for year zero in column E are replicated by the circular and cumulative solution process in all 100 subsequent columns, thereby forming rows of stationary values for the 14 endogenous variables and the nine constants (also for the 30 aggregates, not shown). These all trace out horizontal or flatline graphs, when plotted against 100 years of simulated historical time.

Table 4.5 – Model A Stationary State

	A	B	C	D	E	F	G
1	**A - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	0 00%	160,000	160,000	160,000
4	Seedcorn Invested	Qi	0.00%	0.00%	**40,000**	40,000	40,000
5	Employment	L	0.00%	0.00%	16,000	16,000	16,000
6	Realised Profit Rate	r	0 00%	0.00%	5.0%	5.0%	5 0%
7	**Identities**						
8	Wage Bill	W	0.00%	0.00%	$3,200,000	$3,200,000	$3,200,000
9	Seedcorn Capital	Ka	0 00%	0.00%	$1,111,864	$1,111,864	$1,111,864
10	Foodcorn Capital	Kb	0.00%	0.00%	$1,600,001	$1,600,001	$1,600,001
11	Capital Stock	K	0.00%	0 00%	$2,711,865	$2,711,865	$2,711,865
12	Profit	R	0.00%	0.00%	$135,593	$135,593	$135,593
13	Normal Profit Rate	n	0.00%	0.00%	5 0%	5.0%	5.0%
14	Profitability Gap	a	na	na	0.0%	0.0%	0.0%
15	Foodcorn Supplied	Qs	0.00%	0.00%	115,122	115,122	115,122
16	Price Level	p	0.00%	0.00%	1.000	1.000	1.000
17	Inflation Rate	gp	na	na	0.0%	0 0%	0.0%
18	**Constants**						
19	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
20	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
21	Labour Productivity	λ	0 00%	0.00%	**10**	10	10
22	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
23	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
24	Corn Price	P	0.00%	0 00%	**$27.80**	$27.80	$27.80
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200 00	$200 00
26	Interest Rate	ii	0 00%	0.00%	**4.0%**	4 0%	4.0%
27	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

As viewed on the computer screen, there is a row of tabs at the foot of this (and every other) spreadsheet, indicating six categories of variables and constants whose time paths are plotted. The following list shows the tabs, the categories and some examples of the time series included in each category:

Exp	Expenditures	Gross Product, Consumption, Investment, Saving
Inc	Incomes & Capital	Gross Product, Wage Bill, Gross Surplus, Capital Stock
Hi	High Values	Corn Produced, Seedcorn Invested, Employment
Lo	Low Values	Corn Price, Money Wage, Capital-Labour Ratio

Pct	Percentages	Realised & Normal Profit Rates, Profitability Gap
Rto	Ratios	Saving Ratio, Wage Share, Capital-Output Ratio

Two of these six graphs are reproduced below. They exemplify the flatline behaviour of all variables and constants over the 100-year time-span of the model's stationary-state solution.

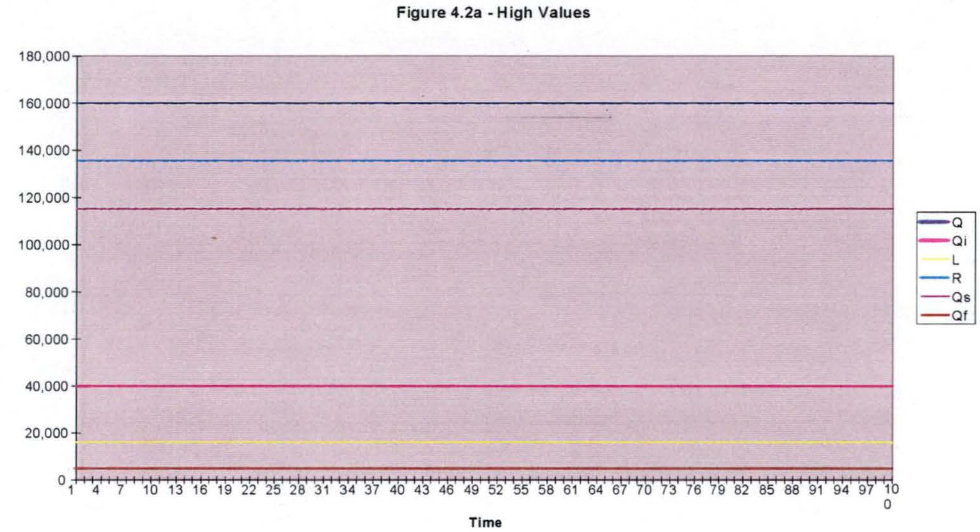


Figure 4.2a above plots six variables in the High Values category.

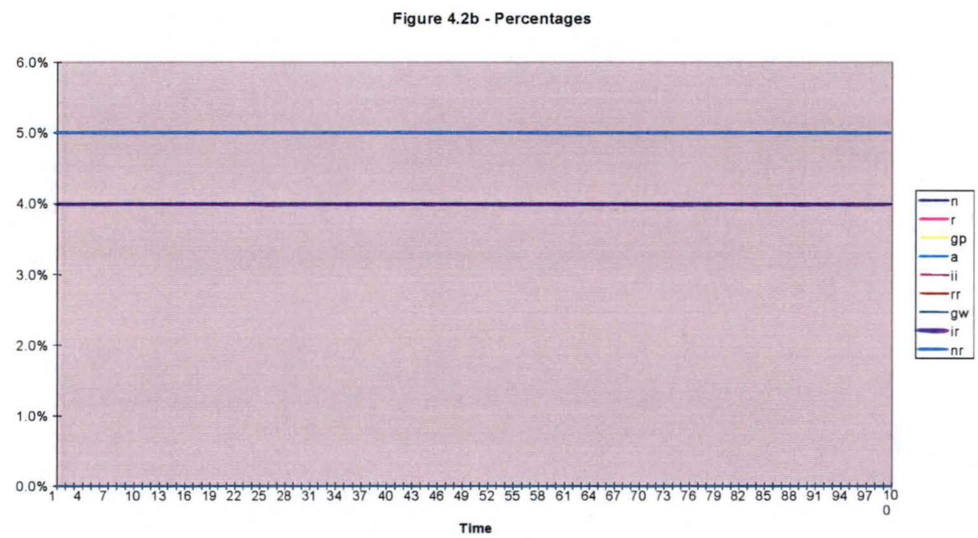


Figure 4.2b above plots nine variables in the Percentages category. Most variables take on a common value of 5% pa, but i = 4% pa and a = 0% pa – a zero profitability gap.

The fact that capitalist farmers, year after year for an entire century, are meeting their expectations while realising their opportunity cost of capital (re = r = n = 5% pa) is what preserves this stationary state. In other words there are never any surprises, so the farmers keep on investing precisely Qi = 40,000 sacks pa of seedcorn after each harvest. So, with no changes in Qio – or in any parameter of the model – the profitability gap remains on zero for 100 years of simulated historical time and tranquil conditions prevail throughout the stationary corn-credit economy.

4.10 Generating the Steady State

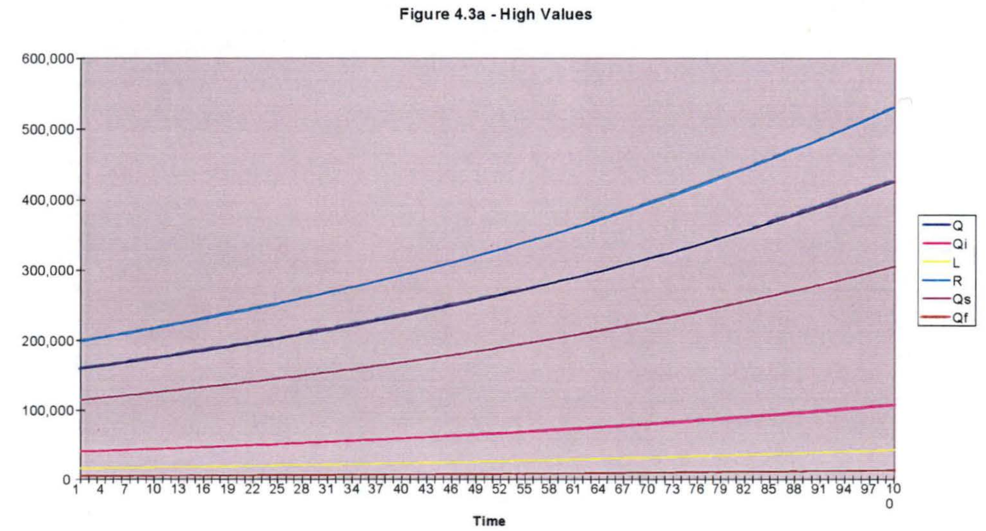
Economic growth implies that net investment must be positive and increasing over time. Lifting an economy out of its stationary-state slumber demands a positive (not a zero) expected profitability gap. In the fully fixprice world of Model A, *steady* exponential growth is achieved simply by raising constant (x) – i.e. P, the corn price – sufficiently to open up a positive and *constant* gap between every r% pa and n% pa. For consistency, the foodcorn retained (Qf) constant should increase annually at the same exponential rate as the economy is targetted to grow, e.g. gQf = gQ = 1% pa. In this way, the economy's dynamic path will be driven by a sequence of positive a% pa differences between r% pa and n% pa – the normal profit rate having been fixed at n = (i + φ) = 5% pa.

This is demonstrated in Table 4.6 below. Raising the corn price to P ≈ \$28.33 and increasing foodcorn retained by gQf = 1% pa makes corn production grow at the *average* exponential rate of gQ = 1% pa, calculated in the ag column over 70 years of simulated historical time. This higher price lifts the realised profit rate to r = 7.2% pa, thus creating a constant positive profitability gap of a = 7.2 – 5 = 2.2% pa. The reaction coefficient of the investment function is fixed at ϕ = 0.4388, so less than half the profitability gap stimulus is passed through, making Qi (hence also Q, L, W, K, R, etc.) grow by 1% pa. This is what propels the corn-credit economy into a steady state of long-period growth. The steady state is what Marx (1885, p 80) termed "Accumulation and Reproduction on an Extended Scale" and Harrod (1948, p 76) referred to as the "Regularly Progressive Economy".

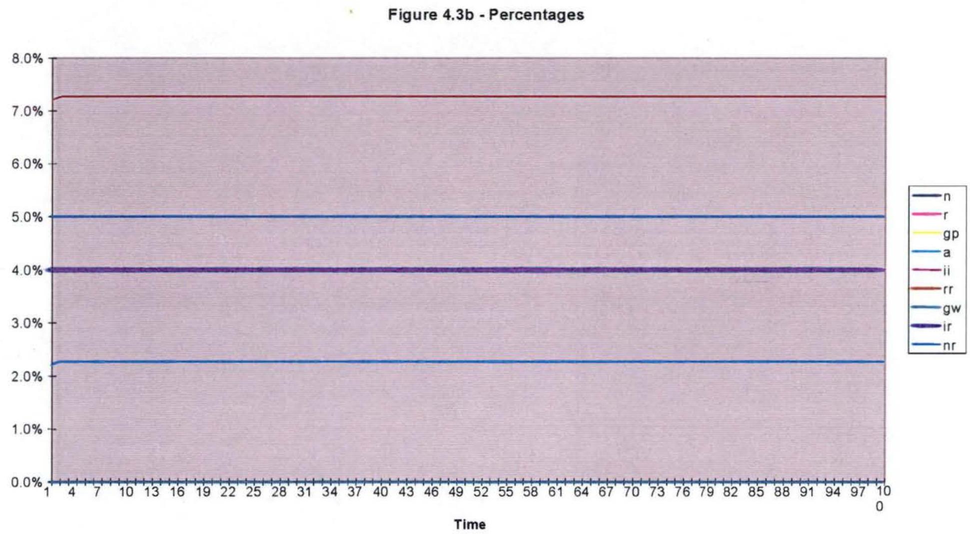
Table 4.6 – Model A Steady State

	A	B	C	D	E	F	G
1	**A - STEADY STATE**	**sn**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.00%	160,000	215,301	426,474
4	Seedcorn Invested	Qi	1.00%	1.00%	**40,000**	**54,361**	**107,680**
5	Employment	L	1.00%	1.00%	16,000	21,530	42,647
6	Realised Profit Rate	r	0.00%	0.00%	7.2%	7.3%	7.3%
7	**Identities**						
8	Wage Bill	W	1.00%	1.00%	$3,200,000	$4,306,018	$8;529,471
9	Seedcorn Capital	Ka	1.00%	1.00%	$1,133,080	$1,524,707	$3,020,179
10	Foodcorn Capital	Kb	1.00%	1.00%	$1,630,530	$2,163,884	$4,285,697
11	Capital Stock	K	1.00%	1.00%	$2,763,610	$3,688,591	$7,305,876
12	Profit	R	1.00%	1.00%	$199,240	$268,103	$531,066
13	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
14	Profitability Gap	a	na	na	2.2%	2.3%	2.3%
15	Foodcorn Supplied	Qs	1.00%	1.00%	115,122	154,299	305,600
16	Price Level	p	0.00%	0.00%	1.000	1.000	1.000
17	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
18	**Constants**						
19	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
20	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
21	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
22	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
23	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
24	Corn Price	P	0.00%	0.00%	**$28.33**	$28.33	$28.33
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
27	Foodcorn Retained	Qf	1.00%	1.00%	**4,878**	6,641	13,194

Some of these steady-state results are plotted in two graphs reproduced below. They are directly comparable with the stationary-state graphs above. Figure 4.3a below reveals *steady exponential growth* at the rate of 1% pa for six key variables.



This growthline behaviour is driven by the constant positive profitability gap which results when P ≈ \$27.80 is raised to P ≈ \$28.33 per sack and foodcorn retained is made to grow at gQf = 1% pa. Figure 4.3b displays flatline behaviour (i.e. zero growth) for nine variables over Model A's 100-year time-span.



The reduced form of Model A shows that lagged seedcorn invested (Qio sacks pa) is the principal driver of this abstract economy. In turn, Qi is driven by the profitability gap and its own lagged value. In Figure 4.3b above, given the constancy of n = 5% pa, the realised profit rate of r = 7.2% pa implies that a = 2.2% pa. This explains why the corn economy is lifted out of the stationary and into the steady state. Given that the money wage is fixed at w = \$200 per worker pa, *all* the monetary benefits from the higher corn price go straight into improving realised profits. Table 4.6 shows farmers' profit or net surplus (R dollars pa) growing at 1% pa. Thus, they have been encouraged to retain ever-increasing stocks of seedcorn out of each annual harvest, thereby generating a steady state of constant exponential growth.

The smoothness and constancy of the time paths plotted in Figures 4.9a and 4.9b may tempt one to imagine that these portray a situation of *equilibrium growth*. Yet with net investment being driven by a *positive gap* between realised and normal profitability, this indicates that farmer-traders are *not* in equilibrium; they *never* remain content with the existing situation after closing their accounts every 31st December. In fact, on the broad Post-Keynesian definition, there is no such thing as an "equilibrium" (let alone an "optimal") time path of investment, outside of the classic stationary state in which nothing more than replacement investment occurs. Only then is the economy in equilibrium, defined above as "a situation of rest, in which the forces leading to change are either absent or countervailing".

In the ergodic world of Neoclassical theory, one might conceive of an *independently* growing demand for output, into which an optimally-growing capital stock can be fitted by pursuing an optimal investment strategy. However, in the nonergodic world of Post-Keynesian theory, causality runs the other way: humans create their own economic futures. Expecting positive pure profits, capitalists invest in excess of capital stock replacement requirements. This makes income, expenditure and production (including its all-important profit component) grow, thereby bringing about something resembling the situation they foresaw. Capitalists then adjust their expectations in the light of realised results and investment outlays are altered accordingly. This is inherently a dynamic disequilibrium process and steady-state growth is only one of many possible outcomes.

4.11 Specimen "Traverses"

The generation of a steady state of positive economic growth from its ancestral stationary state is achieved by suddenly perturbing two parameters of Model A, viz. raising the corn price constant (P) in year zero to create a positive profitability gap and making capitalist consumption of foodcorn (Qf) grow at the same rate as the economy. Economic growth is fuelled by net investment and depends on creating and maintaining this positive profitability gap (a > 0) between r and n. Such prospects of supernormal profitability encourage capitalist farmers to hold back ever more seedcorn from each successive harvest, before releasing the balance of their crops for sale as foodcorn at the following year's weekly foodcorn markets. Effectively, then, a dynamic disequilibrium traverse-like *growth* process has just been initiated, starting from a stationary-state basecase. It takes time for the new steady state to become established.

As noted above, this raises an important theoretical issue concerning how long-period "equilibrium" growth paths are defined in economics. The classic dynamic paths are the stationary state of zero growth (flatline) and the steady state of constant exponential growth (growthline). Yet, given that a profitability gap "genome" has been identified as underpinning most accepted investment functions (see Appendix B), entrepreneurs are only "content" (i.e. in equilibrium) whenever the expected (re% pa) equals the normal (n% pa) profit rate. The static, myopic or naïve expectation function (re = r% pa) is only one of many possible specifications, including adaptive and least-squares expectation functions.

In the pure flexprice barter economies of Neoclassical general equilibrium theory, all firms must be earning normal profits. If they are not, then positive (negative) "pure profits" are present and these will stimulate entry/investment (exit/disinvestment) in all affected industries or sectors. Therefore, levels of investment above or below depreciation seem incompatible with equilibrium *ex definitione*, which calls into question long-standing concepts like the "equilibrium level of net investment"[28] and the "equilibrium rate of economic growth".[29]

That is why the growthline steady state must be viewed as a special type of dynamic disequilibrium traverse path. The steady state is special because it is smooth and continuous, despite the underlying entrepreneurial disequilibrium of r > n% pa. Provided this positive difference (i.e. the profitability gap, a% pa) remains constant, an *illusion* of "dynamic equilibrium" is produced. In fact, however, the *only* genuine long-period equilibrium time path is the flatline stationary state, a tool of analysis first developed by the Classical economists.

The procedure chosen to generate the following specimen "traverses" (in fact, all traverses) must, of necessity, operate *via* the central profitability gap mechanism in the corn-credit models. It resembles the "traverse-like" procedure used above for the *generation* of a reference steady state of 1% pa growth, starting from the *solution* of a reference stationary state of 0% pa growth. By suddenly perturbing only one specific parameter, the *ceteris paribus* condition is enforced, thereby guaranteeing that every movement away from a reference time path (either the stationary-state or the steady-state basecase) must be due to that parameter-change *and to nothing else*. This is the essence of the computer simulation experimental method: rigid enforcement of the *ceteris paribus* assumption. Note also that the pertubation occurs in year 30, not in year zero. This preserves a segment of the basecase or reference path, against which the observed traverse path may be compared.

Two specimen "traverses" are initiated, the first from the stationary-state basecase and the second from the steady-state basecase. Both are sparked off by a sudden, unexpected, unplanned, four percent drop in seedcorn invested during year 30. This Misallocation Scenario involves four percent of the sacks of seedcorn (already earmarked for investment at the end of year 30) being *mistakenly* sold as foodcorn during year 31.

Table 4.7 below displays the economic effects of this unintended misallocation *via* a comparison of spreadsheet columns C and D. Column C (headed rd) shows reference differences between the stationary-state basecase *and itself*, in percentages. For this reference solution, of course, all such differences must be zero. Column D (headed ad)

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[28] Appendix B shows that Dale Jorgenson (1963) faced this problem when he derived an optimal time path for the capital stock, but had to invoke *ad hoc* time-lags and adjustment-costs to obtain the dynamic behaviour of net investment.

[29] Roy Harrod's (1939) "warranted rate of growth" implies equality between planned investment and saving flows. But if the cue for entrepreneurs to invest (in excess of depreciation) is an expectation of earning pure (i.e. supernormal) profits, the economy cannot be in general equilibrium. A state of disequilibrium (r > n) remains, even if the actual, warranted and natural growth rates coincide.

shows actual differences between the traverse time path and the basecase time path, again in percentages, following any parameter-change during year 30. It can be seen that, neglecting the constants, virtually all variables dropped off by around four percent. As would be expected (given the fixed corn price, money wage and interest rate), it is the physical variables (plus those with a volume component) which decreased.

Table 4.7 – Model A Misallocation Scenario from Stationary State

	A	B	C	D	E	F	G
1	**A - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	-3.99%	160,000	153,600	153,733
4	Seedcorn Invested	Qi	0.00%	-3.99%	**40,000**	**38,373**	**38,434**
5	Employment	L	0 00%	-3 99%	16,000	15,360	15,373
6	Realised Profit Rate	r	0.00%	0.06%	5 0%	4.8%	5 0%
7	**Identities**						
8	Wage Bill	W	0.00%	-3.99%	$3,200,000	$3,072,000	$3,074,657
9	Seedcorn Capital	Ka	0.00%	-3.99%	$1,111,864	$1,067,390	$1,068,313
10	Foodcorn Capital	Kb	0.00%	-4.08%	$1,600,001	$1,622,238	$1,534,625
11	Capital Stock	K	0.00%	-4.05%	$2,711,865	$2,689,628	$2,602,938
12	Profit	R	0.00%	-3.99%	$135,593	$130,170	$130,282
13	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
14	Profitability Gap	a	na	na	0 0%	-0.2%	0 0%
15	Foodcorn Supplied	Qs	0.00%	-4.16%	115,122	110,349	110,421
16	Price Level	p	0.00%	0.00%	1.000	1.000	1.000
17	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
18	**Constants**						
19	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
20	Seedcorn Yield	θ	0.00%	0 00%	**4**	4	4
21	Labour Productivity	λ	0.00%	0 00%	**10**	10	10
22	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
23	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
24	Corn Price	P	0.00%	0.00%	**$27.80**	$27.80	$27 80
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4 0%
27	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

Figure 4.4a below shows that the relevant flatline plots suddenly were displaced downward by four percent during year 30, then immediately resumed their horizontal trend through the remaining 70 years of simulated historical time. These instantaneous "dropoffs" can hardly be described as "traverses".

Figure 4.4a - High Values

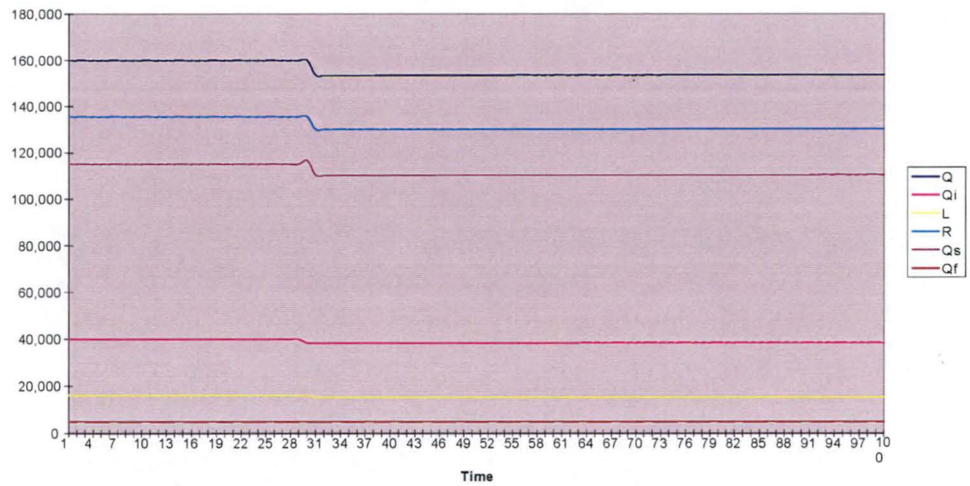


Figure 4.4b below shows how the year-30 misallocation event causes a temporary fall in the realised profit rate during year 31, with no change in the normal profit rate. This is reflected in the a = 0% pa profitability gap turning negative during that same year.

Figure 4.4b - Percentages

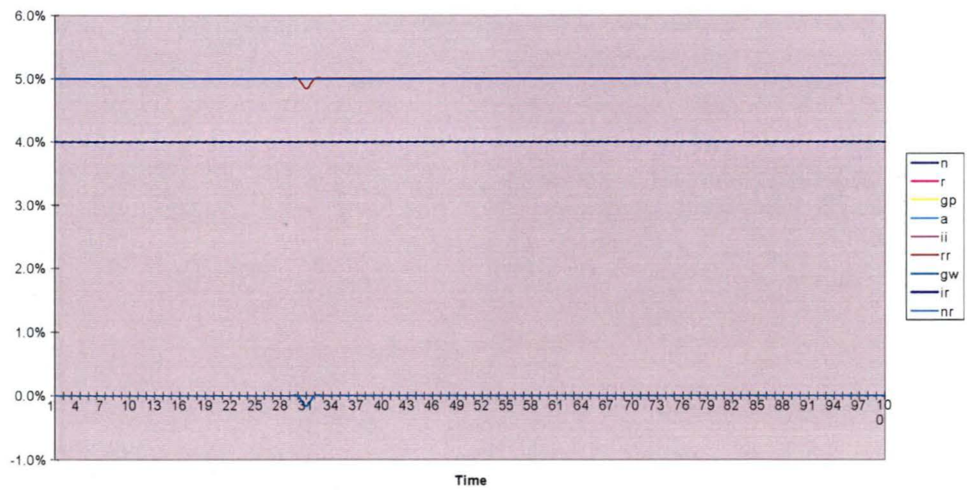


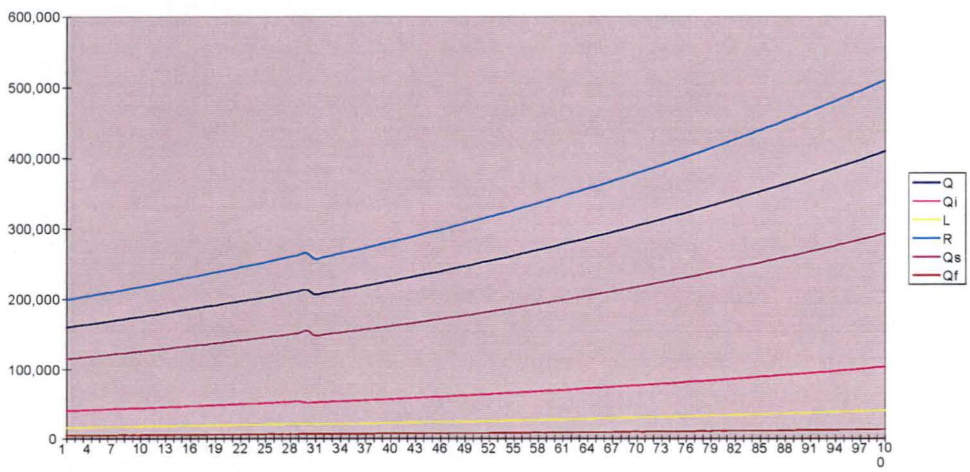
Table 4.8 below displays the economic effects of the same Misallocation Scenario *via* a comparison of spreadsheet columns C and D. But in this case, column C (headed rg) shows the reference growth rates for the steady-state basecase, in percent pa. These annual growth rates, of course, are the same as those reported above when a steady state is generated from its parent stationary state. Column D (headed ag) shows the actual growth rates for the traverse time path, in percent pa, following a change in any variable or parameter during year 30. The two columns are virtually identical because the sole effect of

a sudden four percent drop in seedcorn invested is a *once-for-all* downward displacement in the time paths of most variables. There need be no change in their growth rates.[30]

Table 4.8 – Model A Misallocation Scenario from Steady State

	A	B	C	D	E	F	G
1	**A - STEADY STATE**	**sn**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.00%	160,000	206,689	409,939
4	Seedcorn Invested	Qi	1.00%	1.00%	**40,000**	**52,134**	**103,509**
5	Employment	L	1.00%	1.00%	16,000	20,669	40,994
6	Realised Profit Rate	r	0.00%	0.00%	7.2%	7.0%	7.3%
7	**Identities**						
8	Wage Bill	W	1.00%	1.00%	$3,200,000	$4,133,777	$8,198,778
9	Seedcorn Capital	Ka	1.00%	1.00%	$1,133,080	$1,463,719	$2,903,085
10	Foodcorn Capital	Kb	1.00%	0.99%	$1,630,530	$2,194,378	$4,112,173
11	Capital Stock	K	1.00%	0.99%	$2,763,610	$3,658,097	$7,015,258
12	Profit	R	1.00%	1.00%	$199,240	$257,379	$510,476
13	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
14	Profitability Gap	a	na	na	2.2%	2.0%	2.3%
15	Foodcorn Supplied	Qs	1.00%	1.00%	115,122	147,914	293,236
16	Price Level	p	0.00%	0.00%	1.000	1.000	1.000
17	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
18	**Constants**						
19	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
20	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
21	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
22	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
23	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
24	Corn Price	P	0.00%	0.00%	**$28.33**	$28.33	$28.33
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
27	Foodcorn Retained	Qf	1.00%	1.00%	**4,878**	6,641	13,194

Figure 4.5a - High Values



[30] Similarly, in Robert Solow's (1956) growth model, a perturbation of the saving propensity parameter does not affect his Neoclassical economy's growth rate, only its level of real gross domestic product.

Figure 4.5a above shows that the relevant growthline plots suddenly were displaced downward by four percent during year 31, then immediately resumed their upward trend through the remaining 70 years of simulated historical time.

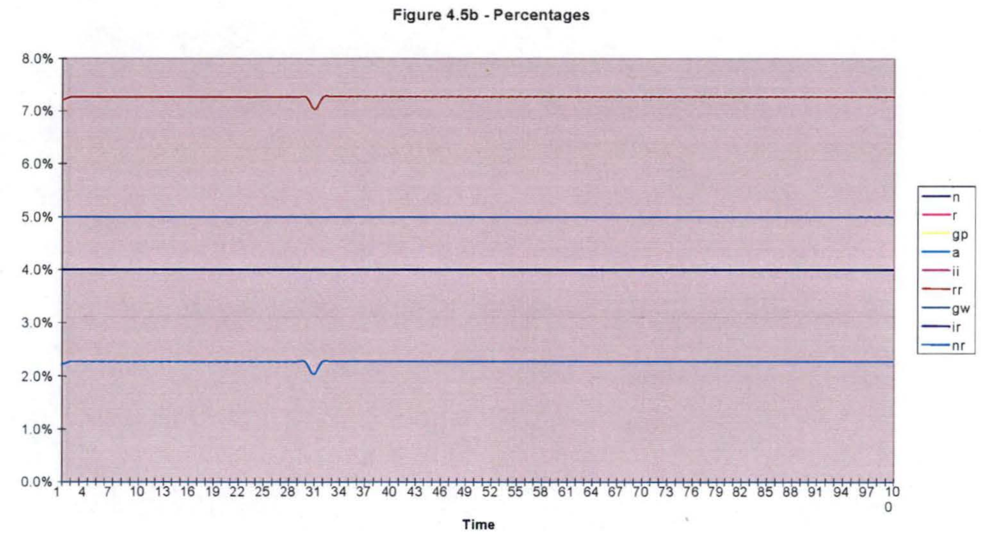


Figure 4.5b above shows how the year-30 misallocation event causes a temporary fall in the realised profit rate during year 31, with no change in the normal profit rate. This is reflected in a dip in the profitability gap during that same year. The steady-state gap is a = 2.3% pa, comprising r = 7.3% pa less n = 5% pa. During year 31 only, r dips to 7% pa and a drops to 2% pa.

4.12 Some Theoretical Implications

First, Post-Keynesian dynamic monetary production models can be "one equation short", just as in Neoclassical static barter exchange models. But, whereas static barter models merely require that any one commodity be arbitrarily chosen as *numéraire*, dynamic monetary models need a highly-specific closure condition, viz. that the expected and normal profit rates be equal, so that entrepreneurs remain content to maintain their investment outlays constant and their capital stocks intact.

Secondly, the fact that the three independent equations of Model A explain much by little hints that economics need not necessarily continue separating micro from macro and real from monetary. The long (and so far fruitless) quest for the microfoundations underpinning macroeconomics may need to be refocussed onto discovering the macroconstraints binding microeconomic behaviour or the macroenvironment within which microeconomics operates.

Thirdly, Wicksell's concept of a "pure credit economy" may be an important reagent for dissolving the alleged split personality of economics; the real and monetary sides of an economy *can* be treated in an integrated fashion. In modern economies, the ratio of outside to inside money is asymptoting to zero and modern central banks are placing primary reliance on the bank, funds, discount, or cash rate of interest as an instrument of monetary policy. These facts strongly suggest that the real world is coming to resemble Wicksell's theoretical construct of a pure credit economy.

Finally, the generality of the investment function "genome" effectively means that the classical economists' concept of a stationary state is the only admissible long-period *equilibrium* time path. While this state can be "solved for", the more recently adopted steady state can only be "generated". The reason is that it depends upon the maintenance of an underlying *disequilibrium* through time, in the shape of a positive expected profitability gap.

4.13 Conclusion

In this chapter the pure fixprice Model A was constructed, solved for a stationary state, then used to generate a steady state before launching two specimen "traverses". The nature of the abstract corn-credit economy was treated comprehensively, using structural-form equations, a flowchart, reduced-form equations, and descriptive passages of text. This depth is justified because so much of Model A survives the subsequent construction stages. In the following Chapter 5, the descriptions of Models B, C and D are more sparse.

A "traverse" experiment was performed on Model A, relative to its stationary-state or primary basecase solution. An unintended misallocation of the year-30 harvest (less seedcorn, more foodcorn) resulted in nothing more than a dropoff in the physical and real variables, plus those having a volume component. The relevant flatline plots suddenly were displaced downward by four percent during year 30, but immediately resumed their horizontal march through the remaining 70 years of simulated historical time.

This dropoff effect also occurred when the "traverse" experiment was repeated from the steady-state basecase. There was a *once-for-all* downward displacement in the time paths of most variables during year 30, with no change in the relevant growth rates. The physical variables, plus those having a volume component, immediately resumed their upward trend through the remaining 70 years of simulated historical time.

An instantaneous dropoff cannot qualify as a "traverse", which requires the passage of time. Pure fixprice behaviour is the *extremum* of the "price stickiness" that Neo-Keynesians and

New Keynesians rely on to generate involuntary unemployment in macroeconomic models. Not surprisingly, Model A confirms their findings, but the more important question is whether progressively larger doses of flexibility will lead to better outcomes with respect to production, employment and consumption.

In the following Chapter 5, three of the constants of Model A (the corn price, money wage and interest rate) are converted into endogenous variables *via* the addition of extra structural-form equations to Models B, C and D, respectively. Thus the pure fixprice model progressively will shed its most restrictive assumptions, but will live on inside the core of ever more flexible representations of a Post-Keynesian corn-credit economy. By the end of Chapter 6, the "more important question" concerning the effects of enhanced flexibility will have been answered.

CHAPTER FIVE

FLEXING THE CORN PRICE, MONEY WAGE AND INTEREST RATE

5.1 Introduction

In this chapter the specifications, flowchart descriptions and reduced forms of the second, third and fourth construction stages of the corn-credit model are reported. The pure fixprice Model A is made progressively more flexible by allowing the corn price, wage rate and interest rate to vary. This is done by replacing each of these three constants with extra equations in the structural forms of Models B, C and D, respectively. In this way, Model A sheds its restrictive fixprice assumptions and becomes incorporated ("nested") as the core of an ever-more flexible representation of a Post-Keynesian corn-credit economy.

As each modified structural form is being specified, changes are noted in the nature of the abstract economy, whose technology and behaviour the preceding stage described. As with Model A, once the reduced form has been derived and discussed, each model is solved numerically for its stationary state of zero growth and a steady state of positive growth is generated from that. Finally, specimen traverses are sparked off along both these classic long-period fully-adjusted time paths, serving as comparators for the dynamic disequilibrium behaviour that follows the seedcorn misallocation event analysed in Chapter 4.

The conclusion is preceded by a discussion of certain theoretical implications stemming from the analysis of Models B, C and D.

5.2 Model B

5.2.1 Structural Form

Model B differs from Model A in that equation (E) in Table 5.1 below now determines the corn price endogenously, rather than P being fixed as an unexplained constant. The other Roman-letter constants (w, i and Qf) remain, but these are destined to vanish as the wage rate, interest rate and consumption by farmers are endogenised in Models C, D and E, respectively.

The Greek-letter constants retain their numerical values throughout all stages of model building. In particular, the reaction coefficient remains at its Model A value of ϕ = 0.4388, until flexing the interest rate in Model D requires it to assume the value ϕ = 0.4432, which is slightly higher. Unlike most other parameter values, the choice of ϕ is *not* arbitrary, the above values being the *only* ones consistent with the achievement and maintenance of a steady state of growth at the demonstration rate of 1% pa.

Outside the steady state, the reaction coefficient controls the corn-credit economy's endogenous cyclical behaviour, which is encountered for the first time in Model B. Unlike most Neoclassical models derived from Frisch (1933) – including those of the "real business cycle" persuasion – this Post-Keynesian model does *not* require a continual stream of random shocks to keep its cycles alive.

Table 5.1 – Structural Form of Model B

Equations			
Corn Produced	Q = θ Qio	sacks pa	(A)
Seedcorn Invested	Qi = (1 + ϕ a) Qio	sacks pa	(B)
Employment	L = Q / λ	workers	(C)
Profit Rate	r = R / K	percent pa	(D)
Corn Price	P = W / Qso	$/sack	(E)
Identities			
Wage Bill	W = w L	dollars pa	(1)
Seedcorn Capital	Ka = P Qio	dollars	(2)
Foodcorn Capital	Kb = P Qso / κ	dollars	(3)
Capital Stock	K = Ka + Kb	dollars	(4)
Profit	R = P Q – W – Ka	dollars pa	(5)
Normal Profit Rate	n = i + φ	percent pa	(6)
Profitability Gap	a = r – n	percent pa	(7)
Foodcorn Supplied	Qs = Q – Qi – Qf	sacks pa	(8)
Price Level	p = P / Pz	ratio	(9)
Inflation Rate	gp = (p / po) – 1	percent pa	(10)
Constants			
Reaction Coefficient	ϕ = **0.4388**	ratio	(a)
Seedcorn Yield	θ = **4**	sacks/sack pa	(b)
Labour Productivity	λ = **10**	sacks/worker pa	(c)
Risk Premium	φ = **1.0**	percent pa	(d)
Capital Turnover	κ = **2.0**	ratio	(e)
Money Wage	w = **200.00**	$/worker pa	(x)
Interest Rate	i = **4.0**	percent pa	(y)
Foodcorn Retained	Qf = **4878**	sacks pa	(z)
Initial Value			
Seedcorn Invested	Qiz = **40,000**	sacks pa	(I)

As before, the search algorithm Solver is utilised to determine the unique long-period equilibrium stationary-state solution of Model B. With no change in the previous numerical parameter values, it is not surprising that the flexible corn price now *floats* to the same level at which it was *fixed* as a Roman-letter constant in Model A, viz. P ≈ $27.80 per sack.

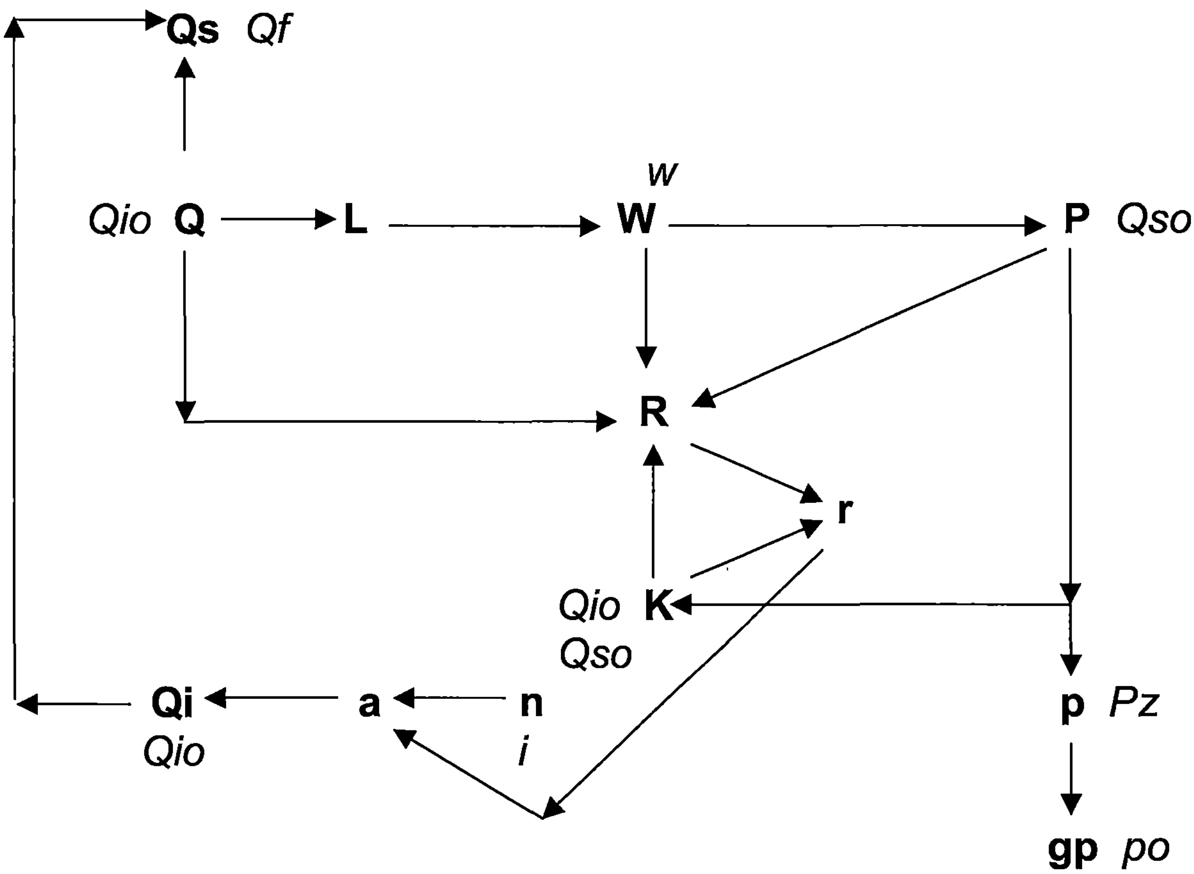
Although Model A needed no assumptions concerning money saving, a "classical saving assumption" is adopted for Models B, C and D. That is, workers do not save any of their money income in these models, opting instead to spend the entire wage bill (W dollars pa) on purchasing the total volume of foodcorn supplied to the weekly markets (Qso sacks pa) and sold for what it will fetch (P dollars/sack) by the farmer-traders.

The new corn price equation (E) originates in the economic growth and income distribution models of Robinson (1956) and Harcourt (1963). It expresses the fact that, during the series of weekly markets, all the economy's stored foodcorn (Qso sacks pa) is sold to the workers in exchange for all the money they earn as wages (W dollars pa). With the workers assumed not to save any money from their wage earnings, this implies that the farmers must receive back as the average corn price (P dollars/sack) *all* of their current wage bill in return for *most* of their previous year's crop, i.e. that part (Qso) not retained by them for investment purposes (Qio) and for their own household consumption (Qf). The Robinson-Harcourt price equation, therefore, is consistent with "Kalecki's dictum" (see below) concerning the direct dependence of capitalists' profits upon their own investment and consumption outlays.

5.2.2 Corn-Credit Economy

The interrelatedness of this recursive dynamic system of structural equations is displayed in Figure 5.1 below. By comparison with the flowchart of Model A, the corn price (P) is now a variable rather than a constant. The arrow from W shows the influence of the wage bill which, together with the predetermined variable Qso, determines the corn price. With non-saving workers being the only participants in the weekly markets for foodcorn, it is inevitable that the average corn price received by the farmers must end up being equal to this year's wage bill (W dollars) divided by last year's foodcorn supplied (Qso sacks).

Figure 5.1 – Flowchart of Model B



The corn-credit model now connects money wages (as both costs and incomes) with the corn price, while still being driven by seedcorn invested (Qio), whose motive force still is the expected profitability gap (a), given by the arrows from r and n.

5.2.3 Reduced Form

As before, Mathematica is used to eliminate all identities before solving Model B. The resulting reduced form is shown in Table 5.2 below, with nothing on the right-hand side of its equations but parameters and predetermined variables. Seven of the model's eight constants and two of its four lagged endogenous variables are present.

Table 5.2 – Reduced Form of Model B

Corn Produced	$Q = \theta Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = \{1 - \phi(i+\varphi) + \phi[\frac{Qio(\theta - 1) - Qso}{Qio + Qso/\kappa}]\}\,Qio$	sacks pa	(B)
Employment	$L = \theta Qio / \lambda$	workers	(C)
Realised Profit Rate	$r = \frac{Qio(\theta - 1) - Qso}{Qio + Qso/\kappa}$	percent pa	(D)
Corn Price	$P = \frac{w\theta Qio}{\lambda Qso}$	dollars/sack	(E)

By comparison with Model A, Model B has one extra equation in its reduced form, viz. equation (E) replaces the formerly constant corn price (P). Thus, all occurrences of P in equations (B) and (D) have vanished, together with all occurrences of the money wage (w) as well. Confining both P and w to the single equation (E) radically simplifies interpretation of the realised profit rate equation (D), as discussed below. As with Model A, one can write the dynamic equilibrium condition $r = n = (i + \varphi)\%$ pa in place of equation (D), plug it into equation (B) and obtain Qi = Qio as the stationary-state solution of Model B.

Equation (D) allows one to interpret the realised rate of profit (r% pa) – defined as dollars of net surplus (R) divided by dollars-worth of capital stock (K) – in purely *physical* terms, i.e. as a flow of corn divided by a stock of corn. Using equation (A), one can rewrite the numerator as Q – Qio – Qso sacks pa, which *resembles* Qf, the volume of foodcorn retained for consumption in farmer households. From identity (8) of the structural form, the constant Qf = Q – Qi – Qs = 4,878 sacks pa. However, in a stationary state, Qi = Qio and Qs = Qso, meaning that the numerator *is* Qf, in fact.

In equation (D) the denominator is simply the sum of the farmers' opening stock of seedcorn and their average stock of foodcorn. So, $r = Qf / [Qio+Qso/\kappa]$ percent pa, in a stationary state. Outside this long-period equilibrium regime, however, equation (D) rules. Its numerator features a difference between *this* year's corn production (Q) and *last* year's seedcorn invested (Qio) and foodcorn supplied (Qso), which accounts for path-dependent fluctuations in the realised profit rate as simulated historical time passes.

The profit rate equation (D) shows that *Kalecki's dictum* holds in Model B. Kalecki (1933, p 79) said that "... capitalists, as a whole, determine their own profits by the extent of their investment and personal consumption". He recognised that "... the common conviction that the more is consumed the less is saved ... is correct with regard to a single capitalist, [but] does not apply to the capitalist class as a whole." Kalecki noted that if some capitalists spend money, either on investment or on consumer goods, their money passes to other capitalists in the form of profits. Thus, investment or consumption by some capitalists simply creates profits for others. As a class, therefore, capitalists gain what they spend. "If – in a closed system – they ceased to construct and consume they could not make any money at all", he concluded.

In the stationary states of Model B where they retain more foodcorn for consumption, for instance, the capitalist farmers realise a higher *rate* of profit. Also, in those where their physical consumption and investment are proportionately higher, farmers achieve a higher *level* of profit – and value of capital stock – with the same profit rate.

The corn price equation (E) shows *why* this is so. Using equation (A) again and rearranging terms, equation (E) can be rewritten as P = $(w/\lambda)(Q/Qso)$ dollars/sack. With constant prime cost of pc = w/λ, the only way open to increase the margin of mn = P – pc (hence also the level of profit) is to raise the ratio of Q to Qso. Equation (A) shows that Q is a multiple (θ) of Qio. On the previous 31st December, *last* year's crop was allocated as seedcorn invested, foodcorn retained and foodcorn supplied (Qo = Qio + Qf + Qso). Had farmers instead raised Qio or Qf or both, then Qso would have been smaller and Q larger, thus achieving a higher ratio of Q (*this* year's crop) to Qso.

Higher Q means more labour and a bigger wage bill. Lower Qso means less foodcorn supplied to the weekly markets. Dividing more wage dollars by fewer foodcorn sacks means that the corn price must rise, thus widening the margin above prime cost and generating more profits for the farmer-traders, in accordance with Kalecki's dictum.

Table 5.2 above shows that Qio appears on the right-hand side of all five reduced-form equations. Therefore, all five endogenous variables (Q, Qi, L, r, and P) are being driven through simulated historical time by the sequence of previous-year values for Qi. The central driving force of Qio is modified by the presence of Qso on the right-hand side of three equations.

Outside the tranquil stationary state, disequilibrium reigns by definition: $r \neq n = (i + \varphi)\%$ pa, hence Qi $\neq$ Qio from equation (B). With complete path-dependence on Qio, the stage is set for some complex dynamic behaviour whenever an initial stationary state is perturbed by any event that drives a wedge between r and n, thus creating a non-zero profitability gap ($a \neq 0$). The "event" that initiates the "specimen traverses" of this chapter is a sudden, unexpected four per cent drop in the volume of seedcorn invested, i.e. an unintended misallocation of corn away from investment (Qi) and towards consumption (Qs) at the end of year 30.

5.2.4 Stationary State

Before the traverse experiments can be performed, the stationary-state condition r = n% pa must be enforced for all 100 columns within Excel, in order to make Model B a "just-determined" system. The set of numerical parameter values this model shares with Model A is retained, including the interest rate and risk premium constants on the right-hand side of n = (i + φ)% pa. Then the foodcorn retained by farmers for their own household consumption (the Qf constant) is manipulated until every year is characterised by r = Qf / [Qio+Qso/κ] = n = 5% pa for a century of simulated historical time, thereby achieving the tranquil long-period equilibrium of a stationary state. Table 5.3 below shows that the *only* volume of foodcorn retained that is consistent with stationarity is a flow of Qf = 4,878 sacks pa, i.e. five percent of the physical capital stock variable [Qio+Qso/κ] = 97,560 sacks. Table 5.3 also shows that the (now flexible) corn price has floated to P ≈ \$27.80 per sack, the same as in the fixprice Model A of Chapter 4.

Table 5.3 – Model B Stationary State

	A	B	C	D	E	F	G
1	**B - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	0.00%	160,000	160,000	160,000
4	Seedcorn Invested	Qi	0.00%	0.00%	**40,000**	40,000	40,000
5	Employment	L	0.00%	0.00%	16,000	16,000	16,000
6	Realised Profit Rate	r	0.00%	0.00%	5.0%	5.0%	5.0%
7	Corn Price	P	0.00%	0.00%	$27.80	$27.80	$27.80
8	**Identities**						
9	Wage Bill	W	0.00%	0 00%	$3,200,000	$3,200,000	$3,200,000
10	Seedcorn Capital	Ka	0 00%	0.00%	$1,111,864	$1,111,864	$1,111,864
11	Foodcorn Capital	Kb	0.00%	0 00%	$1,599,972	$1,599,972	$1,599,972
12	Capital Stock	K	0.00%	0.00%	$2,711,836	$2,711,836	$2,711,836
13	Profit	R	0.00%	0 00%	$135,592	$135,592	$135,592
14	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
15	Profitability Gap	a	na	na	0.0%	0.0%	0.0%
16	Foodcorn Supplied	Qs	0.00%	0.00%	115,122	115,122	115,122
17	Price Level	p	0.00%	0.00%	1 000	1.000	1 000
18	Inflation Rate	gp	na	na	0.0%	0 0%	0 0%
19	**Constants**						
20	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
21	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
22	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
23	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1 0%
24	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4 0%
27	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

Table 5.3 displays years 0, 31 and 100 of this stationary state, as simulated in the Bstat spreadsheet file. This reference solution constitutes the starting point for all subsequent Model B computer runs. The numbers in column E (year zero) are replicated in all 100 subsequent columns, thereby forming rows of stationary values for the 15 endogenous variables and the eight constants, as well as the 31 aggregates (not shown). As with Model A, these all trace out horizontal or flatline graphs, when plotted against a century of simulated historical time, so there is no need to reproduce them here.

As in Model A, the capitalist farmers are fulfilling their long-period expectations, while simultaneously realising their opportunity cost of capital (re = r = ro = n = 5% pa). They experience no surprises as the simulated years pass. Faced with a consistent sequence of zero profitability gaps, farmers keep on investing Qi = Qio = 40,000 sacks pa of seedcorn after each harvest, which ensures that tranquil conditions prevail throughout the corn-credit economy for a full century.

5.2.5 Steady State

A particular growth path is sought, along which corn production increases at the exponential rate gQ = 1% pa over at least 70 years of simulated historical time, i.e. between years 31 and 100. Theory suggests, and experimentation shows, that making foodcorn retained grow by gQf = 1% pa achieves this goal. Table 5.4 below displays the growth rates of this 70-year steady state, as simulated in the Bsted spreadsheet file. Also, the values of all variables are displayed for years 0, 31 and 100. One can see that the physical variables Q, Qi, L, Qs, and Qf all grow at 1% pa, as do the money and real values of W, K and R. All other variables (including the realised profit rate) experience zero growth over the final 70-year period.

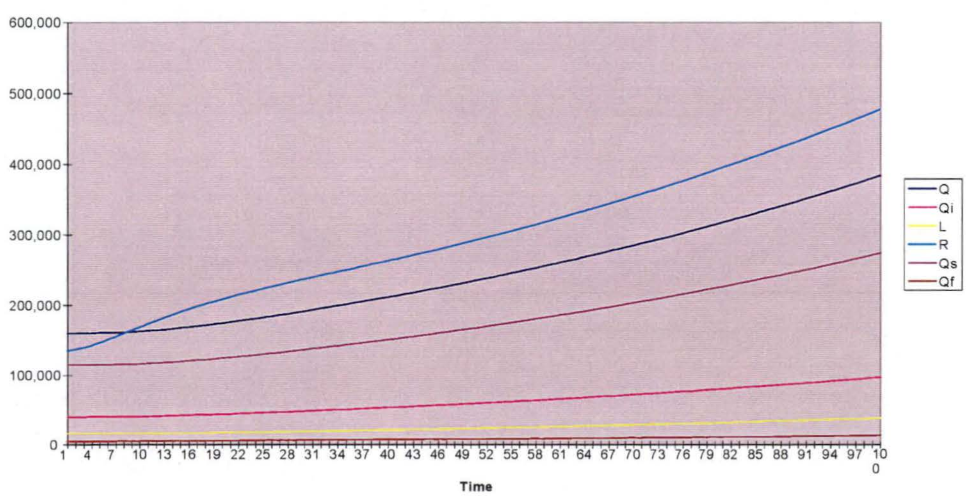
However, sustained growth of gQf = 1% pa during the initial 30-year period progressively lifts the profit rate from r = 5% pa to r = 7.3% pa, thus opening up an expected profitability gap of a = 2.3% pa over and above the normal profit rate of n = 5% pa. It is the *constancy* of this profitability gap which is responsible for the corn-credit economy's smooth exponential growth for most of the century. The higher rate of profit is associated with a corn price that increases from P ≈ \$27.80 to P ≈ \$28.32 per sack. By contrast, in the *fixprice* Model A of Chapter 4, a similar rise in the corn price constant (combined with the same gQf = 1% pa) ushered in a comparable steady state of growth (gQ = 1% pa) immediately. Not even one year of traverse adjustment to the new realities occurred, let alone 30 years. Model B traverses to the steady state because the *flexprice* corn-credit economy needs time to adjust to the progressive increases in Qf which, in accordance with Kalecki's dictum, serve to raise price, profit and profitability.

Table 5.4 – Model B Steady State

	A	B	C	D	E	F	G
1	**B - STEADY STATE**	**sn**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.00%	160,000	193,379	384,282
4	Seedcorn Invested	Qi	1.00%	1.00%	**40,000**	48,824	97,031
5	Employment	L	1.00%	1.00%	16,000	19,338	38,428
6	Realised Profit Rate	r	0.00%	0.00%	5.0%	7.3%	7.3%
7	Corn Price	P	0.00%	0.00%	$27.80	$28.32	$28.32
8	**Identities**						
9	Wage Bill	W	1.00%	1.00%	$3,200,000	$3,867,579	$7,685,648
10	Seedcorn Capital	Ka	1.00%	1.00%	$1,111,864	$1,369,123	$2,721,144
11	Foodcorn Capital	Kb	1.00%	1.00%	$1,599,972	$1,933,756	$3,842,757
12	Capital Stock	K	1.00%	1.00%	$2,711,836	$3,302,878	$6,563,901
13	Profit	R	1.00%	1.00%	$135,592	$239,789	$477,782
14	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
15	Profitability Gap	a	na	na	0.0%	2.3%	2.3%
16	Foodcorn Supplied	Qs	1.00%	1.00%	115,122	137,914	274,057
17	Price Level	p	0.00%	0.00%	1.000	1.019	1.019
18	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
19	**Constants**						
20	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
21	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
22	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
23	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
24	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
27	Foodcorn Retained	Qf	1.00%	1.00%	**4,878**	6,641	13,194

Much of this dynamic behaviour may be viewed in Figures 5.2a and 5.2b below. Figure 5.2a shows that profit or net surplus (R dollars pa) increases faster than Qf, L, Qi, Qs, and Q during the initial period.

Figure 5.2a - High Values



This occurs because the (now highly-flexible) corn price rises endogenously as farmers retain increased volumes of foodcorn for their own consumption (Qf sacks pa), thereby reducing foodcorn supplied to the subsequent year's weekly markets, for which workers bid using their money wage earnings. It takes up to 30 years for employment (L workers) – hence also the wage bill (W dollars pa) and lagged foodcorn supplied (Qso sacks pa) – to "catch up" with each other and stabilise the corn price, net surplus and realised profit rate.

Figure 5.2b - Percentages

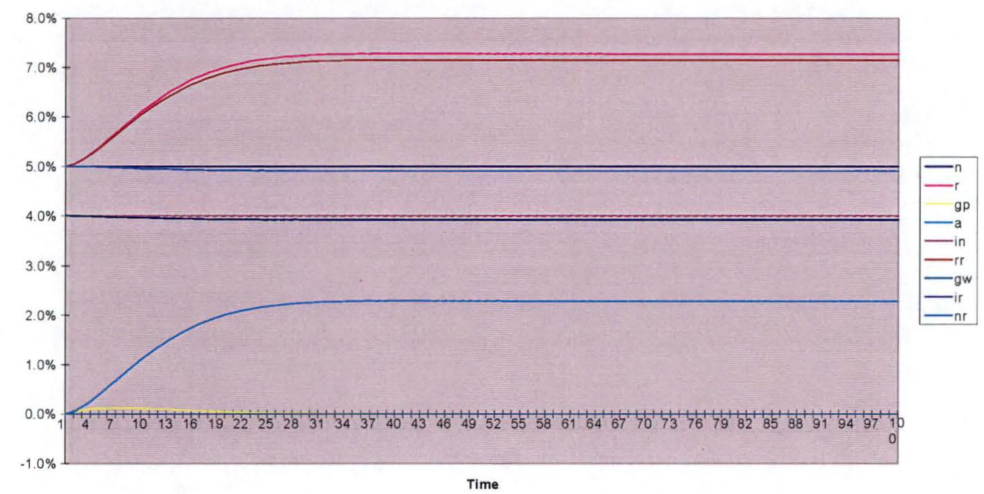
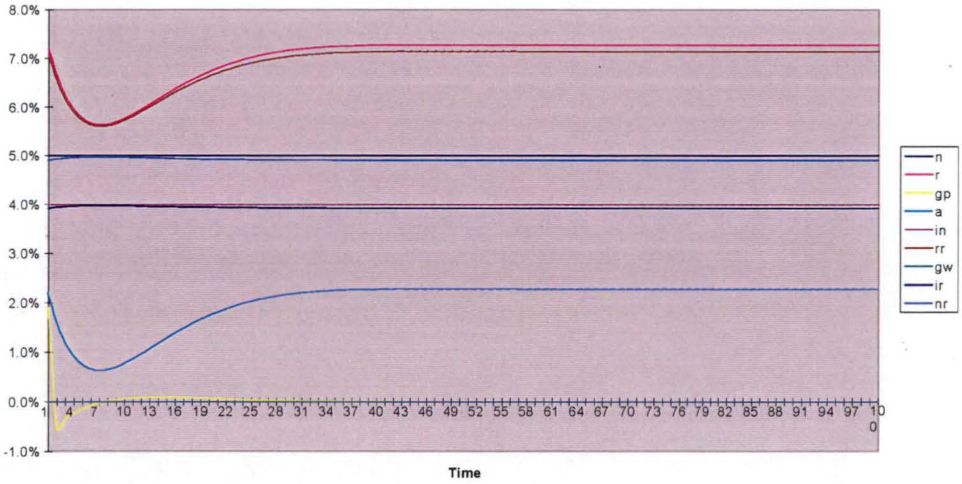


Figure 5.2b above shows that up to 30 years pass before the profit rate (r% pa) and profitability gap (a% pa) adjust to the new reality of foodcorn retained growing at the rate of gQf = 1% pa. This adjustment path from the stationary to the steady state is, of course, a traverse. Its existence explains why all subsequent traverses from the steady state must be initiated no earlier than year 30. Its long duration suggests that even the perfect price flexibility built into Model B cannot guarantee that such an economy will absorb exogenous shocks quickly.

The graph indicates some low-level price inflation (gp > 0% pa) during the initial period and shows that the real profit rate (rr = 7.1% pa) is slightly lower than the realised profit rate (r = 7.3% pa) because of the slightly higher price level.

Finally, an experiment is conducted to test whether Neoclassical "perfect foresight" by farmers selling their produce in the year-1 foodcorn market can *shorten* the traverse. The final, fully-adjusted year-100 corn price (P = $28.3244144563956) is plugged into Model B at year 1, overwriting the Robinson-Harcourt flexprice equation (P = W / Qso) that formerly determined the year-one corn price.

Figure 5.2c - Percentages



However, far from shortening their economy's 30-year traverse, the farmers' new-found prescience actually *lengthens* its duration to 36 years, as shown in Figure 5.2c above.

5.2.6 Specimen Traverses

Two specimen traverses are initiated, the first from the stationary-state basecase and the second from the steady-state basecase. Both traverses are sparked off by a sudden, unexpected, unplanned, four percent drop in seedcorn invested during year 30. As before, this Misallocation Scenario involves four percent of the sacks of seedcorn (already earmarked for investment as circulating capital at the end of year 30) being *mistakenly* released onto the subsequent year's weekly markets, for sale as foodcorn. Year 30 is chosen because (i) a segment of the stationary-state basecase is thereby preserved for comparison with the traverse time path and (ii) the steady-state basecase only began in year 30 because it required a 30-year traverse to generate it in the first place.

Table 5.5 below displays the economic effects of this unintended misallocation *via* a comparison of spreadsheet columns C and D. As in Model A, the rd percentages concern the reference stationary state and the ad percentages measure how the actual traverse path diverges from the basecase. Neglecting the constants, almost all ad percentages are negative, indicating that most of the population (viz. the working class) is worse off with than without the traverse. This is despite the economy's initial year-zero stationary state being reasserted well before year 100, so the traverse beginning in year 31 indicates a net diminution in corn production – as is evident in the graphs presented below. The farmers, however, have benefited in that those ad percentages having most relevance for them have increased, viz. r, P, R, and p.

Table 5.5 – Model B Misallocation Scenario from Stationary State

	A	B	C	D	E	F	G
1	**B - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	-2.41%	160,000	153,600	160,002
4	Seedcorn Invested	Qi	0.00%	-2.35%	**40,000**	37,292	40,000
5	Employment	L	0.00%	-2.41%	16,000	15,360	16,000
6	Realised Profit Rate	r	0.00%	2.72%	5.0%	-1.6%	5.0%
7	Corn Price	P	0.00%	0.12%	$27.80	$26.32	$27.80
8	**Identities**						
9	Wage Bill	W	0.00%	-2.41%	$3,200,000	$3,072,000	$3,200,044
10	Seedcorn Capital	Ka	0.00%	-2.29%	$1,111,864	$1,010,648	$1,111,876
11	Foodcorn Capital	Kb	0.00%	-2.41%	$1,599,972	$1,535,973	$1,599,994
12	Capital Stock	K	0.00%	-2.36%	$2,711,836	$2,546,621	$2,711,870
13	Profit	R	0.00%	0.34%	$135,592	-$40,057	$135,585
14	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
15	Profitability Gap	a	na	na	0.0%	-6.6%	0.0%
16	Foodcorn Supplied	Qs	0.00%	-2.53%	115,122	111,430	115,124
17	Price Level	p	0.00%	0.12%	1.000	0.947	1.000
18	Inflation Rate	gp	na	na	0.0%	-5.3%	0.0%
19	**Constants**						
20	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
21	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
22	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
23	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
24	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
27	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

Two graphs reproduced below show that it takes over 40 years for the economy to adjust to the Misallocation Scenario and traverse back onto its original stationary-state dynamic path.

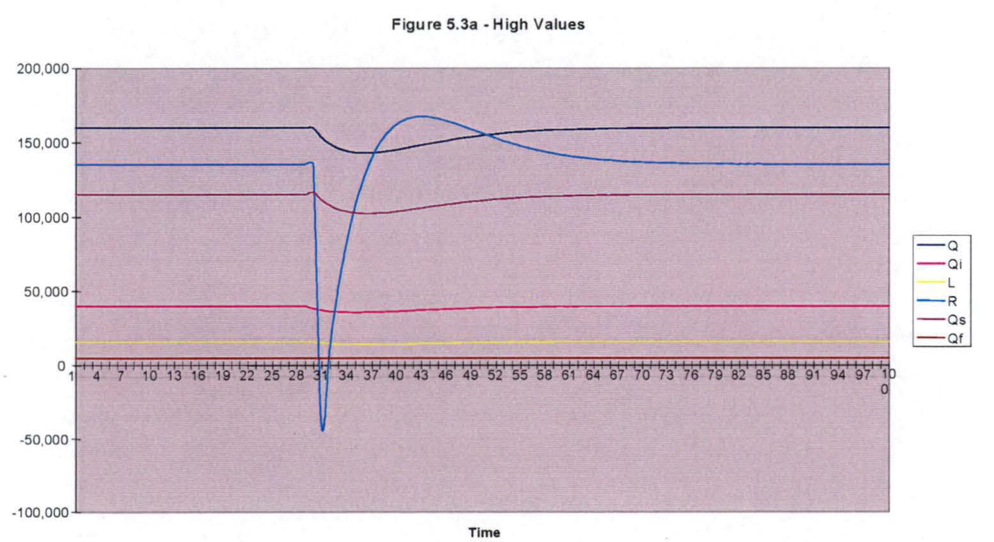


Figure 5.3a above shows that investment (Qi), production (Q), sales (Qs), and employment (L) all turn down in response to the shock, but traverse smoothly – albeit slowly – back onto their original flatline plots. By contrast, profit (R) overcompensates for its initial downswing

by turning upwards and *overshooting* its original time path, before regaining a flatline trajectory.

Figure 5.3b - Percentages

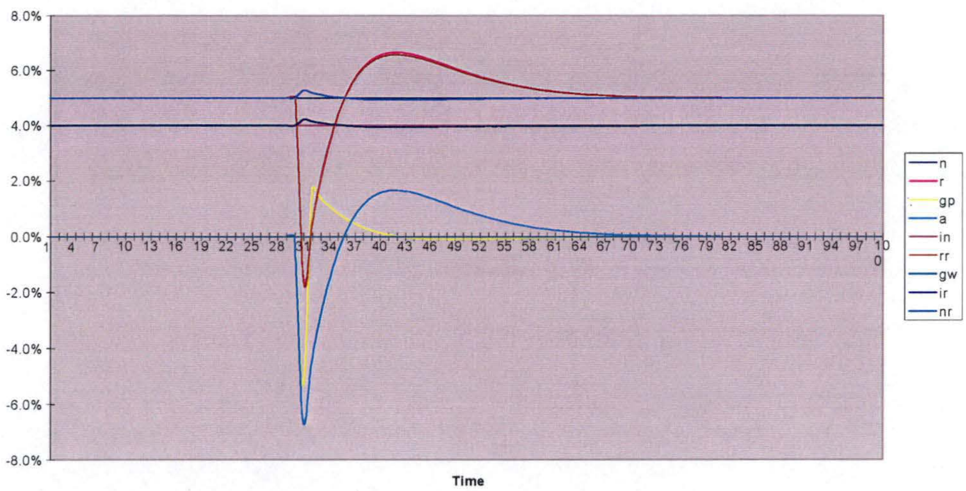


Figure 5.3b above shows that the inflation rate (gp), the nominal (r) and real (rr) profit rates, and the profitability gap (a) also temporarily overcompensate in the same manner as profit. This explains the asymmetry between the effects of this traverse process on the two societal groups, workers and farmers.

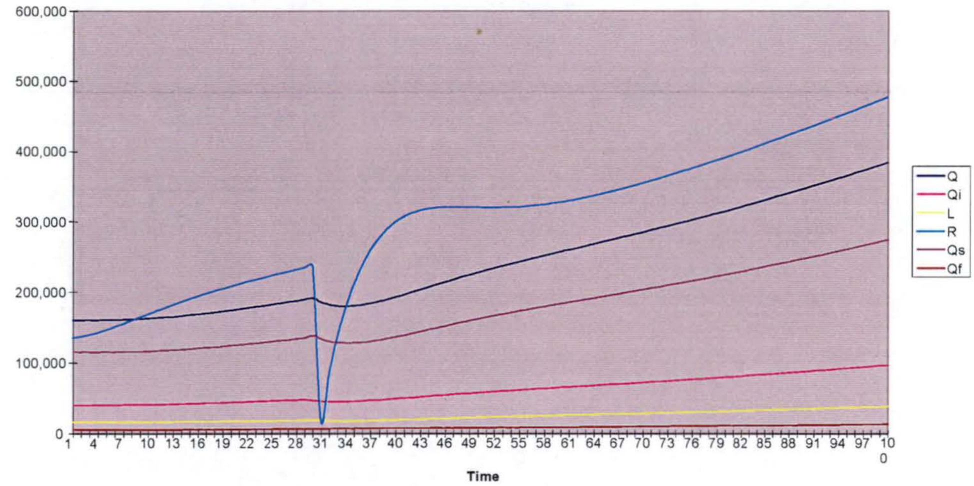
Table 5.6 – Model B Misallocation Scenario from Steady State

	A	B	C	D	E	F	G
1	**B - STEADY STATE**	**sn**	**ag**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.14%	160,000	185,644	384,284
4	Seedcorn Invested	Qi	1.00%	1.14%	**40,000**	45,522	97,032
5	Employment	L	1.00%	1.14%	16,000	18,564	38,428
6	Realised Profit Rate	r	0.00%	0.27%	5.0%	0.6%	7.3%
7	Corn Price	P	0.00%	0.00%	$27.80	$26.81	$28.32
8	**Identities**						
9	Wage Bill	W	1.00%	1.14%	$3,200,000	$3,712,876	$7,685,690
10	Seedcorn Capital	Ka	1.00%	1.14%	$1,111,864	$1,244,166	$2,721,150
11	Foodcorn Capital	Kb	1.00%	1.14%	$1,599,972	$1,856,406	$3,842,778
12	Capital Stock	K	1.00%	1.14%	$2,711,836	$3,100,572	$6,563,928
13	Profit	R	1.00%	1.41%	$135,592	$19,623	$477,759
14	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
15	Profitability Gap	a	na	na	0.0%	-4.4%	2.3%
16	Foodcorn Supplied	Qs	1.00%	1.15%	115,122	133,482	274,059
17	Price Level	p	0.00%	0.00%	1.000	0.964	1.019
18	Inflation Rate	gp	na	na	0.0%	-5.3%	0.0%
19	**Constants**						
20	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
21	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
22	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
23	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
24	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
25	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
26	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
27	Foodcorn Retained	Qf	1.00%	1.00%	**4,878**	6,641	13,194

Table 5.6 above displays the economic effects of the same Misallocation Scenario *via* a comparison of spreadsheet columns C and D. As in Model A, the rg percentages concern the reference *steady* state and the ag percentages concern the actual traverse path. Neglecting the constants, almost all ag percentages are positive, indicating that most of the population is better off with than without the traverse. Well before year 100, the economy's initial year-30 steady state is reasserted, in that the profitability gap is again at its a = 2.3% pa value. However, this particular steady state exhibits a slightly higher growth rate of gQ = 1.14% pa *versus* the original gQ = 1% pa. Thus the traverse due to a misallocation of seedcorn in year 30 actually has *improved* the situation of most residents in the growing economy. As for the farmers, while their profits (R) are growing slightly faster, there is some diminution in several other ag percentages that impinge upon their welfare, viz. r, rr, rs, mn, and m.

Figures 5.4a and 5.4b below show that it takes 33 years for the economy to adjust to the Misallocation Scenario and traverse back onto its original steady-state dynamic path.

Figure 5.4a - High Values



For the steady state, Figure 5.4a above broadly reproduces the stationary-state traverse behaviour displayed in Figure 5.3a above.

Figure 5.4b - Percentages

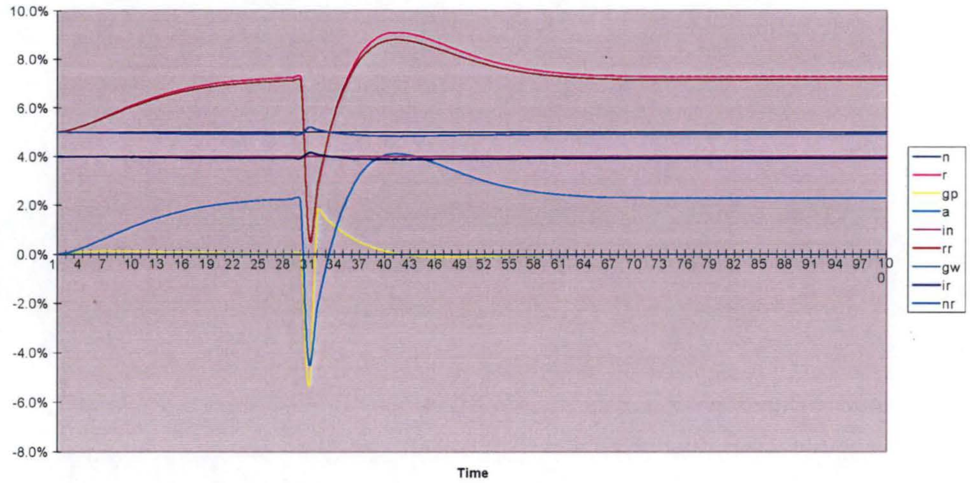


Figure 5.4b above shows that the extent of *overshooting* is not as great for gp, a, r, and rr as it was in the stationary-state traverse.

5.3 Model C

5.3.1 Structural Form

Model C differs from Model B in that equation (F) in Table 5.7 below now determines the money wage endogenously, rather than w being fixed as the unexplained constant (x). The new money wage equation (F) is based on the notion that the relative bargaining strengths of farmers and workers may change over time. As each new year opens, these groups commence wage bargaining with full knowledge of the previous year's wage rate (wo) and inflation rate (gpo), plus the current labour supply situation on each farm.

A second initial value (II) must be added to capture the wage rate in year zero (wz = \$200 per worker pa), this being a salient feature of the economy's history up to the base year. Identity (11) defines a new variable, viz. the employment rate ($e = L / \eta$), which requires the economy's level of employment to be limited, for the first time, by the new workforce constant (f), viz. η = 16,000 workers.

Recall that identity (9) is the price level (p ratio) and identity (10) is the inflation rate (gp% pa). The latter is the growth rate of the former and its one-year lagged value (gpo% pa) now appears on the right-hand side of the new money wage equation (F), together with the new identity (11), i.e. the employment ratio (e) discussed above.

Each of these two explanators of the money wage is multiplied by its own coefficient, viz. constant (g) for the employment ratio and constant (h) for the lagged inflation rate. The arbitrarily-assigned values of these constants are $\varepsilon = 4$ and $\rho = 12$, respectively.

Table 5.7 – Structural Form of Model C

Equations			
Corn Produced	$Q = \theta\ Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = (1 + \phi\ a)\ Qio$	sacks pa	(B)
Employment	$L = Q / \lambda$	workers	(C)
Profit Rate	$r = R / K$	percent pa	(D)
Corn Price	$P = W / Qso$	\$/sack	(E)
Money Wage	$w = wo + \varepsilon\ (e - 1) + \rho\ gpo$	\$/worker pa	(F)
Identities			
Wage Bill	$W = w\ L$	dollars pa	(1)
Seedcorn Capital	$Ka = P\ Qio$	dollars	(2)
Foodcorn Capital	$Kb = P\ Qso / \kappa$	dollars	(3)
Capital Stock	$K = Ka + Kb$	dollars	(4)
Profit	$R = P\ Q - W - Ka$	dollars pa	(5)
Normal Profit Rate	$n = i + \varphi$	percent pa	(6)
Profitability Gap	$a = r - n$	percent pa	(7)
Foodcorn Supplied	$Qs = Q - Qi - Qf$	sacks pa	(8)
Price Level	$p = P / Pz$	ratio	(9)
Inflation Rate	$gp = (p / po) - 1$	percent pa	(10)
Employment Rate	$e = L / \eta$	ratio	(11)
Constants			
Reaction Coefficient	ϕ = **0.4388**	ratio	(a)
Seedcorn Yield	θ = **4**	sacks/sack pa	(b)
Labour Productivity	λ = **10**	sacks/worker pa	(c)
Risk Premium	φ = **1.0**	percent pa	(d)
Capital Turnover	κ = **2.0**	ratio	(e)
Workforce	η = **16,000**	workers	(f)
Employment Rate Coefficient	ε = **0.3**	ratio	(g)
Inflation Rate Coefficient	ρ = **0.1**	ratio	(h)
Interest Rate	i = **4.0**	percent pa	(y)
Foodcorn Retained	Qf = **4878**	sacks pa	(z)
Initial Values			
Seedcorn Invested	Qiz = **40,000**	sacks pa	(I)
Wage Rate	wz = **200.00**	\$/worker pa	(II)

Both farmers and workers know the supply and demand for labour, hence also what the current year's employment ratio (e) is most likely to be. Farmers have Qso sacks of seedcorn stored, so they know what size crop to expect and how much labour will be needed to raise it. It was the workers who carried these same sacks into the barns, thus they hold the same information. Both sides also know the size of the economy's workforce. Opening money wage rate offers by individual farmers are adjusted during the bargaining process until all have secured the labour they need.

When the economy-wide demand for labour is strong, individual farmers raise their money wage offers in an attempt to "poach" workers from other farms or tempt them to work longer hours. But in a soft labour market situation, individual farmers lower their offers. It is assumed that unemployed workers and their families survive on the charity of relations and friends who *are* in gainful employment. If gpo = 0 (zero inflation) and e = 1.0 (full employment), the current average wage rate remains at w = wo. Given the coefficients ε and ρ, there also exist *combinations* of e and gpo that bring about the same result.

Negative inflation (which lowers w) and overfull employment (which raises w) are definite possibilities. A situation of overfull employment (e > 1.0) is handled by offering paid overtime *hours*, which are accounted for in the model as L > η *workers*. If the standard working day is eight hours, then multiplying by eight readily converts these (L – η) "virtual workers" to actual overtime hours. These excess hours (with associated wage payments) are distributed in some more or less equitable fashion over the economy's available workforce. A new aggregate, the unemployment rate, is defined as u = [1 – e]%. If positive, there is unemployment; if negative, there is overfull employment.

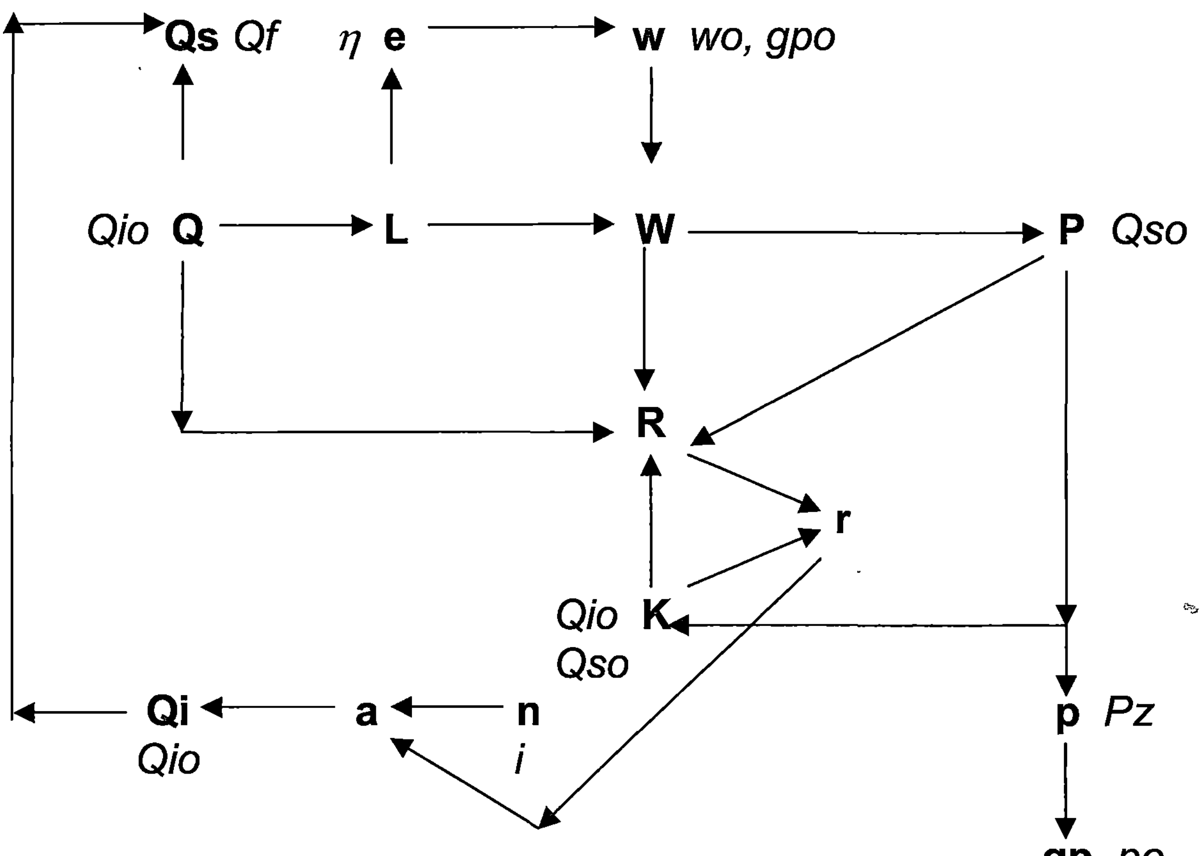
5.3.2 Corn-Credit Economy

The interrelatedness of this recursive dynamic system of structural equations is displayed as a flowchart in Figure 5.5 below. By comparison with the flowchart for Model B in Figure 5.1 above, a labour market module has been added to determine this year's money wage (w) endogenously, rather than simply imposing it as a Roman-letter constant. In this module, w is determined by last year's money wage (wo) being raised (lowered) according as whether the current employment ratio (e) is higher (lower) than one and/or the lagged inflation rate is high (low).

The vertical arrow from L determines e in conjunction with the exogenous workforce, η. The horizontal arrow from e determines w in conjunction with the previous year's money wage (wo) and rate of inflation (gpo) – the lagged value of gp, which appears in the bottom right-hand corner of the flowchart. Two arrows point to the wage bill (W), indicating that it is formed as the product of this year's money wage (w) and employment (L).

This means that workers now are able to raise a Robinsonian "inflation barrier" against excessively high rates of capital accumulation (seedcorn invested). Higher retentions of seedcorn (Qio) push up production (Q > Qo), hence also the economy's demand for labour (L > Lo), employment ratio (e > eo) and wage bill (W > Wo). This occurs in the face of lower foodcorn supplied (Qso), forcing up the corn price (P > Po), the price index (p > po), the inflation rate (gp > gpo), and finally the money wage (w > wo), because both e and gpo have risen.

Figure 5.5 – Flowchart of Model C



5.3.3 Reduced Form

As before, Mathematica is used to eliminate all identities before solving Model C. The resulting reduced form is shown in Table 5.8 below, with nothing on the right-hand side of its equations but the *known* values of its parameters and predetermined variables. Nine of the model's ten constants and all four of its five lagged endogenous variables are present.

This reduced form exhibits only two differences from that of Model B, viz. the money wage equation (F) has been added and, not surprisingly, the corn price equation (E) has had part of its former numerator (the w term) replaced by the right-hand side of the new money wage equation. The former denominator of this corn price equation (λQso) has been retained.

Table 5.8 – Reduced Form of Model C

Corn Produced	$Q = \theta Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = \{1 - \phi\,(i+\varphi) + \phi\dfrac{(\theta - 1)Qio - Qso}{Qio + Qso/\kappa}\}\,Qio$	sacks pa	(B)
Employment	$L = \theta Qio / \lambda$	workers	(C)
Realised Profit Rate	$r = \dfrac{(\theta - 1)Qio - Qso}{Qio + Qso/\kappa}$	percent pa	(D)
Corn Price	$P = \dfrac{[wo + \varepsilon(\theta Qio/\lambda\eta - 1) + \rho gpo]\theta Qio}{\lambda Qso}$	dollars/sack	(E)
Money Wage	$w = wo + \varepsilon(\theta Qio/\lambda\eta - 1) + \rho gpo$	\$/worker pa	(F)

Lagged seedcorn invested (Qio) appears on the right-hand sides of all six reduced-form equations. Therefore, all six endogenous variables (Q, Qi, L, R, P, and w) are being driven through simulated historical time by the sequence of previous-year values for Qi. The central driving force of Qio is amplified or moderated by the presence of other lagged variables, e.g. Qso appears on the right-hand side of three equations. The wage rate determinants (wo and gpo) appear in two equations.

5.3.4 Stationary State

No action is needed to maintain the Model B stationary state condition for all 100 columns within Excel, in order to make Model C "just-determined". With the workforce constant set at η = 16,000 workers, and no change to the Model B determinants of employment (L = 16,000 workers), there is nothing to disturb the economy's tranquillity. So, with the employment ratio being maintained at e = 1.0 (and the lagged inflation rate at gpo = 0% pa), the money wage does not change (w = wo). Thus the results obtained from Model B, including r = n = 5% pa and P = \$27.80 per sack, continue to characterise each year for a century of simulated historical time.

Table 5.9 below displays years 0, 31 and 100 of this stationary state, as simulated in the Cstat spreadsheet file. This reference solution constitutes the starting point for all subsequent Model C computer runs. The numbers in column E (year zero) are replicated in all 100 subsequent columns, thereby forming rows of stationary values for the 17 endogenous variables and the ten constants.

Table 5.9 – Model C Stationary State

	A	B	C	D	E	F	G
1	**C - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	0 00%	160,000	160,000	160,000
4	Seedcorn Invested	Qi	0.00%	0.00%	**40,000**	40,000	40,000
5	Employment	L	0.00%	0 00%	16,000	16,000	16,000
6	Profit Rate	r	0.00%	0.00%	5.0%	5.0%	5.0%
7	Corn Price	P	0.00%	0.00%	$27.80	$27.80	$27 80
8	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
9	**Identities**						
10	Wage Bill	W	0.00%	0 00%	$3,200,000	$3,200,000	$3,200,000
11	Seedcorn Capital	Ka	0.00%	0.00%	$1,111,864	$1,111,864	$1,111,864
12	Foodcorn Capital	Kb	0 00%	0 00%	$1,599,972	$1,599,972	$1,599,972
13	Capital Stock	K	0.00%	0.00%	$2,711,836	$2,711,836	$2,711,836
14	Profit	R	0.00%	0.00%	$135,592	$135,592	$135,592
15	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
16	Profitability Gap	a	na	na	0.0%	0.0%	0.0%
17	Foodcorn Supplied	Qs	0.00%	0 00%	115,122	115,122	115,122
18	Employment Ratio	e	0.00%	0.00%	1.000	1.000	1.000
19	Price Level	p	0 00%	0 00%	1 000	1.000	1.000
20	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
21	**Constants**						
22	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0 4388
23	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
24	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
25	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1 0%
26	Capital Turnover	κ	0.00%	0 00%	**2**	2	2
27	Workforce	η	0.00%	0.00%	**16,000**	16,000	16,000
28	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
29	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
30	Interest Rate	ii	0 00%	0.00%	**4.0%**	4 0%	4.0%
31	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

As with Model B, these all trace out horizontal or flatline graphs, when plotted against 100 years of simulated historical time, so there is no need to reproduce them here.

As in Model B, the farmers are fulfilling their long-period expectations, while simultaneously realising their opportunity cost of capital (re = r = ro = n = 5% pa). They experience no surprises as the simulated years pass. Faced with a consistent sequence of zero profitability gaps, farmers keep on investing Qi = Qio = 40,000 sacks pa of seedcorn after each harvest, which ensures that tranquil conditions prevail throughout the corn-credit economy for a full century.

5.3.5 Steady State

A particular growth path is sought, along which corn production increases at the exponential rate of gQ = 1% pa over at least 70 years of simulated historical time, i.e. between years 31 and 100. As before, this goal is achieved by making foodcorn retained grow by gQf = 1% pa.

With corn production increasing at 1% pa, this implies that employment also grows at gL = 1% pa. Therefore, the Model C workforce also must grow by $g\eta$ = 1% pa, so as to keep the employment ratio at e = 1.0, thereby preserving full employment. Robinson (1962, pp 52-53) has described this particular growth path as "a golden age", in which "near full employment is maintained" and "labour *harmony* may be said to prevail".

However, the corn-credit economy now is undergoing a radical change in the age-structure of its workforce, this transition being the demographic equivalent of an economic traverse. To meet the increasing labour requirements (gL = 1% pa) of the steady states generated in Models A and B, it was assumed that *gastarbeiters* (average-productivity, migrant workers) were sourced from some neighbouring economy experiencing unemployment. Access to this pool of foreign labour enabled average labour productivity to remain constant at λ = 10 sacks/worker pa.

In Model C, however, the economy is constrained to meet increasing labour requirements from its own growing workforce, but this implies a change in labour productivity. Specifically, λ must *fall* during the demographic transition as an influx of young, inexperienced recruits more than offsets the outflux of older, high-productivity retirees. Eventually the age-composition again will stabilise, though at a lower average productivity figure. It is assumed this population process takes 20 years, during which time labour productivity declines smoothly from λ = 10 to λ = 8.884 sacks/worker pa. Twenty years is consistent with empirical studies of demographic transitions where the workforce's age-composition stabilises at a constant ratio of young:old members that is higher than before.

Table 5.10 displays the growth rates of this 70-year steady state, as simulated in the Csted spreadsheet file. Also, the values of all variables are displayed for years 0, 1, 31 and 100. One can see that the physical variables Q, Qi, L, Qs, η, and Qf all grow at 1% pa, as do the money and real values of W, K, R, Y, Rg, C, S, and I. All other variables (including the realised profit rate) experience zero growth over the final 70-year period. Labour productivity attains its new and lower level of λ = 8.884 sacks/worker pa *after* the 20-year demographic transition.

Sustained growth of gQf = $g\eta$ = 1% pa during the initial 30-year period endogenously lifts the realised profit rate from r = 5% pa to r = 7.3% pa, thus opening up a profitability gap of a = 2.3% above the normal profit rate of n = 5% pa. It is the constancy of this profitability gap which is responsible for the corn-credit economy's smooth exponential growth for most of the century.

Table 5.10 – Model C Steady State

	A	B	C	D	E	F	G
1	**C - STEADY STATE**	**sn**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.00%	160,000	193,379	384,282
4	Seedcorn Invested	Qi	1.00%	1.00%	**40,000**	48,824	97,031
5	Employment	L	1.00%	1.00%	16,000	21,767	43,256
6	Realised Profit Rate	r	0.00%	0.00%	5 0%	7.3%	7.3%
7	Corn Price	P	0.00%	0 00%	$27 80	$31.97	$31.95
8	Money Wage	w	0.00%	0.00%	**$200.00**	$200.61	$200.45
9	**Identities**						
10	Wage Bill	W	1.00%	1.00%	$3,200,000	$4,366,665	$8,670,578
11	Seedcorn Capital	Ka	1.00%	1 00%	$1,111,864	$1,545,799	$3,069,863
12	Foodcorn Capital	Kb	1.00%	1.00%	$1,599,972	$2,183,294	$4,335,214
13	Capital Stock	K	1 00%	1 00%	$2,711,836	$3,729,094	$7,405,077
14	Profit	R	1.00%	1.00%	$135,592	$270,733	$539,011
15	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
16	Profitability Gap	a	na	na	0.0%	2 3%	2 3%
17	Foodcorn Supplied	Qs	1.00%	1 00%	115,122	137,914	274,057
18	Employment Ratio	e	0.00%	0 00%	1.000	0.999	1 000
19	Price Level	p	0.00%	0 00%	1.000	1.150	1 150
20	Inflation Rate	gp	na	na	0 0%	0 0%	0 0%
21	**Constants**						
22	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
23	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
24	Labour Productivity	λ	0.00%	0.00%	**10**	9	9
25	Risk Premium	φ	0 00%	0.00%	**1.0%**	1 0%	1.0%
26	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
27	Workforce	η	1.00%	1.00%	**16,000**	21,781	43,277
28	Employment Wage Coefficient	ε	0 00%	0.00%	**4**	4	4
29	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
30	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
31	Foodcorn Retained	Qf	1 00%	1.00%	**4,878**	6,641	13,194

In Model B, the higher profit rate is associated with a corn price that increases by less than two percent. However, in Model C the corn price rises from P = $27.80 to P = $31.95 per sack, an increase of almost 15 percent. The price rise is largely attributable to increased labour input per sack of corn, due to the lower average labour productivity (output/worker) following the demographic transition.

Much of this dynamic behaviour may be viewed as Figures 5.6a through 5.6c below. (The graphs of Qf, L, Qi, Qs, Q, and R are not shown because these time series exhibit similar behaviour to that of Model B, in that the profit or net surplus variable increases faster than the physical variables during the traverse from stationary to steady state.)

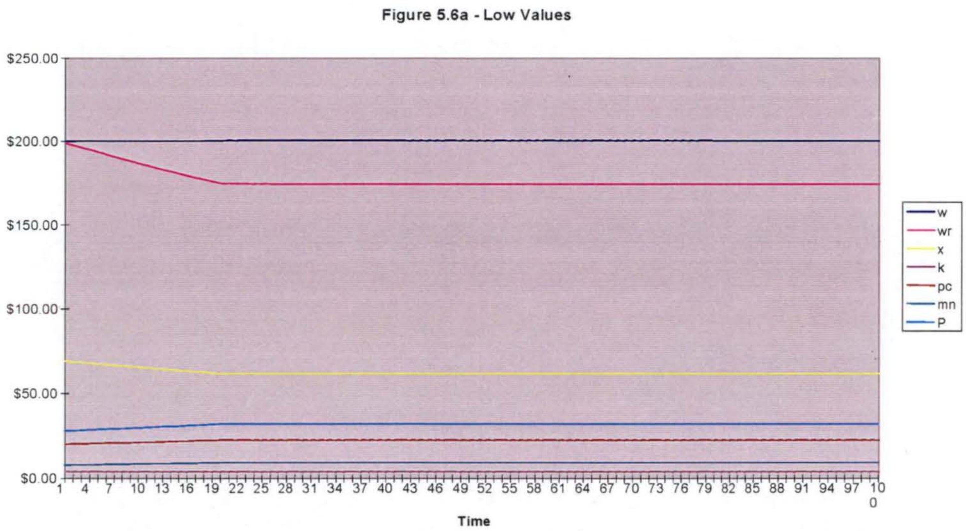


Figure 5.6a above shows how the real wage (wr) falls due to diminishing average labour productivity pushing up prime cost (pc) and the corn price (P). During this period, the falling real wage is associated with a capital-labour ratio that diminishes by 12.6 percent, from x = 69.5 to x = 61.7 sacks/worker, but there is no causation either way. *Both* phenomena are due to the demographic transition. This reduces labour productivity but has no effect on seedcorn yield, which remains constant at θ = 4 sacks of corn produced per sack of seedcorn invested. (Alternatively, the capital-output ratio remains at v = 0.25 sacks/sack.)

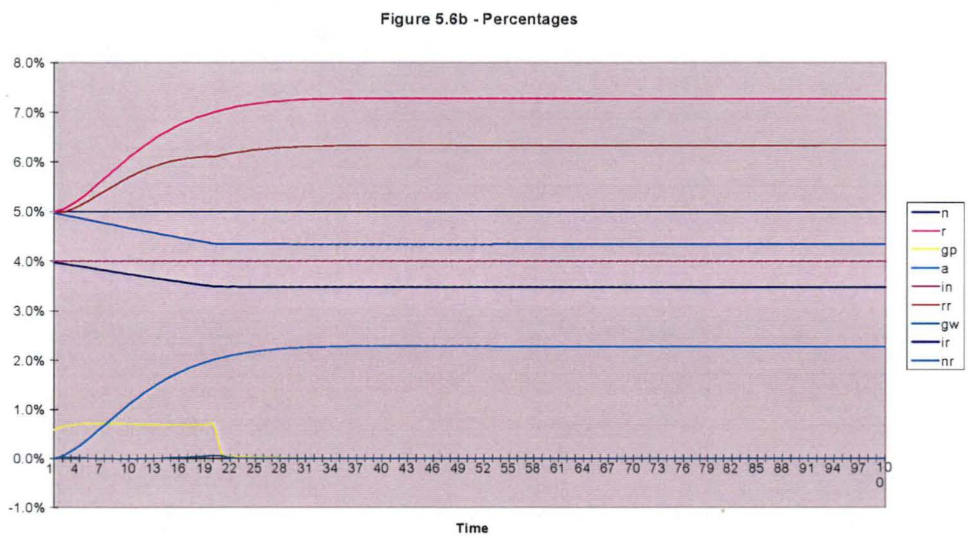


Figure 5.6b above shows the inflation rate largely constant at gp = 0.7% pa during the smooth 20-year demographic transition, afterwards falling to gp = 0% pa. Price inflation adversely affects farmers, not only workers; their real profit rate stabilises at rr = 6.3% pa,

well below its fully-adjusted nominal value of r = 7.3% pa. As in Model B, the realised profit rate and its associated profitability gap (a% pa) take up to 30 years to achieve their fully-adjusted levels.

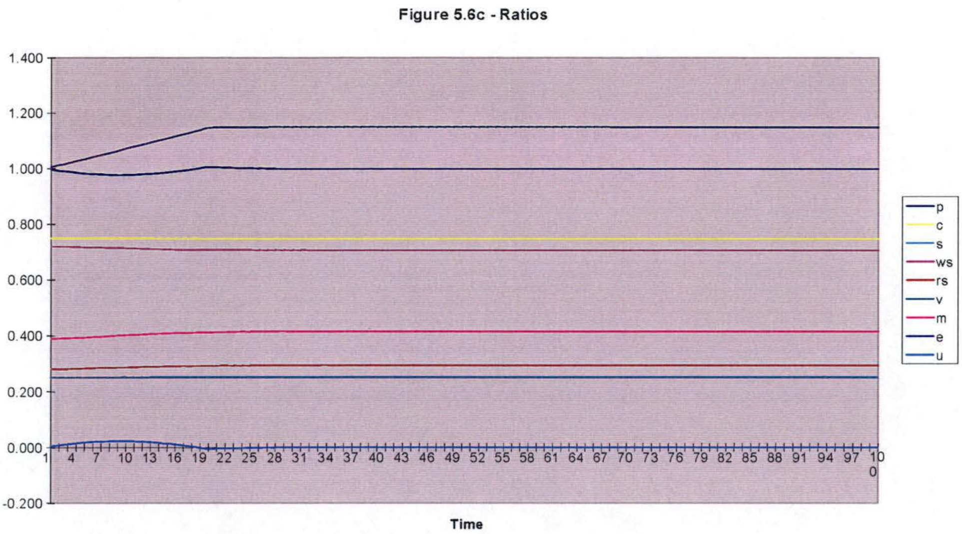


Figure 5.6c above shows the price level rising from p = 1.000 to p = 1.150 while the formerly stationary economy is adjusting to its new steady state. After some initial low-level unemployment, rising investment ensures that employment (L) eventually is restored to equality with the workforce (η) so that the unemployment rate falls back to u = 0% pa. There also is a slight fall in the share of wages (ws) and a complementary rise in the share of gross surplus (rs).

5.3.6 Specimen Traverses

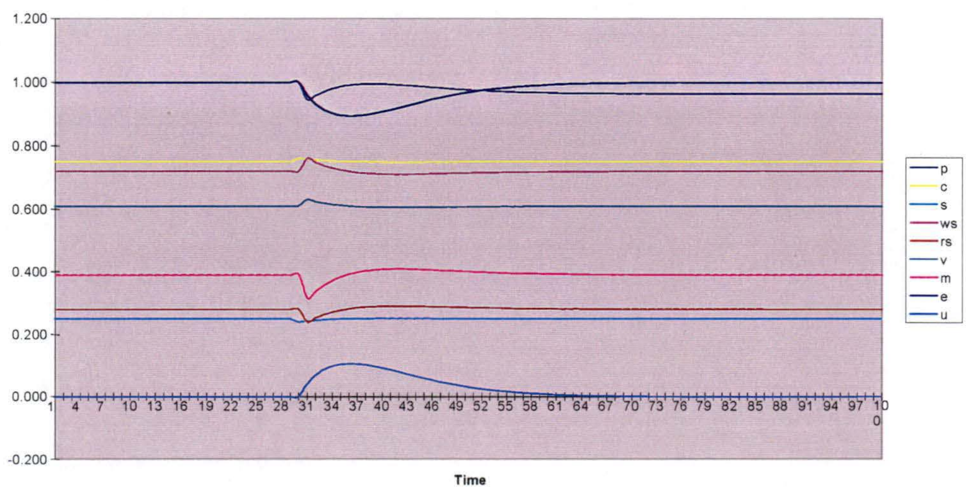
As before, two specimen traverses are initiated, the first from the stationary-state basecase and the second from the steady-state basecase. Both traverses are sparked off by a sudden, unexpected, unplanned, four percent drop in seedcorn invested during year 30. As before, this Misallocation Scenario involves four percent of the sacks of seedcorn (already earmarked for investment as as circulating capital at the end of year 30) being *mistakenly* released onto the subsequent year's weekly markets, for sale as foodcorn.

Table 5.11 below shows how the initial *stationary* state is disrupted by the unintended misallocation. While all physical and real ad percentages register identical falls to those of Model B, the price and nominal percentage falls are far larger. The newly-flexible money wage (w) falls by 3.03% and the corn price (P) by 2.92%. With w falling more than P over 70 years, this lifts the real profit rate (rr) by 5.67%. Unemployment during the traverse is responsible for pushing down the money wage from $200 to $192.85 once the economy has fully adjusted to the year-30 shock.

Table 5.11 – Model C Misallocation Scenario from Stationary State

	A	B	C	D	E	F	G
1	**C - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	-2.41%	160,000	153,600	160,002
4	Seedcorn Invested	Qi	0.00%	-2.35%	**40,000**	37,292	40,000
5	Employment	L	0.00%	-2.41%	16,000	15,360	16,000
6	Realised Profit Rate	r	0.00%	2.72%	5.0%	-1.6%	5.0%
7	Corn Price	P	0.00%	-2.92%	$27.80	$26.30	$26.80
8	Money Wage	w	0.00%	-3.03%	**$200.00**	$199.84	$192.85
9	**Identities**						
10	Wage Bill	W	0.00%	-5.39%	$3,200,000	$3,069,542	$3,085,639
11	Seedcorn Capital	Ka	0.00%	-5.29%	$1,111,864	$1,009,839	$1,072,126
12	Foodcorn Capital	Kb	0.00%	-5.39%	$1,599,972	$1,534,745	$1,542,793
13	Capital Stock	K	0.00%	-5.35%	$2,711,836	$2,544,584	$2,614,918
14	Profit	R	0.00%	-2.81%	$135,592	-$40,025	$130,738
15	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
16	Profitability Gap	a	na	na	0.0%	-6.6%	0.0%
17	Foodcorn Supplied	Qs	0.00%	-2.53%	115,122	111,430	115,124
18	Employment Ratio	e	0.00%	-2.41%	1.000	0.960	1.000
19	Price Level	p	0.00%	-2.92%	1.000	0.946	0.964
20	Inflation Rate	gp	na	na	0.0%	-5.4%	0.0%
21	**Constants**						
22	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
23	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
24	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
25	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
26	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
27	Workforce	η	0.00%	0.00%	**16,000**	16,000	16,000
28	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
29	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
30	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
31	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

Figure 5.7 - Ratios



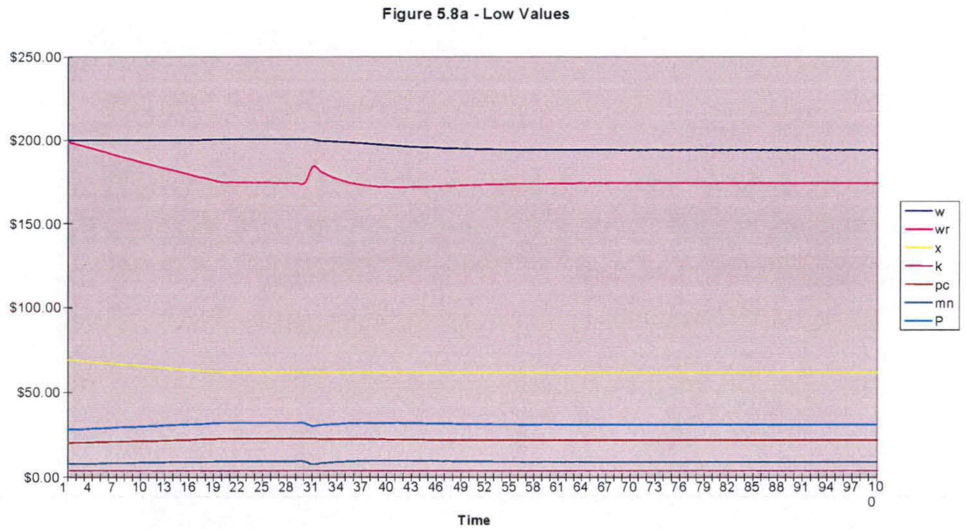
The High Values and Percentages graphs are not shown because they display near-identical behaviour to the comparable Model B graphs, viz. Figures 5.4a and 5.4b above. However, the Ratios graph for Model C contains important labour market information. Figure 5.7 above shows how the unemployment rate peaks at u = 10.6% during the traverse, so that the employment ratio simultaneously troughs at e = 0.894.

Table 5.12 below shows how the initial *steady* state is disrupted by the unintended misallocation. While all physical and real ag percentages register identical rises to those of Model B, the price and nominal *growth rates* are lower, although their fully-adjusted, post-traverse *levels* are far higher. This is because the corn price has risen to P = \$30.95 *versus* P = \$28.32 in Model B, reflecting lower labour productivity following the Model C demographic transition.

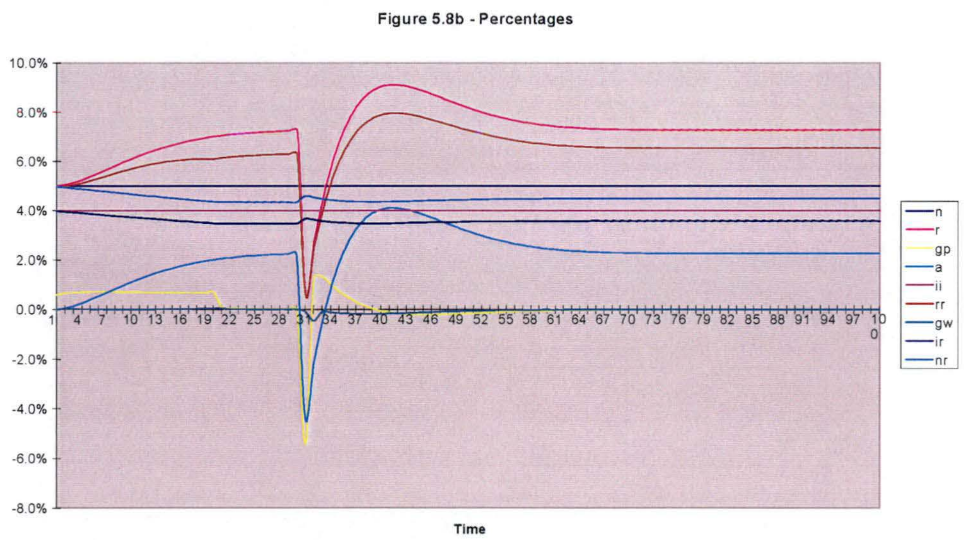
Table 5.12 – Model C Misallocation Scenario from Steady State

	A	B	C	D	E	F	G
1	**C - STEADY STATE**	**sn**	**ag**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.14%	160,000	185,644	384,284
4	Seedcorn Invested	Qi	1.00%	1.14%	**40,000**	45,522	97,032
5	Employment	L	1 00%	1.14%	16,000	20,897	43,256
6	Realised Profit Rate	r	0.00%	0.27%	5.0%	0.6%	7.3%
7	Corn Price	P	0.00%	-0.03%	\$27.80	\$30.24	\$30 95
8	Money Wage	w	0 00%	-0.03%	**\$200.00**	\$200 45	\$194.13
9	**Identities**						
10	Wage Bill	W	1 00%	1.11%	\$3,200,000	\$4,188,657	\$8,397,261
11	Seedcorn Capital	Ka	1.00%	1.11%	\$1,111,864	\$1,403,598	\$2,973,085
12	Foodcorn Capital	Kb	1.00%	1 11%	\$1,599,972	\$2,094,292	\$4,198,558
13	Capital Stock	K	1.00%	1.11%	\$2,711,836	\$3,497,890	\$7,171,642
14	Profit	R	1.00%	1.38%	\$135,592	\$22,137	\$521,992
15	Normal Profit Rate	n	0.00%	0 00%	5 0%	5 0%	5.0%
16	Profitability Gap	a	na	na	0.0%	-4.4%	2.3%
17	Foodcorn Supplied	Qs	1.00%	1.15%	115,122	133,482	274,059
18	Employment Ratio	e	0.00%	0.14%	1 000	0.959	1.000
19	Price Level	p	0.00%	-0.03%	1.000	1.088	1.113
20	Inflation Rate	gp	na	na	0.0%	-5.4%	0.0%
21	**Constants**						
22	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
23	Seedcorn Yield	θ	0.00%	0 00%	**4**	4	4
24	Labour Productivity	λ	0 00%	0 00%	**10**	9	9
25	Risk Premium	φ	0 00%	0.00%	**1.0%**	1.0%	1.0%
26	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
27	Workforce	η	1.00%	1.00%	**16,000**	21,781	43,277
28	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
29	Inflation Wage Coefficient	ρ	0.00%	0 00%	**12**	12	12
30	Interest Rate	ii	0.00%	0 00%	**4.0%**	4.0%	4.0%
31	Foodcorn Retained	Qf	1 00%	1.00%	**4,878**	6,641	13,194

Figures 5.8a through 5.8c below show that it takes up to 34 years for the economy to adjust to the Misallocation Scenario and traverse back onto a steady-state dynamic path.

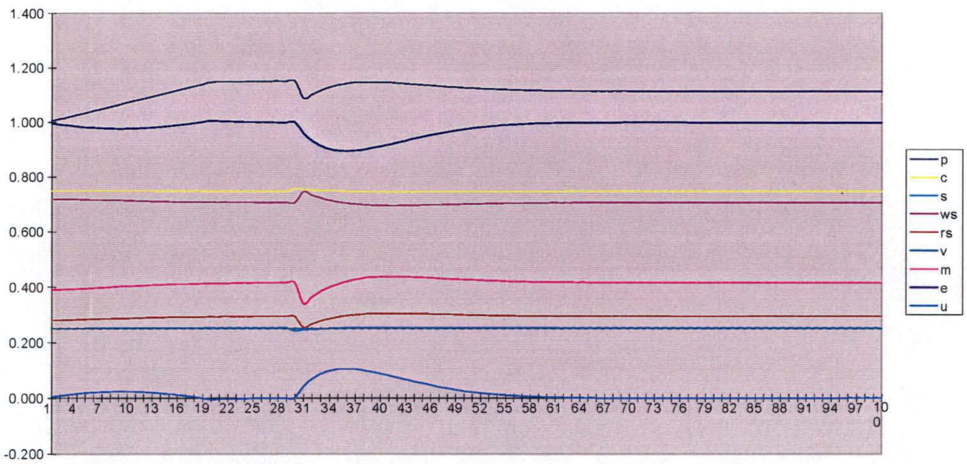


In Figure 5.8a above, it is evident that the productivity-sapping demographic transition lowers the capital-labour ratio while raising the corn price, thus reducing the real wage.



Although Figure 5.8b above shows similar dynamic behaviour of the Model C profit rate and profitability gap to that of Model B, it is interesting because it highlights the gp = 0.7% pa inflation associated with the (smooth) demographic transition and the deflation/inflation that accompanies the (overshooting) economic traverse.

Figure 5.8c - Ratios



The net effect (a substantial rise in the price level, p) is shown in Figure 5.8c above, together with the same sort of peaking of the unemployment rate – and troughing of the employment ratio – that occurs during the stationary-state traverse discussed above.

5.4 Model D

5.4.1 Structural Form

Model D differs from Model C in that equation (G) in Table 5.13 below now determines the interest rate endogenously, rather than i being fixed as an unexplained constant. In addition, the corn price equation (E) is modified to reflect a new assumption, viz. that all interest income (J dollars pa) is spent by its recipients on foodcorn at the weekly markets, just like all wage income (W dollars pa). Moreover, a third initial value (III) must be added to capture the interest rate in year zero (iz = 4% pa), this being a salient feature of the economy's history up to the base year.

Four new identities are required to specify Model D and one existing identity must be modified. Identity (12) is the farmers' average interest-bearing debt outstanding over the current year (D dollars), i.e. a $1/\mu$ fraction of their annual wage bill, with μ = 52 when wages are paid fortnightly. This enables identity (13) to define the farmers' overall debt:assets ratio (d), i.e. what they *owe* the banks (D dollars), divided by the current value of seedcorn and foodcorn capital which they themselves *own* (Ko dollars). In turn, this allows identity (14) to be defined as the growth rate (gd% pa) of the farmers' debt:assets ratio.

Identity (15) is the total interest bill (J dollars pa) paid by farmers to bankers. It services the working capital (D dollars) that farmers must borrow each fortnight to meet their payrolls. Finally, the existing profit identity (5) is modified by subtracting the interest bill, J dollars pa being a new cost of production faced by farmers for the first time in the Model D economy. Identity (5) has been modified to recognise that the farmers' sales revenue (P Q dollars pa) must necessarily cover the value of their seedcorn capital consumption and their wage bill, plus interest paid to the bankers, before they can declare a profit (R dollars pa).

The interest rate equation now determines the (nominal) interest rate on bank loans of money for working capital (i% pa) endogenously. Equation (G) shows the influence of growth in the debt:assets ratio (gd% pa) on the interest rate charged by banks. This explanator is multiplied by its coefficient ($\delta = 0.1$), the new constant (i) in Table 5.13 below. Whenever gd = 0, the current average interest rate will remain unchanged at i = io. Growth in the farmers' debt:assets ratio is used by bankers as an indicator of the risk that further loans might not be serviced on time or may even be repudiated. Faster (slower) growth of d means this risk increases (decreases) over time.

In this corn-credit economy with a flexible interest rate (i% pa), farmers build up their opportunity cost of capital or target rate of return or normal profit rate on a mixed objective/subjective basis. They face the ruling rate of interest on bank loans as an *objective* market-determined price of money, one that already includes a component to cover the risk of default on servicing or repayment of debt. Farmers then add the *subjective* risk premium (φ = 1% pa) that they believe characterises their own profit-yielding investments in corn production (as opposed to the bankers' risk of making interest-yielding placements in money loans) to determine the true opportunity cost of capital sunk into the farming industry, viz. n = $(i + \varphi)$% pa.

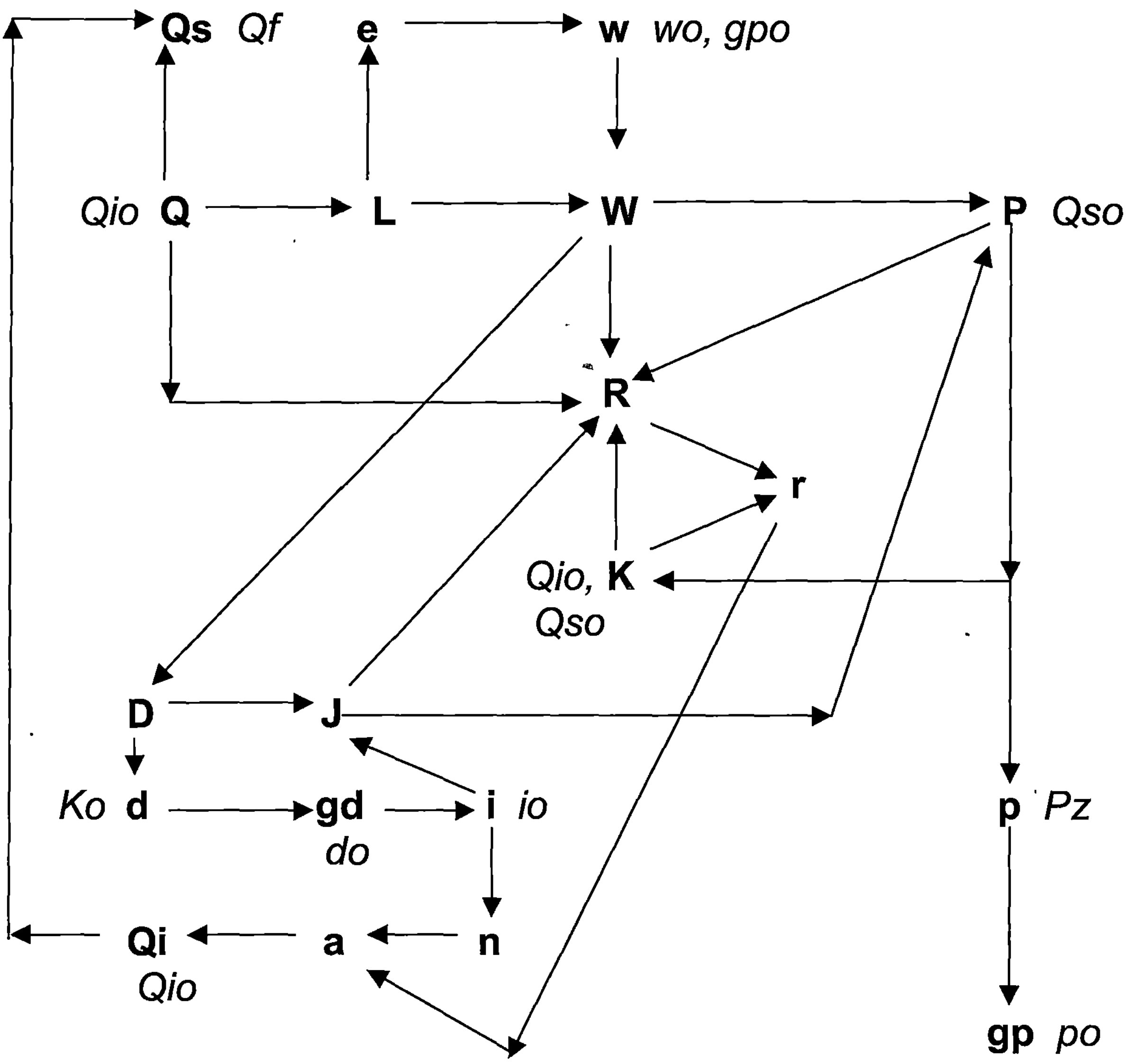
Table 5.13 – Structural Form of Model D

Equations			
Corn Produced	$Q = \theta\, Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = (1 + \phi\, a)\, Qio$	sacks pa	(B)
Employment	$L = Q / \lambda$	workers	(C)
Profit Rate	$r = R / K$	percent pa	(D)
Corn Price	$P = (W + J) / Qso$	$/sack	(E)
Money Wage	$w = wo + \varepsilon\,(e - 1) + \rho\, gpo$	$/worker pa	(F)
Interest Rate	$i = io + \delta\, gd$	percent pa	(G)
Identities			
Wage Bill	$W = w\, L$	dollars pa	(1)
Seedcorn Capital	$Ka = P\, Qio$	dollars	(2)
Foodcorn Capital	$Kb = P\, Qso / \kappa$	dollars	(3)
Capital Stock	$K = Ka + Kb$	dollars	(4)
Profit	$R = P\, Q - W - Ka - J$	dollars pa	(5)
Normal Profit Rate	$n = i + \varphi$	percent pa	(6)
Profitability Gap	$a = r - n$	percent pa	(7)
Foodcorn Supplied	$Qs = Q - Qi - Qf$	sacks pa	(8)
Price Level	$p = P / Pz$	ratio	(9)
Inflation Rate	$gp = (p / po) - 1$	percent pa	(10)
Employment Rate	$e = L / \eta$	ratio	(11)
Average Debt	$D = W / \mu$	dollars	(12)
Debt:Assets Ratio	$d = D / Ko$	ratio	(13)
D:A Ratio Growth Rate	$gd = (d / do) - 1$	percent pa	(14)
Interest Bill	$J = i\, D$	dollars pa	(15)
Constants			
Reaction Coefficient.	ϕ = **0.4388**	ratio	(a)
Seedcorn Yield	θ = **4**	sacks/sack pa	(b)
Labour Productivity	λ = **10**	sacks/worker pa	(c)
Risk Premium	φ = **1.0**	percent pa	(d)
Capital Turnover	κ = **2.0**	ratio	(e)
Workforce	η = **16,000**	workers	(f)
Employment Rate Coefficient	ε = **4**	ratio	(g)
Inflation Rate Coefficient	ρ = **12**	ratio	(h)
D:A Ratio Growth Coefficient	δ = **0.01**	ratio	(i)
Wage Bill Turnover	μ = **52**	ratio	(j)
Foodcorn Retained	Qf = **4878**	sacks pa	(z)
Initial Values			
Seedcorn Invested	Qiz = **40,000**	sacks pa	(I)
Wage Rate	wz = **200.00**	$/worker pa	(II)
Interest Rate	iz = **4.0**	percent pa	(III)

5.4.2 Corn-Credit Economy

The interrelatedness of this recursive dynamic system of structural equations is displayed as a flowchart in Figure 5.9 below. By comparison with the flowchart for Model C in Figure 5.5 above, a banking sector module has been added to determine the interest rate (i% pa) endogenously, rather than simply imposing it as a Roman-letter constant. In this module, the bankers raise (lower) last year's interest rate (io% pa) according as whether the farmers' debt:assets ratio growth rate (gd% pa) – an indicator of lender's risk – is high (low) this year.

Figure 5.9 – Flowchart of Model D



The oblique arrow from W to D indicates that farmers finance their fortnightly wage bills by incurring debt to the extent of $D = W / \mu$ dollars, averaged over the crop year. Arrows moving anticlockwise from D to d to gd to i show that bankers raise the nominal interest rate (i > io) on such loans whenever the *security* is impaired by the risk of a higher debt (D) to assets (Ko) ratio, where d > do implies this ratio grows by gd percent.[31] Arrows from D and from i show how the interest bill paid by farmers is calculated as J = i D dollars pa. This amount is shared between depositors for lending their money and bankers for offering the service.

The arrow from J to P indicates that recipients of the economy's interest bill (just like workers who receive its wage bill) spend all the money on foodcorn at the weekly markets. Relative to Model C, this implies a higher corn price because, in Model D, more money (W + J dollars) is offered for the same quantity of foodcorn supplied (Qso sacks). This interest bill (J), being an extra money cost of doing business as a farmer, will reduce the profit realised in agriculture (R = P Q – W – Ka – J dollars), as shown by the arrow from J to R. Finally, the arrow from i to n indicates that the normal profit rate is now an endogenous variable, rather than the sum of two constants.

In Model D, it is the richest farmers who are acting as the bankers, by accepting deposits and advancing this money (in the form of overdrafts) to their fellow farmers, thus allowing the latter to meet their fortnightly payrolls. So, while all farmers still directly retain Qf = 4,878 sacks pa of foodcorn for consumption within their own households, the richest farmer-bankers among them manage to consume *more* because they are *also* participating in the weekly foodcorn markets. These bankers share the economy's interest bill (J dollars pa) with their depositors.

There is no need to know the "spread" between borrowing and lending rates, because bankers and depositors alike are assumed to bid all J dollars of their interest earnings (in competition with the workers bidding all W dollars of their wage earnings) for the available Qso sacks pa of foodcorn released onto the weekly markets. This raises the corn price above the level determined in Models A through C, since these earlier stages represent economies in which no borrowing or lending of money at interest occurs (except informally between farmers at an interest rate fixed by tradition at i = 4% pa).

In their capacity as bankers, the richest farmers are sensitive to changes in the risk that interest and/or principal may not be repaid on time – or even at all. This risk is proxied by the growth rate of the debt:equity ratio (gd% pa). Kalecki (1937) called this the "principle of increasing risk", as noted above. The coefficient (δ) of the debt:assets ratio growth rate is an index of the bankers' degree of sensitivity to lender's risk.

[31] This is an application of Kalecki's (1937) "Principle of Increasing Risk".

Having assets (Ko) as collateral reduces risk, thus the $d = D / Ko$ ratio is preferred to the $D = W / \mu$ average. Yet it is the pace at which d is changing which makes bankers feel less (or more) secure about farmers' prospects for servicing their debts.

The average debt ($D = W / \mu$ dollars) carried by all farmers over the current year is the economy's fortnightly wage bill. Their overdrafts are extinguished every fortnight, after the proceeds of two weekly foodcorn markets have been banked. It is not only ordinary farmers who must bear interest on money committed to corn production. Even the rich farmer-bankers will recognise interest on their fortnightly wage bills as an *opportunity cost* which must be entered into their farm ledger accounts as a charge, otherwise their costs of production would be understated.

Most Post-Keynesians agree with Basil Moore (1988) that "money is credit-created and demand-driven", albeit in the conventional context of money as a medium of exchange and a store of value. In these corn-credit models, by contrast, money is merely a unit of account and a standard of value. Nonetheless, *Moore's dictum* still holds true.

To appreciate why money is *demand-driven*, recall that workers are paid fortnightly throughout each crop year, then spend all their wages at the weekly foodcorn markets. Almost immediately, the farmer-vendors deposit this same money (*qua* sales receipts) in their own bank accounts. This offsets the recent mass withdrawals by workers and extinguishes their own overdrafts on a fortnightly basis.

Thus, at the turn of each fortnight in a stationary-state economy, it could be argued that *money momentarily goes out of existence*. The workers have paid out all their money to the sellers of foodcorn. The farmer-traders have none because all these sales receipts have been used to repay their overdrafts. The bankers have no money either, because all they do is shuffle *other people's money* between depositors and borrowers, earning their "spread" on the turn. In Model D, the economy's entire stock of the money of account dies at the end of one fortnight and is reborn at the start of the next.

Just how much money will be born next *year* depends on the farmers' new wage bill (W dollars), which determines what fresh bank overdraft they must carry ($D = W / \mu$ dollars). The economy's bankers (rich farmers with large stocks of corn, who know how the "money-go-round" works) always stand willing to supply this demand, their only concern being what interest rate they should charge for the service. This interest rate fluctuates with the economy's aggregate debt:assets ratio growth rate, gd% pa being a proxy for lender's risk.

Because the workers spend their wages weekly (soon after the banks have lent those same wages to their employers in the form of overdrafts), there is practically no tension between the supply of money deposits and the demand for money loans. The supply of deposits is purely passive, therefore the active force *driving* money can be nothing other than the farmers' *demand* for loans, in accordance with Moore's dictum.

To appreciate why money is *credit-created*, note that rational workers will have no dealings with any banks that ordinary farmers attempt to set up. All financial systems operate on the basis of trust and this one is no different. The reason the richest farmers are able to attract both depositors and borrowers is their known rock-solid asset-backing, in an economy whose only tangible assets are stocks of corn. By definition, the richest farmers annually commit to storage far more seedcorn and foodcorn capital, subsequently (and consequently) raising far larger crops and storing a lot more corn than ordinary farmers do.

In a stationary state it is obvious that the same "quantity of money" will simply be recreated, year after year. In a steady state, however, a *larger* quantity of money is needed during each new year, but this poses no problem. All money is created by the stroke of a pen, i.e. a rich and trusted farmer-lender (a "banker") simply makes a book-keeping entry in his accounts receivable or "debtors" ledger. Having assessed the "credit-worthiness" of the loan applicant, the banker decides to "extend credit" in the form of an "overdraft facility" at an interest rate appropriate to the degree of lender's risk. By this action, a paper asset is created out of thin air and transferred as a liability to the borrowing farmer's accounts payable or "creditors" ledger. Within one fortnight, the farmer-borrower distributes all this "new money" in favour of each worker whose name appears in the farm's "payroll ledger". Upon receipt of their wage cheques, the farm's workers spend this money on foodcorn, allowing the farmer-trader to return this money to the banker(s) who so recently (and costlessly) created it.

If a poor farmer tried to "create money", that farm's workforce would refuse to work for such low-quality credits accumulating against their names in the farm's payroll ledger. The knowledge that one of the economy's richest farmer-bankers ultimately is guaranteeing their employer's wage bill will boost workers' confidence that such high-quality credits will be acceptable to all vendors of foodcorn when they attempt to purchase foodcorn.

5.4.3 Reduced Form

As before, Mathematica is used to eliminate all identities before solving Model D. The resulting reduced form is shown in Table 5.14 below, with nothing on the right-hand side of its equations but the *known* values of its parameters and predetermined variables. Ten of the model's 11 constants and seven of its eight lagged endogenous variables are present.

This reduced form exhibits only three differences from that of Model C, viz. the interest rate equation (G) has been added while both the seedcorn invested and corn price equations (B and E respectively) have changed. All three equations are long, complex and difficult to interpret.

Lagged seedcorn invested (Qio) appears on the right-hand sides of all seven reduced-form equations. Therefore, all seven endogenous variables (Q, Qi, L, r, P, w, and i) are being driven through simulated historical time by the sequence of previous-year values for Qi. The fact that Qio^2 (and even higher powers, up to the fourth) helps drive seedcorn invested, the corn price and the interest rate indicates a high degree of complexity characterising Model D's cyclical behaviour.

Table 5.14 – Reduced Form of Model D

Corn Produced	$Q = \theta Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = [1 - \phi\{io-\delta-[(\theta-1)Qio-Qso] / (Qio+Qso/\kappa) + \varepsilon\delta\theta^2 Qio^2 / \eta\lambda^2\mu doKo + \delta\theta(wo-\varepsilon+\rho gpo)Qio / \lambda\mu doKo - \varphi\}]\, Qio$	sacks pa	(B)
Employment	$L = \theta Qio / \lambda$	workers	(C)
Realised Profit Rate	$r = \dfrac{(\theta - 1)Qio - Qso}{Qio + Qso/\kappa}$	percent pa	(D)
Corn Price	$P = [\theta Qio\{\varepsilon\theta Qio + \eta\lambda(wo-\varepsilon+\rho gpo)\}\{\varepsilon\delta\theta^2 Qio^2 + \eta\lambda^2\mu(io-\delta+\mu)doKo + \eta\lambda\delta\theta(wo-\varepsilon+\rho gpo)Qio\}] / \eta^2\lambda^4\mu^2 doKoQso$	dollars/sack	(E)
Money Wage	$w = wo + \varepsilon(\theta Qio/\lambda\eta - 1) + \rho gpo$	\$/worker pa	(F)
Interest Rate	$i = \dfrac{\varepsilon\delta\theta^2 Qio^2 + \eta\lambda^2\mu(io-\delta)doKo + \eta\lambda\delta\theta(wo-\varepsilon+\rho gpo)Qio}{\eta\lambda^2\mu doKo}$	percent pa	(G)

The central driving force of Qio is amplified or moderated by the presence of other lagged variables, e.g. Qso appears on the right-hand side of three equations. The wage rate determinants (wo and gpo) appear in four equations and the determinants of the interest rate (io, do and Ko) in three.

5.4.4 Stationary State

No action is needed to maintain the stationary state condition r = n% pa for all 100 columns within Excel, in order to make Model D "just-determined". Relative to Model C, the sole effect of making the interest rate endogenous is an increase in the corn price from P = \$27.80 to P = \$27.82 per sack. This occurs because the recipients of interest (depositors and bankers) are now bringing J dollars pa *more* money to the weekly markets to bid for the *same* volume of lagged foodcorn supplied (Qso) as in Model C. The higher corn price does not raise the profit rate above r = ro = 5% pa, as previously realised by the farmer-traders of Model C; their unit cost of production has risen by precisely the same dollar amount.

The new debt:assets ratio variable floats to d = 0.023 in year zero and remains at this level for the entire simulated century, i.e. it does not grow (gd = gdo = 0% pa). With this growth rate being an explanator of the endogenous interest rate, there can be no change from its initial value of i = io = iz = 4% pa, nor in the normal profit rate of n = no = 5% pa. So, the economy's long-period stationary-state dynamic equilibrium (a = r – n = 0% pa) is not disturbed and tranquillity still prevails.

Table 5.15 below displays years 0, 31 and 100 of this stationary state, as simulated in the Dstat spreadsheet file. This reference solution constitutes the starting point for all subsequent Model D computer runs. The numbers in column E (year zero) are replicated in all 100 subsequent columns, thereby forming rows of stationary values for the 22 endogenous variables and the 11 constants. As with Model C, these all trace out horizontal or flatline graphs, when plotted against 100 years of simulated historical time, so there is no need to reproduce them here.

As in Model C, the farmers are fulfilling their long-period expectations, while simultaneously realising their opportunity cost of capital (re = r = ro = n = 5% pa). They experience no surprises as the simulated years pass. Faced with a consistent sequence of zero profitability gaps, farmers keep on investing Qi = Qio = Qiz = 40,000 sacks pa of seedcorn after each harvest, which explains why there are no changes in the volume of corn production or in sales of foodcorn, relative to Model C.

Table 5.15 – Model D Stationary State

	A	B	C	D	E	F	G
1	**D - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	0.00%	160,000	160,000	160,000
4	Seedcorn Invested	Qi	0.00%	0.00%	**40,000**	40,000	40,000
5	Employment	L	0.00%	0.00%	16,000	16,000	16,000
6	Realised Profit Rate	r	0.00%	0.00%	5.0%	5.0%	5.0%
7	Corn Price	P	0.00%	0.00%	$27.82	$27.82	$27.82
8	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200.00
9	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
10	**Identities**						
11	Wage Bill	W	0.00%	0.00%	$3,200,000	$3,200,000	$3,200,000
12	Seedcorn Capital	Ka	0.00%	0.00%	$1,112,719	$1,112,719	$1,112,719
13	Foodcorn Capital	Kb	0.00%	0.00%	$1,601,203	$1,601,203	$1,601,203
14	Capital Stock	K	0.00%	0.00%	$2,713,922	$2,713,922	$2,713,922
15	Profit	R	0.00%	0.00%	$135,696	$135,696	$135,696
16	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
17	Profitability Gap	a	na	na	0.0%	0.0%	0.0%
18	Foodcorn Supplied	Qs	0.00%	0.00%	115,122	115,122	115,122
19	Employment Ratio	e	0.00%	0.00%	1.000	1.000	1.000
20	Price Level	p	0.00%	0.00%	1.000	1.000	1.000
21	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
22	Average Debt	D	0.00%	0.00%	$61,538	$61,538	$61,538
23	Debt:Assets Ratio	d	0.00%	0.00%	0.023	0.023	0.023
24	D:A Ratio Growth Rate	gd	na	na	0.0%	0 0%	0.0%
25	Interest Bill	J	0.00%	0.00%	$2,462	$2,462	$2,462
26	**Constants**						
27	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4432**	0.4432	0.4432
28	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
29	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
30	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
31	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
32	Workforce	η	0.00%	0.00%	**16,000**	16,000	16,000
33	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
34	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
35	D A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0.1	0.1
36	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
37	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

5.4.5 Steady State

As before, a particular growth path is sought, along which corn production increases at the exponential rate of gQ = 1% pa over at least 70 years of simulated historical time, i.e. between years 31 and 100. As in Model C, this goal is achieved by making foodcorn retained grow by gQf = 1% pa and, in addition, by making the workforce grow by $g\eta$ = 1% pa. Once again, during the inevitable demographic transition, labour productivity declines smoothly from λ = 10 to λ = 8.884 sacks/worker pa over the first two decades.

Table 5.16 below displays the growth rates of this 70-year steady state, as simulated in the Dsted spreadsheet file. Also, the values of all variables are displayed for years 0, 31 and 100. One can see that the physical variables Q, Qi, L, Qs, η, and Qf all grow at 1% pa, although the financial variables D and J grow slightly slower, as do the money variables W, K, R, Y, Rg, C, S, and I. All other variables, including the realised profit rate, experience zero (or slightly negative) growth over the final 70-year period. Labour productivity at the level of λ = 8.884 sacks/worker pa is attained after the 20-year demographic transition.

Table 5.16 – Model D Steady State

	A	B	C	D	E	F	G
1	**D - STEADY STATE**	**sn**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.00%	160,000	192,025	382,926
4	Seedcorn Invested	QI	1.00%	1.00%	**40,000**	48,503	96,689
5	Employment	L	1.00%	1.00%	16,000	21,615	43,104
6	Realised Profit Rate	r	-0.01%	-0.01%	5.0%	7.4%	7.3%
7	Corn Price	P	-0.01%	-0.01%	$27.82	$31.89	$31.67
8	Money Wage	w	-0.01%	-0.01%	**$200.00**	$199.74	$198.45
9	Interest Rate	ii	0 00%	0.00%	**4.0%**	4.0%	4.0%
10	**Identities**						
11	Wage Bill	W	0.99%	0 99%	$3,200,000	$4,317,334	$8,553,914
12	Seedcorn Capital	Ka	0 99%	0.99%	$1,112,719	$1,531,027	$3,031,416
13	Foodcorn Capital	Kb	0.99%	0.99%	$1,601,203	$2,160,298	$4,280,190
14	Capital Stock	K	0.99%	0.99%	$2,713,922	$3,691,326	$7,311,606
15	Profit	R	0.98%	0.98%	$135,696	$272,410	$533,721
16	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
17	Profitability Gap	a	na	na	0 0%	2.4%	2.3%
18	Foodcorn Supplied	Qs	1.00%	1.00%	115,122	136,881	273,043
19	Employment Rate	e	0.00%	0 00%	1.000	0.992	0.996
20	Price Level	p	-0.01%	-0.01%	1.000	1.146	1.138
21	Inflation Rate	gp	na	na	0 0%	0.0%	0.0%
22	Average Debt	D	0.99%	0.99%	$61,538	$83,026	$164,498
23	Debt:Assets Ratio	d	0.00%	0.00%	0 023	0.023	0 023
24	D:A Ratio Growth Rate	gd	na	na	0 0%	0.0%	0.0%
25	Interest Bill	J	0.99%	0.99%	$2,462	$3,338	$6,614
26	**Constants**						
27	Reaction Coefficient	ϕ	0.00%	0 00%	**0.4388**	0.4388	0.4388
28	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
29	Labour Productivity	λ	0.00%	0.00%	**10**	9	9
30	Risk Premium	φ	0.00%	0 00%	**1.0%**	1.0%	1.0%
31	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
32	Workforce	η	1 00%	1.00%	**16,000**	21,781	43,277
33	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
34	Inflation Wage Coefficient	ρ	0 00%	0.00%	**12**	12	12
35	D:A Ratio Growth Coefficient	δ	0.00%	0 00%	**0.1**	0.1	0.1
36	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
37	Foodcorn Retained	Qf	1 00%	1.00%	**4,878**	6,641	13,194

As in Model C, sustained growth of gQf = gη =1% pa during the initial traverse period endogenously lifts the realised profit rate from r = 5% pa to r = 7.3% pa, thus opening up a profitability gap of a = 2.3% pa above the normal profit rate of n = 5% pa. It is the constancy of this profitability gap which is responsible for the corn-credit economy's smooth exponential growth for most of the century. Model D takes 34 years to traverse to its fully-adjusted profitability gap, compared with 30 years for Model C.

In Model C, the higher profit rate is associated with a corn price that increases from $27.80 to $31.95 per sack, an increase of almost 15 percent. In Model D, the initially higher corn price of $27.82 rises by almost 14 percent to $31.67 per sack. As before, the price rise is largely attributable to increased labour input per sack of corn, due to the lower average labour productivity (output/worker) following the demographic transition.

A comparison of the volume of corn produced (Q sacks pa) during year 100 in Tables 5.15 (Model C) and 5.25 (Model D) shows that production is significantly lower in the latter. Thus the year-100 full employment situation of Model C is not repeated, as Model D's unemployment rate of u = 0.4% in year 100 testifies. This is despite the 70-year growth of production (gQ = 1% pa) being the same in both models, indicating that the year-31 base from which growth is measured in all models also is lower in Model D. These and the other differences identified above are due solely to the introduction of debt (D dollars), interest (J dollars pa) and the flexible interest rate (i% pa), which rises to 4.1% pa during the traverse before regaining its initial value of 4% pa.

The flexible interest rate of Model D adversely affects production and employment as the stationary-state economy adjusts to a steady-state of gQ = 1% pa growth. This occurs because the very process of traversing from the stationary state (a = 0% pa) to the steady state (a = 2.3% pa) lifts the economy's interest rate – hence also its normal profit rate – above their previously constant levels, thus making any given profitability gap more difficult to achieve. Robinson (1962, pp 53-54) has described this type of growth path as "a limping golden age", an age in which a "... steady rate of accumulation of capital [takes] place below full employment".

Nonetheless, Model D displays much the same dynamic behaviour as Model C, plotted as Figures 5.10a through 5.10c below.

Figure 5.10a - Low Values

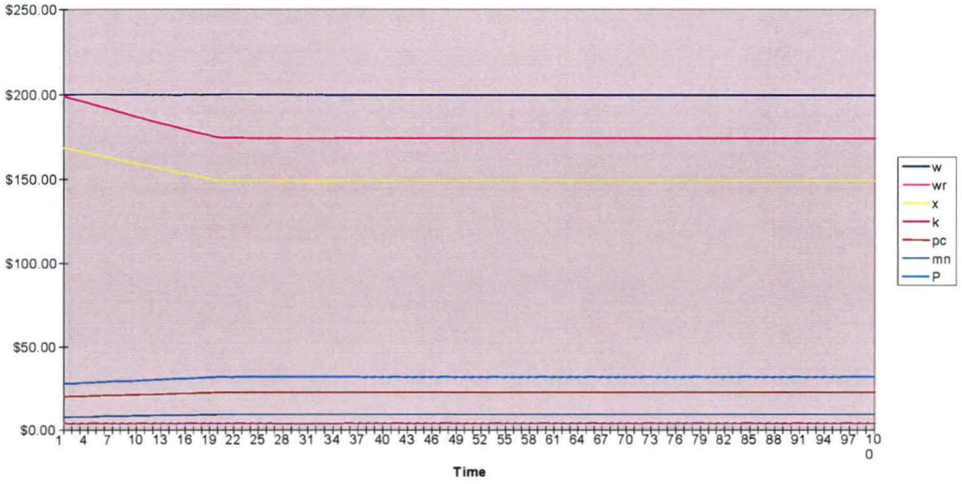


Figure 5.10a above shows how the real wage (wr) falls due to diminishing average labour productivity pushing up prime cost (pc) and the corn price (P).

Figure 5.10b - Percentages

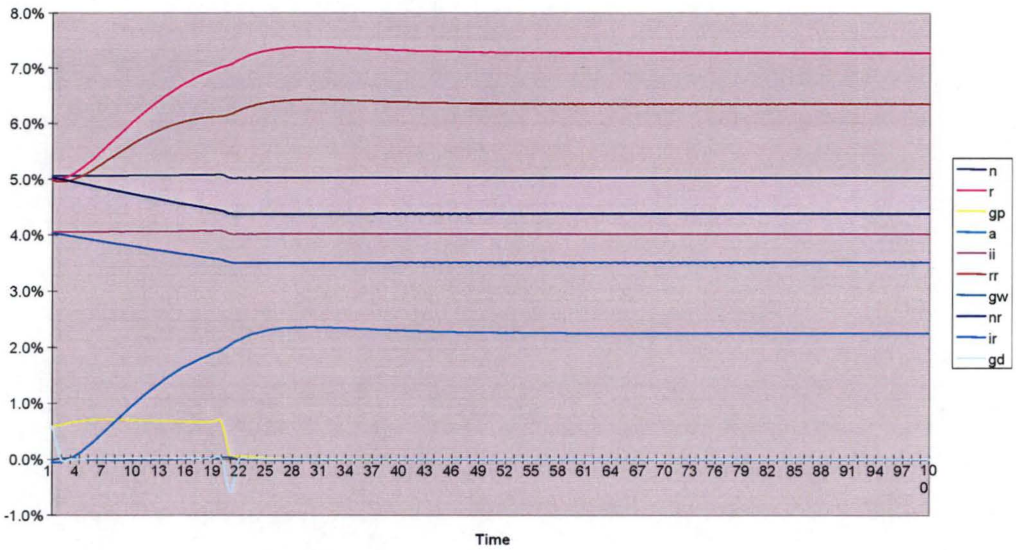


Figure 5.10b above shows the inflation rate largely constant at gp = 0.7% pa during the smooth 20-year demographic transition, afterwards falling to gp = 0% pa. Such price inflation adversely affects farmers, not only workers; their real profit rate stabilises at rr = 6.4% pa, well below its fully-adjusted nominal value of r = 7.3% pa.

Figure 5.10c - Ratios

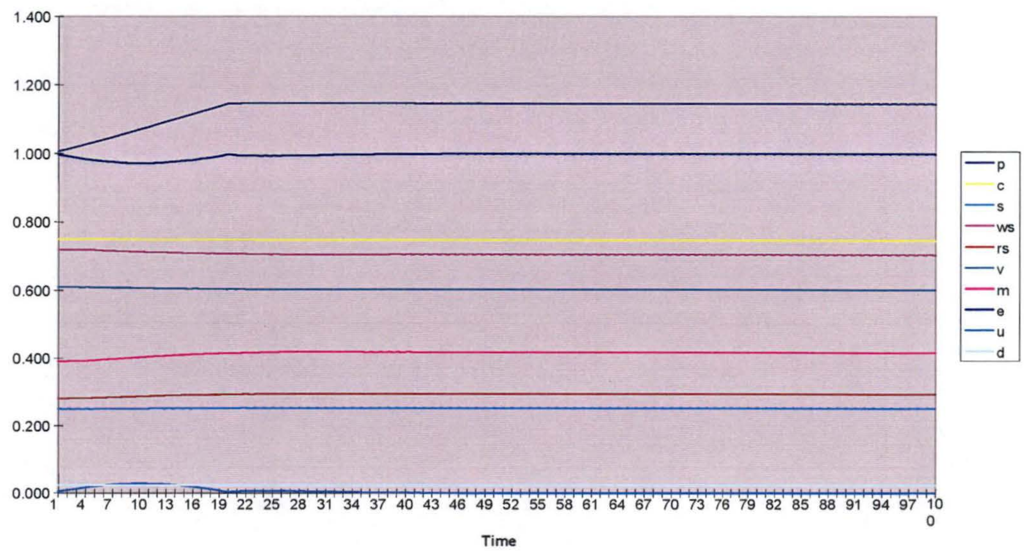


Figure 5.10c above shows the price level rising from p = 1.000 to p = 1.147 during the traverse, then progressively falling to p = 1.138 by year 100. After reaching an

unemployment rate of u = 2.9% during the traverse, rising investment ensures that employment (L) eventually is restored to equality with the workforce (η) so that the situation stabilises at an unemployment rate of u = 0.4% pa. There also is a slight fall in the share of wages (ws) and a complementary rise in the share of gross surplus (rs).

5.4.6 Specimen Traverses

As before, two specimen traverses are initiated, the first from the stationary-state basecase and the second from the steady-state basecase. Both are sparked off by a sudden, unexpected, unplanned, four percent drop in seedcorn invested during year 30. This Misallocation Scenario involves four percent of the sacks of seedcorn being mistakenly released onto the post-harvest foodcorn market.

Table 5.17 – Model D Misallocation Scenario from Stationary State

	A	B	C	D	E	F	G
1	**D - STATIONARY STATE**	**sn**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	-2.45%	160,000	153,600	159,603
4	Seedcorn Invested	Qi	0.00%	-2.39%	**40,000**	37,361	39,901
5	Employment	L	0.00%	-2.45%	16,000	15,360	15,960
6	Realised Profit Rate	r	0.00%	2.62%	5.0%	-1.6%	5.0%
7	Corn Price	P	0.00%	-2.85%	$27.82	$26.32	$26.81
8	Money Wage	w	0.00%	-2.96%	**$200.00**	$199 84	$192.73
9	Interest Rate	ii	0.00%	0.09%	**4.0%**	3.6%	4.0%
10	**Identities**						
11	Wage Bill	W	0.00%	-5.36%	$3,200,000	$3,069,542	$3,075,955
12	Seedcorn Capital	Ka	0.00%	-5.26%	$1,112,719	$1,010,537	$1,069,700
13	Foodcorn Capital	Kb	0.00%	-5.36%	$1,601,203	$1,535,805	$1,539,138
14	Capital Stock	K	0.00%	-5 32%	$2,713,922	$2,546,341	$2,608,838
15	Profit	R	0.00%	-2.86%	$135,696	-$40,053	$130,773
16	Normal Profit Rate	n	0.00%	0.07%	5.0%	4.6%	5.0%
17	Profitability Gap	a	na	na	0.0%	-6.2%	0.0%
18	Foodcorn Supplied	Qs	0.00%	-2.57%	115,122	111,361	114,824
19	Employment Ratio	e	0.00%	-2.45%	1.000	0.960	0.998
20	Price Level	p	0.00%	-2.85%	1.000	0.946	0.964
21	Inflation Rate	gp	na	na	0.0%	-5.4%	0.0%
22	Average Debt	D	0.00%	-5.36%	$61,538	$59,030	$59,153
23	Debt:Assets Ratio	d	0.00%	-0.10%	0.023	0.022	0.023
24	D:A Ratio Growth Rate	gd	na	na	0.0%	-4.1%	0.0%
25	Interest Bill	J	0.00%	-5.28%	$2,462	$2,121	$2,374
26	**Constants**						
27	Reaction Coefficient	ϕ	0 00%	0.00%	**0.4388**	0.4388	0 4388
28	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
29	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
30	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
31	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
32	Workforce	η	0.00%	0.00%	**16,000**	16,000	16,000
33	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
34	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
35	D:A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0.1	0 1
36	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
37	Foodcorn Retained	Qf	0.00%	0.00%	**4,878**	4,878	4,878

Table 5.17 above shows how the initial *stationary* state is disrupted by the unintended misallocation event. All ad percentages are negative, except for those associated with rates of profit, interest and markup. With the money wage (w) falling more than the corn price (P) over 70 years, this lifts the nominal profit rate (r) by ad = 3.02% and the real profit rate by even more. Unemployment and the falling price level, which persist beyond the traverse period, are responsible for pushing down the money wage from w = \$200 to w = \$192.73 by year 100. By contrast, although the interest rate dips to i = 3.6% pa as the traverse commences, it regains its stationary-state value of i = 4% pa within three years.

The High Values and Percentages graphs are not shown because they display near-identical behaviour to the comparable Model C and Model B graphs, cf. Figures 5.8a and 5.8b above. However, the Ratios graph for Model D is reproduced below because it contains important labour market information.

Figure 5.11 - Ratios

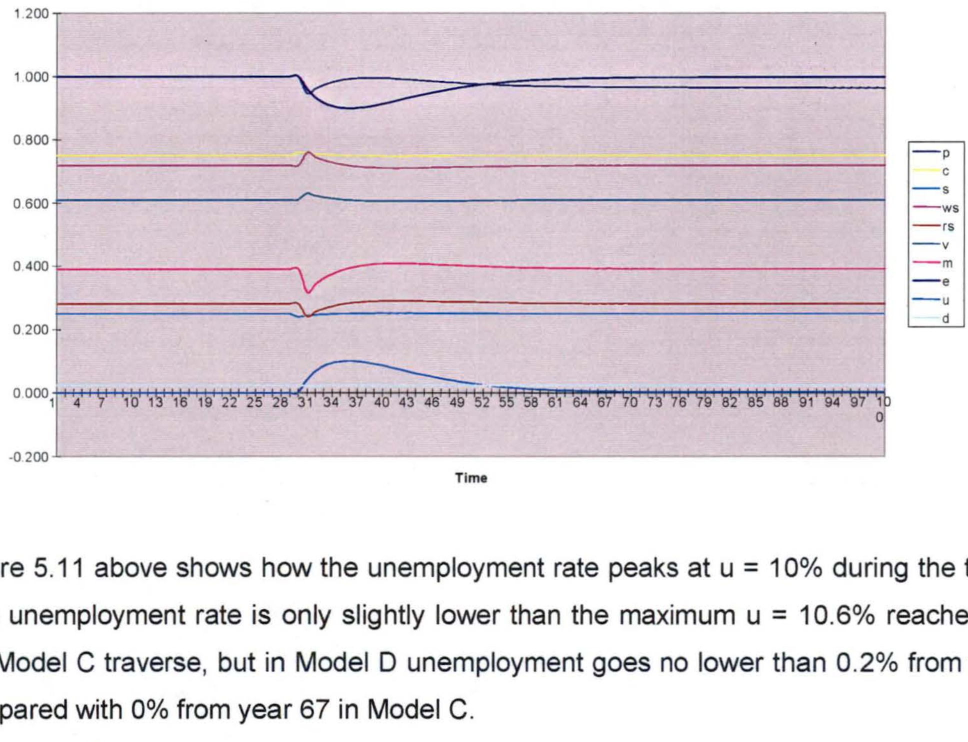


Figure 5.11 above shows how the unemployment rate peaks at u = 10% during the traverse. This unemployment rate is only slightly lower than the maximum u = 10.6% reached during the Model C traverse, but in Model D unemployment goes no lower than 0.2% from year 88, compared with 0% from year 67 in Model C.

Table 5.18 below shows how the initial *steady* state is disrupted by the unintended misallocation. All ag percentages are slightly lower than those of Model C, so all physical variables reach lower fully-adjusted levels following the traverse, e.g. corn produced is Q = 382,087 during year 100 in Model D, which is 2,197 sacks pa lower than in Model C. Even though the corn price (P) starts off slightly higher in Model D, all money variables of Model D also reach lower fully-adjusted levels in year 100, due to their lower volume components.

So, despite realised profitability traversing to the same r = 7.3% pa rate in both models, the introduction of banks, debt and interest has made the economy of Model D less prosperous than that of Model C, in terms of both being "engines of provision" for the material needs of their populations.

Table 5.18 – Model D Misallocation Scenario from Steady State

	A	B	C	D	E	F	G
1	**D - STEADY STATE**	**sn**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	1.00%	1.13%	160,000	184,344	382,087
4	Seedcorn Invested	Qi	1.00%	1.13%	**40,000**	45,305	96,477
5	Employment	L	1.00%	1.13%	16,000	20,750	43,009
6	Realised Profit Rate	r	-0.01%	0.23%	5.0%	0.7%	7.3%
7	Corn Price	P	-0.01%	-0.04%	$27.82	$30.16	$30.65
8	Money Wage	w	-0.01%	-0.04%	**$200.00**	$199.58	$192.04
9	Interest Rate	ii	0.00%	0.02%	**4.0%**	3.6%	4.0%
10	**Identities**						
11	Wage Bill	W	0.99%	1.09%	$3,200,000	$4,141,346	$8,259,278
12	Seedcorn Capital	Ka	0.99%	1.09%	$1,112,719	$1,390,060	$2,927,312
13	Foodcorn Capital	Kb	0.99%	1.09%	$1,601,203	$2,072,076	$4,132,771
14	Capital Stock	K	0.99%	1.09%	$2,713,922	$3,462,136	$7,060,082
15	Profit	R	0.98%	1.32%	$135,696	$25,957	$516,251
16	Normal Profit Rate	n	0.00%	0.02%	5.0%	4.6%	5.0%
17	Profitability Gap	a	na	na	0.0%	-3 9%	2.3%
18	Foodcorn Supplied	Qs	1.00%	1.14%	115,122	132,398	272,416
19	Employment Rate	e	0.00%	0.13%	1.000	0.953	0.994
20	Price Level	p	-0 01%	-0.04%	1.000	1.084	1.102
21	Inflation Rate	gp	na	na	0.0%	-5.4%	0.0%
22	Average Debt	D	0.99%	1.09%	$61,538	$79,641	$158,832
23	Debt:Assets Ratio	d	0.00%	0.01%	0.023	0.022	0.023
24	D:A Ratio Growth Rate	gd	na	na	0.0%	-4.1%	0.0%
25	Interest Bill	J	0.99%	1.11%	$2,462	$2,877	$6,407
26	**Constants**						
27	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4388**	0.4388	0.4388
28	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
29	Labour Productivity	λ	0.00%	0.00%	**10**	9	9
30	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
31	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
32	Workforce	η	1.00%	1.00%	**16,000**	21,781	43,277
33	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
34	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
35	D:A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0 1	0.1
36	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
37	Foodcorn Retained	Qf	1.00%	1.00%	**4,878**	6,641	13,194

The Low Values and Percentages graphs are not shown because they display near-identical behaviour to the comparable Model C graphs, cf. Figures 5.8a and 5.8b above. However, the Ratios graph for Model D contains important labour market information.

Figure 5.12 - Ratios

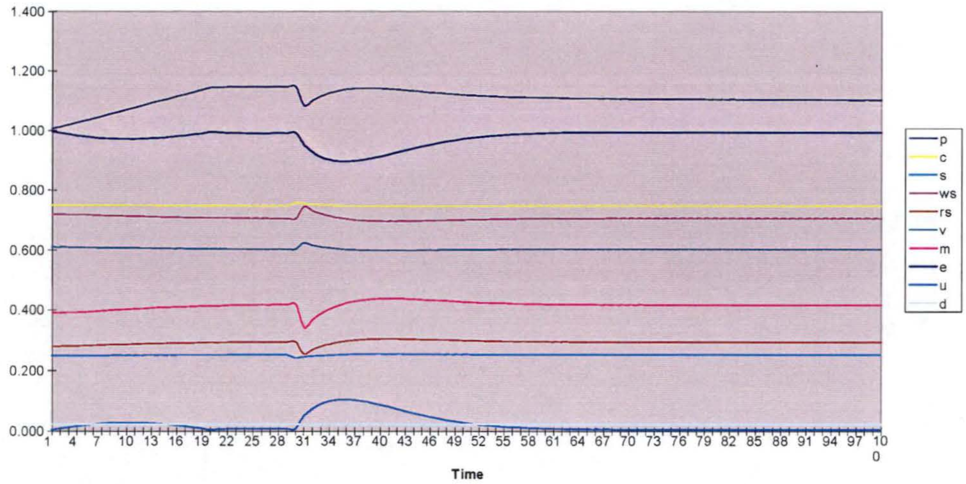


Figure 5.12 above shows that unemployment rate peaks at 10.4%, close to the 10.5% of Model C with its fixed interest rate. However, in Model D the unemployment rate never goes any lower than 0.6% from year 66, compared with zero unemployment from year 67 onwards in Model C.

5.5 Some Theoretical Implications

At the heart of Models B, C and D lies the farmers' aggregate response to any gaps that open up between the profit rate they expect to realise (re = r% pa) and the opportunity cost of capital or normal profit rate (n% pa) they need to earn. This response is measured by the farmers' average reaction coefficient (ϕ), which determines how much more (or less) seedcorn they retain from their crop for investment purposes this year (Qi) as compared with last year (Qio).

This key profitability gap may be altered, *inter alia*, by the volumes of lagged seedcorn invested and farmer consumption, respectively Qio and Qf. *Ceteris paribus*, this shows how *all* increased retentions of corn by farmers last year, whether for investment or personal consumption, have the effect of denying supplies to this year's weekly foodcorn markets. This forces the dollar price of corn that consumers must pay (hence also the profitability that farmers realise from investing in seedcorn) above what it otherwise would be. The reason is that workers do not save any of their money incomes. The economy's entire wage bill (W dollars pa) is bid for all available supplies of foodcorn. This volume (Qso sacks) is lower because more Qio and Qf is assumed to have been retained from last year's crop. Therefore, this year's price of foodcorn – which also is the opportunity cost of seedcorn – floats to whatever higher level clears the market, viz. P = W / Qso dollars/sack.

Given the money wage, the economy's entire *money* gross surplus (Rg dollars) is generated by nothing other than the farmers' *physical* retentions of corn (Qio and Qf) pushing the price of corn above its prime cost of production, thus creating a profit margin. Kalecki's dictum – "Workers spend what they get; capitalists get what they spend"[32] – definitely holds true in the corn-credit economies of Models B and C. In Model D, the assumption that J dollars of interest income also is spent on foodcorn raises the corn price and margin slightly more.

Regarding the realised profit *rate*, farmers must recognise the opportunity cost of their opening stock of seedcorn capital (Ka = P Qio dollars), all of which was sown to produce this year's crop. Because they also are traders, farmers hold Kb = P Qso / κ dollars worth of foodcorn capital as well. So, when making up their financial accounts, farmers value their crop at Y = P Q, then compute their gross surplus as Rg = Y – W, their net surplus as Rn = Rg – Ka, and their profit as R = Rn – J, where J = 0 in Models B and C. Dividing realised profit by the value of capital stock (K = Ka + Kb), farmers determine their realised profit rate as r = R / K percent pa, on average.

The only reason farmers register "surprise" whenever there is a departure from r = n% pa is because they do not realise that it is their own (collective) corn retention decisions that generate r% pa in the first place. As competitive atomistic economic agents, all the farmer-traders ever see is an *apparently* blind impersonal force ("the market") dictating prices, generating profits, revaluing capital stocks, and realising profit rates. If only they would combine – not to rig prices or reduce wages, but to rationally *plan* the appropriate level and smooth growth of their own aggregate seedcorn retentions – the economy no longer need suffer booms and slumps. This fact is brought home forcefully when the smooth and rapid instrumental traverse of Chapter 7 modifies the disruptive and drawn-out *laisser faire* traverse.

Since investment and farmer consumption (plus the corn price that converts these physical into their corresponding value aggregates) are determined by the farmers' own corn retention decisions, it is clear who ultimately – albeit collectively and unconsciously – determines the corn-credit economy's behaviour in terms of cycles, distribution and growth. Employees do nothing other than work, earn wages, spend them, and consume foodcorn; they don't even save. There is no government to wreak havoc in the economy with inappropriate taxing, spending, public debt, and financing policies. Likewise, there are no importers, exporters or foreign investors fouling the nest. The banker *rentiers* do nothing more than extend credit, assess lender's risk and charge an appropriate interest rate. The economy's evolution,

[32] *Vide* Kalecki (1933, p 79) for the original statement of his dictum.

therefore, is entirely in the hands of the farmer *entrepreneurs*, in these corn-credit models of pure *laisser faire* investment, production and exchange. If any individual or group of farmer-traders feels aggrieved about corn prices, profits and profitability being too low in "the market", they should look to the aggregate behaviour of all their fellow entrepreneurs for the explanation.

In these Post-Keynesian models, farmer-capitalists directly determine physical investment (plus their own consumption) and indirectly determine the corn price (P). This, when multiplied by seedcorn invested (Qi sacks pa), yields nominal investment (I dollars pa). The balance of this year's crop (Q – Qi – Qf) is foodcorn supplied (Qs sacks pa), which represents society's provision for next year's consumption by workers (all W dollars are spent) and, in Model D, interest recipients (all J dollars are spent). Thus there is simply no room for any kind of individual or class saving *behaviour*, in either real or monetary terms. "Real saving" by farmers is merely a secondary, indirect and implied outcome of their primary efforts to accumulate physical capital, in the expectation of realising a higher-than-normal profit rate.

Although workers consume all their money incomes, it can be argued that they nonetheless *do* save. In fact, the workers are *forced* to reduce their consumption (i.e. to save in real terms) every time the farmers decide to increase their physical retentions of corn for investment or consumption purposes. This occurs because their decision reduces the supplies of foodcorn flowing onto next year's weekly markets and raises the corn price against workers bidding with their wage dollars.

Throughout history, this potent (and often invisible) "forced saving" mechanism frequently has been used to transfer real resources from a population of producers to an elite intent on "investing" in temples, tombs, wars of conquest, monuments, and the like, while they themselves continue consuming goods and services at levels that maintain their existing lifestyle. As Keynes (1930b, p 132) points out:

> It has been usual to think of the accumulated wealth of the world as having been painfully built up out of the voluntary abstinence of individuals from the immediate enjoyment of consumption ... But it should be obvious that more abstinence is not enough by itself to build cities ... It is enterprise which builds and improves the world's possessions ... the outgoings of enterprise may be found either out of thrift or at the expense of the consumption of the average consumer.

In conclusion, then, "saving" need have nothing whatever to do with "frugality" or "thrift" or "lacking" or "waiting" or "abstinence" on the part of investors, for the real burden of foregone consumption always can be shifted onto others.

Dynamic adjustment processes take time and lots of it. By contrast with the instantaneous adjustments of the fixprice Model A, for the flexprice Models B, C and D it takes around 30 years to move smoothly from the stationary to the steady state of 1% pa economic growth and another 34 years to complete the traverse ignited by a mere four per cent misallocation of output away from investment (seedcorn) and towards consumption (foodcorn).

The traverse to the steady state is initiated by making the economy's workforce (η) and/or capitalist consumption (Qf) grow at the exponential rate of 1% pa. Given the value of the reaction coefficient (ϕ), the principal effect of flexing the corn price in Model B is to convert the instantaneous jumps of Model A into smooth, convergent traverses onto new fully-adjusted dynamic paths, whether stationary or steady.

By flexing the money wage in Model C, an interaction with the corn price is introduced and, with it, the potential for a wage-price spiral. Both unemployment and over-full employment can occur during the traverse, and price flexibility need not preserve labour market clearing along the new fully-adjusted dynamic paths. This result is at odds with the doctrines of the Neo-Keynesians and the New Keynesians, who rely on the "stickiness" of prices and/or wages and/or interest rates as the explanation for involuntary unemployment.

The reduced form of Model D shows that flexing the interest rate suddenly introduces several new lagged determinants of the volume of seedcorn invested, in addition to the existing Qio and Qso explanatory variables, viz. io, do, Ko, wo, and gpo. This increase in the model's complexity is amplified by its principal driver variable (Qio) being raised to the second and fourth powers in the reduced-form corn price equation.

5.6 Conclusion

In this chapter the pure fixprice Model A of Chapter 4 was made progressively more flexible by allowing its constant corn price, money wage and interest rate to vary according to structural equations that can be justified by economic theory. Several insights into income distribution, saving, the traverse process, and the consequences of greater flexibility were achieved, e.g. one concomitant of an attempt to accumulate and grow faster is a (temporarily) higher flexible interest rate, which dampens the growth actually achieved.

An appreciation was gained of the true nature of saving; in real terms, saving always adjusts to permit *whatever* level of investment capitalists decide to undertake. The required volume of "non-consumption" materialises, regardless of whether this saving is voluntary or forced. High rates of accumulation can always be funded by "crowding out" consumers *via* the increased prices of wage goods. The only effect of voluntary saving of money by consumers is to dampen these price rises.

In Chapter 6, three restrictive assumptions are dropped. These are that (a) workers do not save, (b) farmers consume a fixed, or constantly growing, volume of foodcorn and (c) all interest income is spent on foodcorn. In their place, a conventional demand function for foodcorn is specified, with own-price, cross-price and income elasticities, and *all* consumers compete to purchase their foodcorn requirements at the weekly markets. In this chapter, experimentation goes well beyond the standard specimen traverses. As Model E represents the final, fully-flexprice development of this simple Post-Keynesian corn-credit economy, the model is thoroughly analysed to reveal the dynamics of its traverse behaviour.

CHAPTER SIX

A FLEXPRICE CORN ECONOMY

6.1 Introduction

The fifth and final stage in constructing a fully-flexprice Post-Keynesian corn-credit model is presented in this chapter. Although the preceding Model D already represents a flexprice corn economy – in that all prices (for labour, credit and corn) are determined endogenously – there remains an important inflexibility. This concerns the rigid consumption behaviour of workers, bankers and depositors, who are assumed to spend *all* their wage and interest incomes at the weekly foodcorn markets. Furthermore, farmers still consume a fixed volume of foodcorn, no matter how profitable their production and trading operations. In this chapter, Model E is designed to overcome these limitations and represent a corn-credit economy exhibiting maximum flexibility while retaining its essential simplicity.

Model E is identical with Model D, except that it has an alternative equation for flexing the corn price. Instead of being determined simply by dividing wage and interest income by available foodcorn, this money price now flexes with the lagged volume of foodcorn supplied, with *total* household income and with the price of a substitute commodity, viz. bank deposits representing the stock of money savings. The new flexprice equation is derived from a standard constant-elasticities demand function for foodcorn, i.e. a log-linear equation which is homogeneous of degree zero in its three substantive parameters: the price, income and cross elasticities of demand. In this manner, the consumption spending and money saving behaviour of *all* economic agents is rendered highly flexible.

The specification, description, spreadsheet realisation, and stationary-state solution of Model E are presented below. Subsequently, this zero-growth state is used as the launching pad for an attempt at generating a steady state of constant positive growth, as already achieved in Models A through D. Those variables which have no feedback onto the model's behaviour (i.e. the derived "aggregates") are listed. Purely for comparison with Model D, the standard specimen traverse for the Misallocation Scenario is run in the stationary-state context. With model development complete, a large set of parameter-shock experiments is performed to generate a [50 x 13] sensitivity matrix, which is used to select traverses for further study. Dynamic stability is assessed by varying the reaction coefficient (ϕ) to determine the "range of convergence" (within the model's wider "range of viability"), then phase diagrams are plotted for several pairs of variables to show how they are mutually associated.

The observed traverses of Model E evolve through simulated historical time in an abstract closed agrarian capitalist economy with no government and a population of farmers, bankers and workers who consume, save and invest in accordance with their own expectations and incomes, all being price-takers and none possessing the market power of a price-maker. This is the Neoclassical *desideratum*: a pure *laisser faire* world of flexible prices and unfettered liberty, yet the traverses of Model E do *not* support Neoclassical predictions concerning maximal growth, full employment and price level stability in such an "ideal" world.

6.2 Structural Form

In Model E, saving/dissaving by the household sector is the dollar residual after consumption expenditure out of household income from all sources, viz. the wages, interest and *lagged profit* received by workers, bankers and farmers. Profit income is lagged by one year because farmers realise their profits only *after* their current year's production and trading accounts are finalised. In the same way as wage and interest recipients, profit recipients consume and save out of what they *know* they have already earned.

Household saving/dissaving now is *jointly* determined with household consumption. Any increase (decrease) in money household income must be allocated between a greater (lesser) volume of foodcorn consumed and more (fewer) dollars deposited in household bank accounts. This makes bank deposits a *substitute* for foodcorn in corn-credit economies like these, in which every household's consumption exceeds its subsistence minimum (as is assumed at all five stages of model construction). Each household's saving motive is *not* to amass monetary wealth, but to gain for itself a larger share in the interest bill being collected from the farmers by the bankers, who immediately credit most of this value to their depositors' accounts. (The bankers keep the rest as recompense for their own risk-bearing and labour.) Thus individual saving efforts merely *redistribute interest income* between households but cannot alter the total.

Like its predecessor, Model E comprises a system of seven simultaneous equations, of which only six are independent. The equation defining the realised profit rate (r% pa) is *dependent*, being an identity built from other endogenous variables.

Model E is classified as a recursive dynamic system because it contains first-order difference equations, as can be seen in Table 6.1 below. In this table, nine lagged endogenous variables (Qio, Qso, wo, po, gpo, io, Ko, do, and Ro) appear on the right-hand sides of several structural-form equations and identities.

Table 6.1 – Structural Form of Model E

Equations			
Corn Produced	$Q = \theta\ Qio$	sacks pa	(A)
Seedcorn Invested	$Qi = (1 + \phi\ a)\ Qio$	sacks pa	(B)
Employment	$L = Q / \lambda$	workers	(C)
Profit Rate	$r = R / K$	percent pa	(D)
Corn Price	$P = \exp[\ \{\ln Qso - \alpha\ - \chi\ \ln(1/i) - \gamma\ \ln Yh\} / \beta\ ]$	\$/sack	(E)
Money Wage	$w = wo + \varepsilon\ (e - 1) + \rho\ gpo$	\$/worker pa	(F)
Interest Rate	$i = io + \delta\ gd$	percent pa	(G)
Identities			
Wage Bill	$W = w\ L$	dollars pa	(1)
Seedcorn Capital	$Ka = P\ Qio$	dollars	(2)
Foodcorn Capital	$Kb = P\ Qso / \kappa$	dollars	(3)
Capital Stock	$K = Ka + Kb$	dollars	(4)
Profit	$R = P\ Q - W - Ka - J$	dollars pa	(5)
Normal Profit Rate	$n = i + \varphi$	percent pa	(6)
Profitability Gap	$a = r - n$	percent pa	(7)
Foodcorn Supplied	$Qs = Q - Qi$	sacks pa	(8)
Price Level	$p = P / Pz$	ratio	(9)
Inflation Rate	$gp = (p / po) - 1$	percent pa	(10)
Employment Ratio	$e = L / \eta$	ratio	(11)
Average Debt	$D = W / \mu$	dollars	(12)
Debt:Assets Ratio	$d = D / Ko$	ratio	(13)
D:A Ratio Growth Rate	$gd = (d / do) - 1$	percent pa	(14)
Interest Bill	$J = i\ D$	dollars pa	(15)
Household Income	$Yh = W + J + Ro$	dollars pa	(16)
Constants			
Reaction Coefficient	ϕ = **0.4432**	ratio	(a)
Seedcorn Yield	θ = **4**	sacks/sack pa	(b)
Labour Productivity	λ = **10**	sacks/worker pa	(c)
Risk Premium	φ = **1**	percent pa	(d)
Capital Turnover	κ = **2**	ratio	(e)
Workforce	η = **16,000**	workers	(f)
Employment Rate Coefficient	ε = **4**	ratio	(g)
Inflation Rate Coefficient	ρ = **12**	ratio	(h)
D:A Ratio Growth Coefficient	δ = **0.01**	ratio	(i)
Wage Bill Turnover	μ = **52**	ratio	(j)
Intercept Coefficient	α = **0.2157**	ratio	(k)
Price Elasticity of Demand	β = **-3**	ratio	(l)
Cross Elasticity of Demand	χ = **2**	ratio	(m)
Income Elasticity of Demand	γ = **1**	ratio	(n)
Initial Values			
Seedcorn Invested	Qiz = **40,000**	sacks pa	(I)
Wage Rate	wz = **200.00**	\$/worker pa	(II)
Interest Rate	iz = **4.0**	percent pa	(III)

The numerical values chosen for Model E's 17 parameters – three of which are initial values given by history – are shown in bold type. There are now conventional Greek-letter symbols for *all 14 constants*, the previous Roman-letter symbols having vanished. This indicates that no trace of fixprice behaviour remains in the flexprice corn-credit model's final structural form. Investors are unconstrained, bankers independently set the interest rate, workers negotiate the money wage with their employers, and – significantly – consumers are *sovereign* with respect to their consumption/saving choices. Therefore, the Model E corn-credit economy now exhibits *complete* price flexibility.

Identities (1) through (16) are easily eliminated from this structure. If this were to be done, their substance would reappear on the right-hand sides of equations (A) through (G), making them difficult to interpret. Yet six independent technical and behavioural equations are not sufficient to determine Model E's seven unknowns, viz. Q, Qi, L, r, P, w, and i. Equation (D) is an identity defining the realised profit rate, hence it is *not* independent. This means the system is under-determined. So, to select from among the resulting infinite number of possible solutions, Model E is closed by searching for a parameter-set which will ensure the profitability gap remains fixed at a = 0% pa over 100 years of simulated historical time. This is the model's long-period dynamic equilibrium condition.

Closure is achieved by retaining the 13 parameters which Model E has in common with Model D, leaving only four new foodcorn demand function constants to be specified: α, β, χ, and γ. To ensure a large consumer response to changes in the dollar price of foodcorn (P dollars/sack), the *price elasticity of demand* is fixed fairly high, at $\beta = -3$. To guard against "Keynesian income effects" predominating over "Neoclassical price effects" *per assumptione*, the *income elasticity of demand* (γ) is set much lower at $\gamma = +1$. To guarantee the absence of any money illusion on the part of consumers, the foodcorn demand function is made homogeneous of degree zero. This requirement ($\beta + \chi + \gamma = 0$) fixes the *cross elasticity of demand* at $\chi = +2$. Finally, the intercept constant is varied until, at the value of $\alpha \approx 0.2157$, the computed profitability gap remains on a = 0% pa so that a tranquil state of dynamic equilibrium prevails for a full century.

This ensures that the stationary state will continue *indefinitely*, because farmers keep seeing their long-period profitability expectations being fulfilled (re = r% pa), while simultaneously realising a profit rate that precisely meets their required or target rate of return (r = n% pa), i.e. their opportunity cost of capital. These long-period equilibrium closure conditions keep farmers content to go on accumulating Qi = 40,000 sacks pa of seedcorn after every harvest for 100 years of simulated historical time. This procedure (i.e. setting r = n% pa) for closing an under-determined dynamic Post-Keynesian monetary model is comparable with choosing the nth commodity as *numéraire* (i.e. setting Pn = 1) to close an under-determined static Neoclassical barter model of general equilibrium.

6.3 Components of the Model

In Table 6.1, there are six independent equations and one dependent equation to determine the model's seven unknowns. Secondly, there are 16 identities, whose only role is to make the right-hand sides of the equations more readable; it is a simple task to eliminate all of them from the structural form of Model E. Thirdly, there are 14 constants, parameters whose Greek-letter symbols indicate that no trace of fixprice behaviour remains in this model of a pure flexprice corn-credit economy. Finally, there are three initial values, the model's parameters for seedcorn invested, the money wage and the interest rate in the base year, year zero (indicated by "z"). However, the year-zero corn price (Pz) which appears in identity (9) is *not* a parameter; its value is computed by the pricing formula in column E of the Estat spreadsheet.

6.3.1 Equations

The only change from Model D is in equation (E) for the price of foodcorn (P dollars/sack) in both models. Formerly, farmers retained Qf = 4,878 sacks pa of foodcorn for consumption within their own households during the coming year, supplying the *balance* of their crop (net of seedcorn invested) to be sold at next year's weekly foodcorn markets. In Model E, by contrast, farmers earmark the *whole* of their crop (net of seedcorn invested) to be sold at the following year's foodcorn markets, retaining *none* for themselves. To meet their household consumption requirements each year, farmers now compete "on a level playing field" with their own workers to purchase the economy's available supply of foodcorn (Qso sacks pa).

At the equilibrium corn price (P dollars/sack) – for a given nominal interest rate (i% pa) and money household income (Yh dollars pa) – the variable quantity of foodcorn demanded (Qd sacks pa) must have come into equality with the fixed quantity of lagged foodcorn supplied (Qso sacks pa). Hence, an economy-wide demand function, whose constant elasticities represent averages of individual behaviours (the AIB assumption), can be written in log-linear form as

$$\ln Qd = \alpha + \beta \ln P + \chi \ln(1/i) + \gamma \ln Yh = \ln Qso \text{ sacks pa}$$

to ensure that Qd = Qso after P has completed its process of adjustment.

The P and (1/i) price terms indicate that consumers must balance the satisfactions to be derived from both foodcorn and bank deposits, when allocating their given money incomes between these two commodities. Using the interest rate *reciprocal* as "the price of bank deposits" – rather than the interest rate itself – associates higher prices with lower quantities of bank deposits, in accordance with the Law of Demand. Thus higher bank deposit "prices" are equivalent to lower returns to saving.

Movements in the *stock* of household bank deposits need not be tracked by Model E. The first reason is that farmers and workers are assumed to consider only their household *income* (which includes interest) – and not their monetary *wealth* (held as bank deposits) – when making their consumption decisions. Individual households desire bank deposits only for the interest incomes they yield. The second reason is that the stock of bank deposits has *no* monetary implications in a Wicksellian pure credit economy.

Replacing Qd with Qso in the foodcorn demand function and manipulating to get P onto its left-hand side yields

$$P = \exp[\,\{\ln Qso - \alpha\ - \chi\ \ln(1/i) - \gamma\ \ln Yh\}\,/\,\beta\,] \qquad \text{dollars/sack,}$$

which is equation (E) in Table 6.1 above. Apart from the interest rate reciprocal, the corresponding corn price equation of Model D has similar determinants. However, lagged foodcorn supplied (Qso sacks pa) is larger by the amount formerly retained by farmers for their own consumption, i.e. by Qf = 4,878 sacks pa. Also, the wage bill (W dollars pa) and interest bill (J dollars pa) have been rolled in with the lagged profit variable (Ro dollars pa) to form the new household income aggregate (Yh dollars pa).

6.3.2 Identities

There are two changes from Model D, viz. a redefinition of foodcorn supplied (Qs) and a definition of the new variable, household income (Yh). The Roman-letter constant for foodcorn retained (Qf) no longer appears on the right-hand side of identity (8) and a new identity (16) defines household income as the sum of wages (W), interest (J) and lagged profit (Ro).

6.3.3 Constants

By comparison with Model D, the last surviving Roman-letter constant (Qf) has vanished and four demand function constants have been added. These are the intercept constant (α), the price elasticity of demand (β), the cross elasticity of demand (χ), and the income elasticity of demand (γ) for foodcorn.

6.3.4 Initial Values

There are no changes because Model E requires the same three initial values as Model D (Qiz, wz and iz) to encapsulate its complete history up to and including year zero.

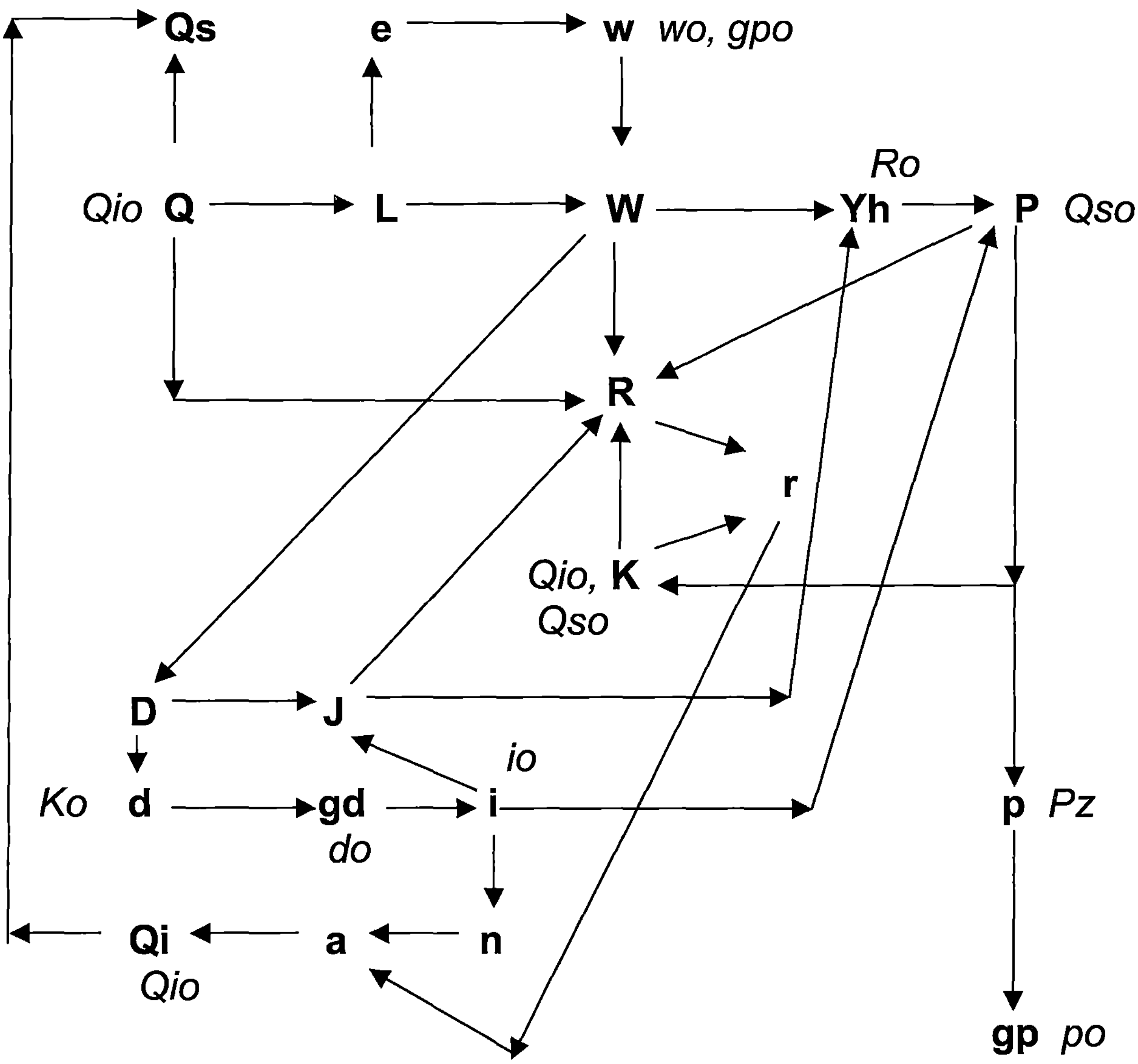
6.4 Corn-Credit Economy Description

The interrelatedness of this recursive dynamic system of structural equations and identities is displayed as a flowchart in Figure 6.1 below.

By comparison with the flowchart for Model D in the previous chapter, the quantity of foodcorn supplied (Qs) is now entirely endogenous, i.e. the Roman-letter constant (Qf) of Figure 5.5 has vanished. No longer is the corn price (P) determined by *most* household income, as previously shown by arrows originating at W and J in the Model D flowchart. In Model E, this flexible own-price is determined by fixed supply and variable demand in the weekly foodcorn markets. Figure 6.1 below shows that P now depends on the lagged quantity of foodcorn supplied, on *all* household income (including lagged profit) and on the price of the substitute commodity (bank deposits).

In Model E, two endogenous variables (W, J) and one lagged endogenous variable (Ro) define household income (Yh), which helps determine the endogenous corn price (P), in combination with the endogenous interest rate reciprocal (1/i) and the lagged endogenous flow of foodcorn supplied (Qso) onto the weekly markets for foodcorn. The parameters of the demand function (α, β, χ, γ) are not shown, in conformity with the treatment of such Greek-letter constants in the flowcharts of Models A through D.

Figure 6.1 – Flowchart of Model E



6.5 Nature of the Corn-Credit Economy

Like its predecessors, Model E represents an abstract Post-Keynesian monetary production or corn-credit economy that is closed (autarkic) with no government (anarchic) and is history-bound and path-dependent. As in the Model D economy, the corn price, money wage and interest rate are flexible, but there is one important difference. All consumers (including farm households) now compete "on a level playing field" to purchase the supplies of last year's foodcorn that farmer-traders release from their granaries on a weekly basis to sustain the workforce engaged in raising this year's new crop of corn, together with the bankers and themselves.

Previously, workers, bankers and those holding bank deposits had no power over the disposition of their wage and interest incomes, all of which were spent at the weekly foodcorn markets. Likewise, farmers were given no choice but to set aside in their barns, and subsequently consume, Qf = 4,878 sacks of foodcorn annually. In Model E, however, everyone earning wages and/or interest and/or profits enjoys *consumer sovereignty* over the disposition of their money incomes. They decide how to allocate each year's income between spending and saving; bank deposits thus become a *substitute* for foodcorn, in this economy where every household earns more than enough to subsist.

The analyses that follow suggest that consumer sovereignty is a powerful force which, when combined with the fact that entrepreneurs choose for society its rate of capital accumulation, can lead to great instability in the production and employment aggregates that the population depends on for its provisioning.

6.6 Aggregates

Accounting definitions for 32 macroeconomic aggregates are set out in Table 6.2 below, together with the units in which they are measured. None of these values has *any* feedback effects on the 23 endogenous variables determined by the structural-form equations and identities.

Table 6.2 – Aggregates of Model E

Gross Product	Y = P Q	dollars pa
Gross Surplus	Rg = Y – W	dollars pa
Net Surplus	Rn = Rg – Ka	dollars pa
Real Gross Product	Yr = Y / p	constant dollars pa
Real Wage Bill	Wr = W / p	constant dollars pa
Real Gross Surplus	Rgr = Rg / p	constant dollars pa
Real Net Surplus	Rnr = Rn / p	constant dollars pa
Real Profit	Rr = R / p	constant dollars pa
Consumption	C = P (Q – Qi)	dollars pa
Consumption Ratio	c = C / Y	ratio
Real Consumption	Cr = C / p	constant dollars pa
Real Household Income	Yhr = Yh / p	constant dollars pa
Saving	S = Y – C	dollars pa
Saving Ratio	s = S / Y	ratio
Real Saving	Sr = S / p	constant dollars pa
Investment	I = P Qi	dollars pa
Real Investment	Ir = I / p	constant dollars pa
Real Interest Rate	ir = i / p	percent pa
Real Wage	wr = w / p	constant $/worker pa
Real Profit Rate	rr = r / p	percent pa

Wage Bill Share	ws = W / Y	ratio
Gross Surplus Share	rs = Rg / Y	ratio
Investment Multiplier	k = Yr / Ir	ratio
Real Capital Stock	Kr = K / p	constant dollars
Capital-Output Ratio	v = Kr / Yr	ratio
Capital-Labour Ratio	x = Kr / L	ratio
Money Wage Growth Rate	gw = w / wo – 1	percent pa
Real Normal Profit Rate	nr = n / p	percent pa
Unemployment Rate	u = 1 – e	percent
Prime Cost	pc = w / λ	dollars/sack
Margin	mn = P – pc	dollars/sack
Markup	m = mn / pc	ratio

Although many are self-evident, these derived aggregates can be useful for checking that national accounting identities (such as S = I dollars pa) are consistent with the structural-form variables endogenously determined by Model E. In fact, *everything* that occurs in this model economy after year zero ultimately stems from (a) the three initial values (Qiz, wz, iz); (b) the biology, technology and behaviour expressed by the independent equations which determine the values of the key variables Q, Qi, L, P, w, and i; and (c) the long-period dynamic equilibrium condition that expected and normal profitability must coincide (r = n% pa).

6.7 Reduced Form

The Wolfram Mathematica computer program was *unable* to derive a reduced form from the structural form of Model E. The complexity of Model D's reduced form (as shown in Table 5.14) has been increased significantly by adding the new, four-parameter, log-linear corn price equation of Model E. Also, there are two lagged variables in this new equation; Qso enters explicitly and Ro implicitly. However, there are no grounds for believing that all seven endogenous variables (Q, Qi, L, r, P, w, and i) should not continue being "driven" by the sequence of previous-year values for Qi, with the central driving force of Qio being amplified or moderated by the presence of other lagged variables, such as Qso and Ro.

Model E's reduced form must exceed four dimensions, otherwise Mathematica would have derived it. Instead, the program attempted a z-transform (into a quintic or higher) and failed. Now there do exist polynomials beyond the quartic which nonetheless can be solved. Apparently the reduced form of Model E is not a member of this class.

6.8 Spreadsheet Realisation

The structural form of Model E was programmed into a Microsoft Excel spreadsheet and given the computer filename Estat (see Appendix D, with enclosed CD-ROM). The spreadsheet formulae for the equations, identities and constants of Table 6.1 above are set out in Table 6.3 below. Formulae for the aggregates of Table 6.2 above are not shown.

Table 6.3 – Model E Spreadsheet Formulae

	A	B	C	D	E	F
1	**E - STATIONARY STATE**	**SN**	**rd**	**ad**	**0**	**=+E1+1**
2	**Equations**					
3	Corn Produced	Q				=+F29*E4
4	Seedcorn Invested	Qi			**40000**	=+E4*(1+F28*F17)
5	Employment	L				=+F3/F30
6	Profit Rate	r				=+F15/F14
7	Corn Price	P				=+EXP((LN(E18)-F38-F40*LN(1/F9)-F41*LN(F26))/F39)
8	Money Wage	w			**200**	=+E8+F35*E21+F34*(F19-1)
9	Interest Rate	ii			**0.04**	=+E9+F36*F24
10	**Identities**					
11	Wage Bill	W				=+F8*F5
12	Seedcorn Capital	Ka				=+F7*E4
13	Foodcorn Capital	Kb				=+F7*E18/F32
14	Capital Stock	K				=+F12+F13
15	Profit	R				=+F7*F3-F11-(F7*E4)-F25
16	Normal Profit Rate	n				=+F9+F31
17	Profitability Gap	a				=+F6-F16
18	Foodcorn Supplied	Qs				=+F3-F4
19	Employment Ratio	e				=+F5/F33
20	Price Level	p				=+F7/$E7
21	Inflation Rate	gp				=+F20/E20-1
22	Average Debt	D				=+F11/F37
23	Debt Assets Ratio	d				=+F22/E14
24	D A Ratio Growth Rate	gd				=+F23/E23-1
25	Interest Bill	J				=+F9*F22
26	Household Income	Yh				=+F11+F25+E15
27	**Constants**					
28	Reaction Coefficient	φ			**0.4432**	=+E28
29	Seedcorn Yield	θ			**4**	=+E29
30	Labour Productivity	λ			**10**	=+E30
31	Risk Premium	φ			**0.01**	=+E31
32	Capital Turnover	κ			**2.00000000000007**	=+E32
33	Workforce	η			**16000**	=+E33
34	Employment Wage Coefficient	ε			**4**	=+E34
35	Inflation Wage Coefficient	ρ			**12**	=+E35
36	D·A Ratio Growth Coefficient	δ			**0.1**	=+E36
37	Wage Bill Turnover	μ			**52**	=+E37
38	Intercept Constant	α			**0.21573414123849**	=+E38
39	Price Elasticity of Demand	β			**=-E40-E41**	=+E39
40	Cross Elasticity of Demand	χ			**2**	=+E40
41	Income Elasticity of Demand	γ			**1**	=+E41

The rows of Table 6.3 are numbered 1 through 41 and the columns are tagged A through F. Columns A and B list the long and short names, respectively, of all variables and constants in the model. The formulae contained in columns C and D are suppressed for clarity. Column E (year zero) holds the three initial values and all 14 constants. All its year-zero formulae also are suppressed for clarity, but they are based on those shown in the next column. Column F displays formulae for the model's equations, identities and constants for year one. The (missing) columns for years two through 100 simply continue the pattern established in column F.

Note that the three initial values (**bold** type) in year zero are given by history as 40,000 sacks pa of seedcorn invested, a $200/worker pa money wage and a 4% pa interest rate. Apart from the 14 constants (also **bold** type), all other year-zero values are *computed* rather than specified. *Prima facie*, the whole of column E should have been filled with known historical base-period data. Yet, as this is not an empirical model, there are no historical data. So, reliance is placed on the assumption that, in any given year, history cannot be internally inconsistent. That is why the model's standard set of equations and identities (not shown in column E) is used to compute all remaining year-zero values.

The spreadsheet realisation of Model E is reminiscent (albeit not an implementation) of Gunnar Myrdal's (1957, p 30) "... circular causation in the cumulative processes of economic change". The *circularity* resides in the columns, where the 23 structural-form equations simultaneously determine the endogenous variables. The *cumulation* occurs along the rows, where the time path that each of these variables (plus the 32 aggregates) traces out is dependent on the recursive levels of Qio, Qso, wo, po, gpo, io, Ko, do, and Ro shown in each previous column and on the initial values Qiz, wz and iz shown in column E. This mix of "circular and cumulative" (or "simultaneous and recursive") causation will become more obvious once the stationary state is discussed and Model E experiences population growth.

6.9 Solving for the Stationary State

As described above, by varying a single parameter – the intercept constant (α) of the foodcorn demand function – a long-period stationary-state equilibrium condition is enforced for all 100 columns within the Excel spreadsheet, in order to make Model E "just-determined" and obtain its numerical solution.

Table 6.4 below displays years 0, 31 and 100 of this century-long stationary state, as simulated in the Estat spreadsheet file. This reference solution constitutes the starting point for all subsequent Model E computer runs, including the sensitivity analysis of all but four of the 17 parameters and the traverses initiated by perturbing them. The entry of farmers into the foodcorn markets, spending out of their lagged profits, is largely what causes the corn price to rise to P = $27.85, compared with the Model D outcome of $27.82 per sack.

All data in column E (year zero, the base year) are replicated dynamically by the circular and cumulative solution process in all 100 subsequent columns, thereby forming rows of stationary values for all 55 variables. As with Model D, these all trace out horizontal or flatline graphs, when plotted against a century of simulated historical time, so there is no need to reproduce them here. These time-series document a Post-Keynesian stationary equilibrium *process*, not a sequence of Neoclassical static equilibrium *positions*.

Table 6.4 – Model E Stationary State

	A	B	C	D	E	F	G
1	**E - STATIONARY STATE**	**SN**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.00%	0 00%	160,000	160,000	160,000
4	Seedcorn Invested	Qi	0.00%	0.00%	**40,000**	40,000	40,000
5	Employment	L	0.00%	0 00%	16,000	16,000	16,000
6	Profit Rate	r	0 00%	0 00%	5.0%	5 0%	5.0%
7	Corn Price	P	0.00%	0 00%	$27.85	$27.85	$27 85
8	Money Wage	w	0.00%	0.00%	**$200.00**	$200.00	$200 00
9	Interest Rate	ii	0.00%	0 00%	**4.0%**	4.0%	4.0%
10	**Identities**						
11	Wage Bill	W	0.00%	0 00%	$3,200,000	$3,200,000	$3,200,000
12	Seedcorn Capital	Ka	0 00%	0 00%	$1,113,900	$1,113,900	$1,113,900
13	Foodcorn Capital	Kb	0 00%	0 00%	$1,670,849	$1,670,849	$1,670,849
14	Capital Stock	K	0 00%	0.00%	$2,784,749	$2,784,749	$2,784,749
15	Profit	R	0.00%	0.00%	$139,237	$139,237	$139,237
16	Normal Profit Rate	n	0.00%	0 00%	5.0%	5.0%	5.0%
17	Profitability Gap	a	na	na	0.0%	0.0%	0.0%
18	Foodcorn Supplied	Qs	0 00%	0 00%	120,000	120,000	120,000
19	Employment Ratio	e	0.00%	0 00%	1.000	1.000	1.000
20	Price Level	p	0 00%	0.00%	1.000	1 000	1.000
21	Inflation Rate	gp	na	na	0.0%	0.0%	0.0%
22	Average Debt	D	0.00%	0.00%	$61,538	$61,538	$61,538
23	Debt:Assets Ratio	d	0 00%	0.00%	0.022	0 022	0.022
24	D:A Ratio Growth Rate	gd	na	na	0.0%	0 0%	0 0%
25	Interest Bill	J	0.00%	0 00%	$2,462	$2,462	$2,462
26	Household Income	Yh	0.00%	0.00%	$3,341,699	$3,341,699	$3,341,699
27	**Constants**						
28	Reaction Coefficient	ϕ	0.00%	0 00%	**0.4432**	0.4432	0.4432
29	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
30	Labour Productivity	λ	0.00%	0 00%	**10**	10	10
31	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
32	Capital Turnover	κ	0.00%	0 00%	**2**	2	2
33	Workforce	η	0 00%	0.00%	**16,000**	16,000	16,000
34	Employment Wage Coefficient	ε	0 00%	0.00%	**4**	4	4
35	Inflation Wage Coefficient	ρ	0.00%	0 00%	**12**	12	12
36	D:A Ratio Growth Coefficient	δ	0 00%	0.00%	**0.1**	0.1	0.1
37	Wage Bill Turnover	μ	0 00%	0.00%	**52**	52	52
38	Intercept Constant	α	0.00%	0.00%	**0.2157**	0 2157	0 2157
39	Price Elasticity of Demand	β	0 00%	0.00%	**-3.0**	-3 0	-3.0
40	Cross Elasticity of Demand	χ	0 00%	0.00%	**2.0**	2.0	2.0
41	Income Elasticity of Demand	γ	0.00%	0.00%	**1.0**	1.0	1.0

6.10 Generating the "Unsteady State"

Unlike stationary states, steady states cannot be "solved for" but must be "generated" within the dynamic model economy. They are *not* long-period equilibrium states, but exhibit a very special kind of *disequilibrium*. This entails long-term positive constancy of the crucial profitability gap: a = (r – n) > 0% pa. It is the lure of above-normal expected profitability that induces capitalists to keep on investing a steadily-growing volume of seedcorn as each year passes. The farmers are led to increase their annual retentions of seedcorn by the higher-than-expected and higher-than-normal profit rates they consistently realise, year after year.

Contra Harrod (1939) with his unique "warranted rate", the *only* requirement for ongoing steady-state growth is a profitability gap that always remains positive and constant. Each such gap is associated with its own unique growth rate of capital accumulation. Given the ruling interest rate (i% pa) and risk premium (φ% pa), this is the one gQi% pa rate capable of *realising* for farmers a profit rate sufficiently above the opportunity cost of capital to keep them accumulating precisely (1 + gQi) more seedcorn than they did the previous year. However, if the profitability gap refuses to remain positive and constant, an "unsteady state" will be the inevitable outcome and this is just what occurs in Model E.

In Model A, a constant positive gap is achieved *directly* by the simple expedient of raising the fixed corn price by *fiat* to enhance realised profitability. All other models in the sequence use an *indirect* approach. In Model B, the volume of foodcorn retained for consumption by farmers is made to grow by gQf = 1% pa, while in Models C and D, one *also* has to make the workforce grow at gη = 1% pa. In all four models, these actions cause the exponential growth rates of seedcorn invested (gQi) and of corn produced (gQ) to remain constant at 1% pa. Thus, classic steady states are shown to *exist* when the corn price is either fixed (Model A) or else flexes with variations in consumer expenditure outlays being directed at changing volumes of foodcorn supplied to the market (Models B, C and D).

From the Model D experience, one might expect that merely setting gη = 1% pa is all that should be required to usher in a steady state of growth in Model E. However, that proves to be incorrect. Despite having mostly identical parameters to Model D, the highly-sensitive *consumer behaviour* introduced by the new flexprice equation renders Model E exceptionally cyclical. For the Model E reaction coefficient ϕ = 0.4432 (as also in Model D) there are damped cycles, indicating that the economy *eventually* must converge on a steady state. For the elasticities β = -3, χ = 2 and γ = 1, chosen for the foodcorn demand function, experimentation establishes that there exists *no* value of the reaction coefficient within the range 0.3724 ≤ ϕ ≤ 0.8218 that permits Model E to traverse into a steady state in less than 100 years. Outside these limits – the "range of viability" for Model E – the economy suffers a catastrophic collapse and cannot operate at all.

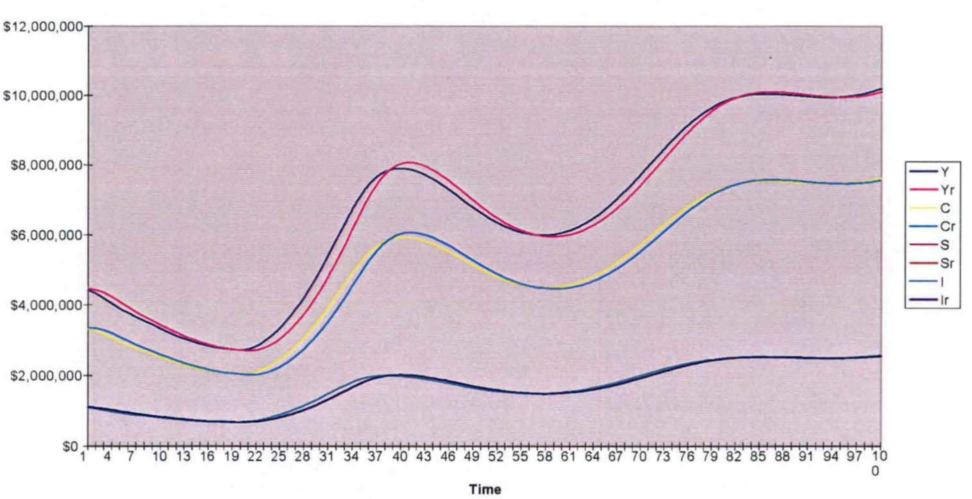
Table 6.5 above displays years 0, 31 and 100 of this century-long "unsteady state", as simulated in the Ested spreadsheet file. Clearly the cyclical behaviour mentioned above depresses economic growth to some extent, i.e. the 1% pa exogenous workforce growth is not matched by endogenous growth in seedcorn investment, employment and corn production. These, together with related variables, grow at rates of less than 0.8% pa, so the employment ratio falls to 0.944 in year 100. Consequently, the money wage falls to less than $177 and the corn price rises to $28.10 in the same year, lifting the profit rate to 6.6% pa.

Table 6.5 – Model E "Unsteady State"

.	A	B	C	D	E	F	G
1	**E - STEADY STATE**	**SN**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.78%	0.78%	160,000	173,776	362,877
4	Seedcorn Invested	Qi	0.76%	0.76%	**40,000**	47,383	91,380
5	Employment	L	0.78%	0.78%	16,000	19,561	40,847
6	Profit Rate	r	na	na	5 0%	25.2%	6 6%
7	Corn Price	P	-0.03%	-0 03%	$27.85	$31.17	$28.10
8	Money Wage	w	0.03%	0.03%	**$200.00**	$166.84	$176.83
9	Interest Rate	ii	0.00%	0.00%	**4.0%**	3 8%	4.0%
10	**Identities**						
11	Wage Bill	W	0.81%	0.81%	$3,200,000	$3,263,451	$7,222,978
12	Seedcorn Capital	Ka	0.76%	0 76%	$1,113,900	$1,354,294	$2,549,404
13	Foodcorn Capital	Kb	0.81%	0.81%	$1,670,849	$1,806,627	$3,794,717
14	Capital Stock	K	0 79%	0.79%	$2,784,749	$3,160,921	$6,344,121
15	Profit	R	na	na	$139,237	$797,072	$419,716
16	Normal Profit Rate	n	0.00%	0.00%	5.0%	4.8%	5.0%
17	Profitability Gap	a	na	na	0.0%	20.5%	1 6%
18	Foodcorn Supplied	Qs	0.79%	0.79%	120,000	126,393	271,497
19	Employment Ratio	e	-0.21%	-0.21%	1.000	0.898	0.944
20	Price Level	p	-0.03%	-0.03%	1.000	1.119	1.009
21	Inflation Rate	gp	na	na	0.0%	-0.2%	0.2%
22	Average Debt	D	0 81%	0.81%	$61,538	$62,759	$138,903
23	Debt:Assets Ratio	d	0.00%	0 00%	0 022	0.022	0 022
24	D:A Ratio Growth Rate	gd	na	na	0.0%	0.2%	0.0%
25	Interest Bill	J	0.81%	0 81%	$2,462	$2,359	$5,519
26	Household Income	Yh	0.75%	0.75%	$3,341,699	$3,997,940	$7,624,663
27	**Constants**						
28	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4432**	0.4432	0 4432
29	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
30	Labour Productivity	λ	0.00%	0 00%	**10.00**	8.88	8.88
31	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
32	Capital Turnover	κ	0 00%	0.00%	**2**	2	2
33	Workforce	η	1.00%	1.00%	**16,000**	21,781	43,277
34	Employment Wage Coefficient	ε	0 00%	0.00%	**4**	4	4
35	Inflation Wage Coefficient	ρ	0 00%	0 00%	**12**	12	12
36	D:A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0.1	0 1
37	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
38	Intercept Constant	α	0.00%	0 00%	**0.2157**	0 2157	0.2157
39	Price Elasticity of Demand	β	na	na	**-3.0**	-3.0	-3.0
40	Cross Elasticity of Demand	χ	0 00%	0.00%	**2.0**	2 0	2.0
41	Income Elasticity of Demand	γ	0.00%	0 00%	**1.0**	1.0	1.0

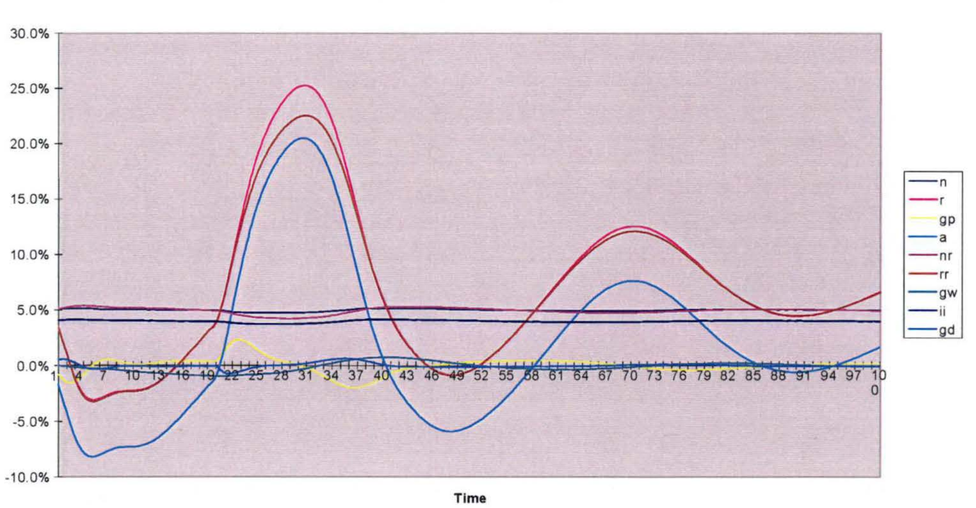
Several time paths are plotted in the three graphs reproduced below. The expenditure aggregates in Figure 6.2a below undergo two complete cycles, the second being of lesser amplitude. The stylised fact of an asymmetric "sawtooth" pattern in business cycles is evident: long-duration upswings in economic activity are followed by downswings of shorter duration.

Table 6.2a - Expenditures



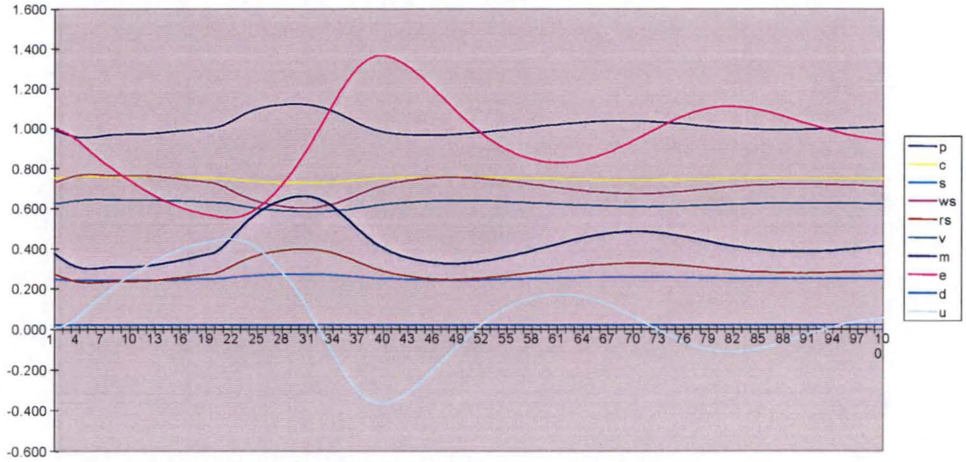
As is appropriate for two ultimate "drivers" of the economy, the best-defined cycles are seen in Figure 6.2b below, which plots the behaviour of the profit rate and the profitability gap.

Table 6.2b - Percentages



In Figure 6.2c below, the unemployment rate also experiences two damped cycles, the first of which peaks at u = 44.7 percent of the workforce. The least volatile cyclical behaviour is exhibited by the gross product shares of the wage bill and gross surplus, which is consistent with another stylised fact: the long-term empirical stability of distributive shares in capitalist economies.

Table 6.2c - Ratios



As an experiment, the time-span of Model E was doubled to 200 years. It was observed that *188 years* passed before the economy settled down into a fully-adjusted steady state exhibiting a constant profitability gap of a = 2.4% pa. By contrast, the traverses which generated a steady state from its parent stationary state in Models B, C and D lasted some *30 years*.

6.11 Specimen Traverse

As for earlier models, a traverse is initiated from the stationary state basecase. However, no specimen traverse can be computed, even from the experimental 200-year steady state investigated above. Workforce growth in the highly-sensitive Model E generates an "Unsteady State" and a fully-adjusted reference basecase is unavailable. A 12-year fully-adjusted dynamic path is simply not long enough for experimentation that is comparable with that performed upon the earlier models.

Once again, the Misallocation Scenario is run from the opening stationary state, involving four percent of the sacks of seedcorn (already earmarked for investment as circulating capital at the end of year 30) being *mistakenly* released onto the weekly markets of year 31, for sale as foodcorn. Traverses are initiated during year 30 to preserve a segment of the basecase flatline time path, against which the plotted traverse behaviour can be compared.

Table 6.6 – Model E Misallocation Scenario from Stationary State

	A	B	C	D	E	F	G
1	**E - STATIONARY STATE**	**SN**	**rd**	**ad**	**0**	**31**	**100**
2	**<u>Equations</u>**						
3	Corn Produced	Q	0 00%	-0.50%	160,000	153,600	154,587
4	Seedcorn Invested	Qi	0 00%	-0 49%	**40,000**	39,452	38,761
5	Employment	L	0.00%	-0.50%	16,000	15,360	15,459
6	Profit Rate	r	0.00%	1 34%	5.0%	10.8%	5.7%
7	Corn Price	P	0.00%	-0 07%	$27.85	$29.39	$27.83
8	Money Wage	w	0.00%	-0.19%	**$200.00**	$199.84	$198 72
9	Interest Rate	ii	0.00%	0.10%	**4.0%**	3.6%	4.0%
10	**<u>Identities</u>**						
11	Wage Bill	W	0.00%	-0 68%	$3,200,000	$3,069,542	$3,071,973
12	Seedcorn Capital	Ka	0.00%	-0.59%	$1,113,900	$1,128,608	$1,075,559
13	Foodcorn Capital	Kb	0.00%	-0.58%	$1,670,849	$1,786,962	$1,608,209
14	Capital Stock	K	0.00%	-0.58%	$2,784,749	$2,915,570	$2,683,768
15	Profit	R	0 00%	1.60%	$139,237	$314,160	$152,337
16	Normal Profit Rate	n	0 00%	0.08%	5 0%	4.6%	5.0%
17	Profitability Gap	a	na	na	0.0%	6.2%	0.7%
18	Foodcorn Supplied	Qs	0.00%	-0 50%	120,000	114,148	115,825
19	Employment Ratio	e	0.00%	-0.50%	1.000	0 960	0.966
20	Price Level	p	0.00%	-0 07%	1.000	1.055	0.999
21	Inflation Rate	gp	na	na	0.0%	5 5%	0 0%
22	Average Debt	D	0 00%	-0.68%	$61,538	$59,030	$59,076
23	Debt:Assets Ratio	d	0.00%	-0.17%	0.022	0 021	0.022
24	D:A Ratio Growth Rate	gd	na	na	0 0%	-4.1%	0.0%
25	Interest Bill	J	0.00%	-0 52%	$2,462	$2,121	$2,367
26	Household Income	Yh	0.00%	-0.59%	$3,341,699	$3,210,900	$3,222,826
27	**<u>Constants</u>**						
28	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4432**	0 4432	0 4432
29	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
30	Labour Productivity	λ	0.00%	0.00%	**10**	10	10
31	Risk Premium	φ	0 00%	0 00%	**1.0%**	1.0%	1.0%
32	Capital Turnover	κ	0 00%	0.00%	**2**	2	2
33	Workforce	η	0 00%	0.00%	**16,000**	16,000	16,000
34	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
35	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
36	D:A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0.1	0 1
37	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
38	Intercept Constant	α	0.00%	0 00%	**0.2157**	0.2157	0 2157
39	Price Elasticity of Demand	β	0.00%	0 00%	**-3.0**	-3.0	-3.0
40	Cross Elasticity of Demand	χ	0 00%	0.00%	**2.0**	2.0	2.0
41	Income Elasticity of Demand	γ	0.00%	0.00%	**1.0**	1 0	1.0

Table 6.6 above displays the economic effects of this unintended misallocation *via* a comparison of columns C and D in the perturbed Estat spreadsheet, Estatmal. Column C (headed rd) shows zero percentage reference differences because there *can be none* between the reference stationary-state basecase and itself. Column D (headed ad) shows actual differences between the traverse time path and the basecase time path, again in percentages, following the fall of 4% in seedcorn invested during year 30. It can be seen that the effect of the money wage (w) plus employment (L) falling further than the corn price (P) and corn production (Q) is to raise the profit rate (r) by 1.34 percent.

These actual differences give no indication of how *violently* this specimen traverse behaves, particularly during its first ten years, as can be seen in Figures 6.3a through 6.3c below.

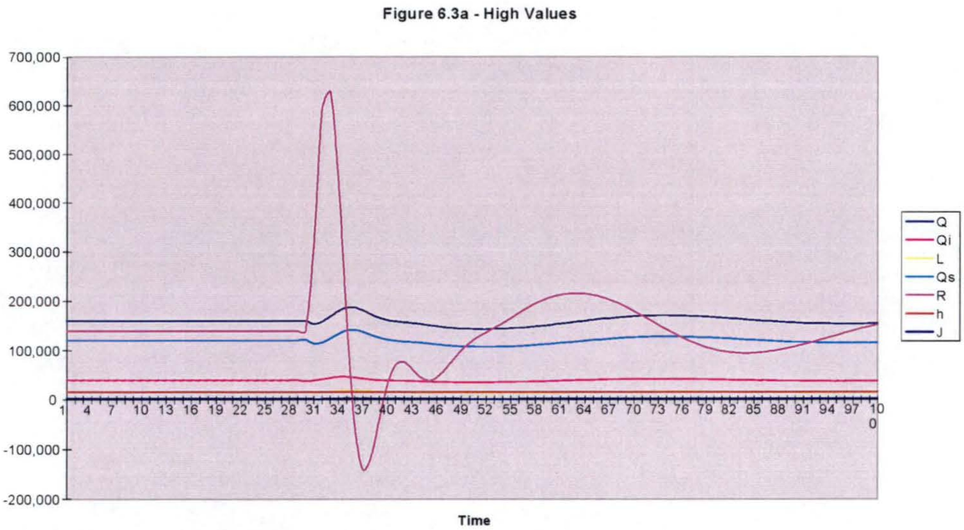


Figure 6.3a above reveals a massive initial swing in profit (R), which initiates a long cycle of far lower amplitude.

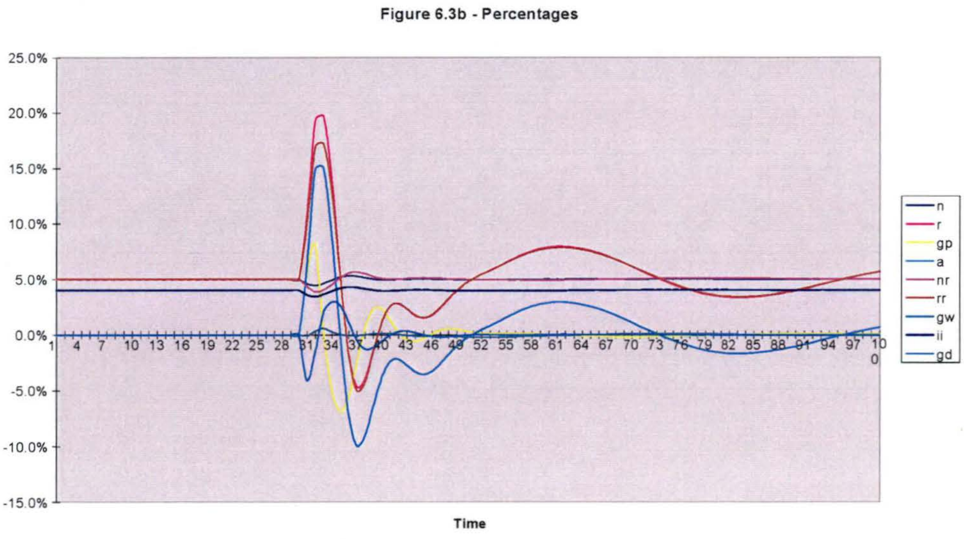
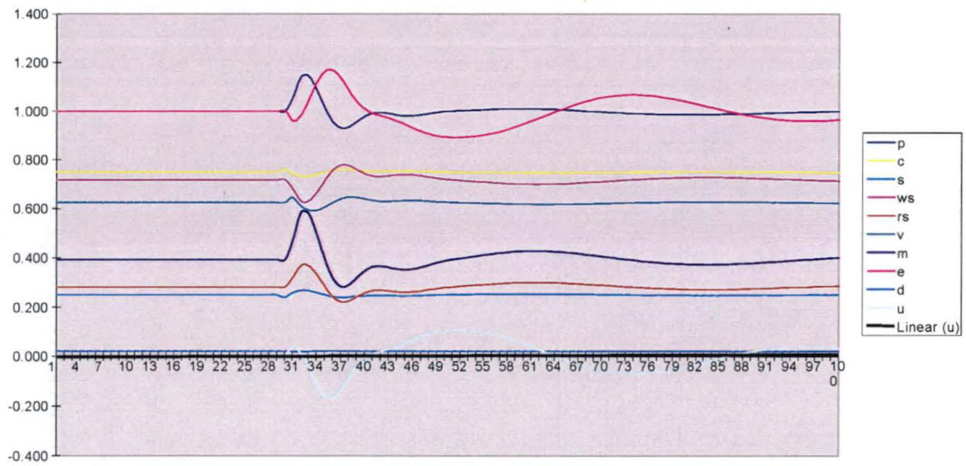


Figure 6.3b above shows the same for the profit rate (r) and the profitability gap (a).

The ensuing cycles are convergent or *damped*, as confirmed by the behaviour of the unemployment rate (u) in Figure 6.3c below. This guarantees ultimate convergence to a terminal stationary state, albeit in far more than the 70 years already traversed. There exists a final stationary state, but the traverse path by which it is approached is of extremely long duration. This explains why the stationary-state sensitivity analysis performed below has to extend over *200 years* of simulated historical time.

Figure 6.3c - Ratios



6.12 Analysing the Sensitivities

For consumers, Model E changes their previous inflexible rules of (a) retaining Qf sacks pa of foodcorn (if farmers) and (b) spending all their wage income (if workers) and interest income (if depositors or bankers) on the Qso sacks pa of foodcorn supplied to the weekly markets. In Models B, C and D, the corn price is flexible, but the rules are not. In Model E, the new rule for all consumers involves the flexible allocation of one's household income (whether derived from wages, interest or lagged profit) between purchasing foodcorn and building bank deposits. In making their allocation decision between consumption and saving, these (now-sovereign) consumers look to the corn price, the interest rate reciprocal and their total household income.

This final instalment of increased flexibility has a massive effect on economic outcomes. The economy becomes extremely unstable and takes more than *six times as long* to traverse from a stationary to a steady state when workforce growth occurs. It is important to determine just how *sensitive* the Model E corn-credit economy is to changes in all the parameters that control its evolution through simulated historical time. The Estat spreadsheet is extended by another century to create a new spreadsheet. Estat200 is used to perform this sensitivity analysis, starting in year 1 of its reference flatline stationary-state basecase and not finishing until year 200. The reason for adding an extra 100 years is to allow plenty of time for the damped cycles of Model E to settle down into a terminal stationary state. Sensitivities are determined after multiplying the values of parameters, one at a time, by 1.001 and allowing each resultant traverse to run its course over the next two simulated centuries.

Table 6.7 below reports the results. The tranquillity of the basecase stationary state is shattered by separately imposing a +0.1% *ceteris paribus* perturbation on each of the model's initial values and constants during year 1. The resulting percentage differences (between the terminal-stationary-state data of year 200 and the initial-stationary-state solution values for the same year, i.e. the Effects) are divided by 0.1% (i.e. the Cause) and reported as a Sensitivity Matrix of 17 parameter columns, each containing 55 Elasticity rows, one for each model variable.

Each column is headed by the short name of the particular initial value or constant whose year-1 change produces these ratio measures of the model's sensitivity:

$$\text{Elasticity} = \%\Delta\ \text{Effect} \ / \ \%\Delta\ \text{Cause}$$

For each Cause there are 55 Effects, so the above formula is applied 55 times to generate each column of Table 6.7 below. The Effects are percentage changes in all 55 model variables and the Cause is a +0.1% change in the value of one specific parameter, yielding a column of 55 Elasticities.

There is room for 935 elasticities in the [55 x 17] Sensitivity Matrix. However, four parameter columns and five variable rows are marked "na" (meaning "not applicable") because these particular elasticities cannot be computed, leaving [50 x 13] = 650 elasticities which can. The reason is two-fold. First, the four constants ϕ, ε, ρ, and δ, respectively, multiply the variables a, (e – 1), gpo, and gd, all of which are zero in the opening stationary state and remain so after *ceteris paribus* perturbations of their associated parameters. Secondly, with the five variables a, gp, gd, u, and gw being zero in the opening stationary state, there is no possibility of computing the numerators of their associated elasticity ratios.

Despite a +0.1% perturbation being used to generate the positive (+x) and negative (-y) elasticities in Table 6.7, a conventional interpretation of the numerical results (reading down each column) would be: a *one* per cent increase in that column's parameter (say, Qiz) produces a +x% increase (or a –y% decrease) in each row variable (Q, Qi, L, r, P, ..., m) of that particular column.

Table 6.7 – Model E Sensitivity Matrix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	**sn**	**Qiz**	**wz**	**iz**	**ϕ**	**θ**	**λ**	**φ**	**κ**	**η**	**ε**	**ρ**	**δ**	**μ**	**α**	**β**	**χ**	**γ**
2																		
3	**Q**	0.02	0 01	-0 01	na	0.00	1 00	0.00	-0 03	1 01	na	na	na	0 04	0.00	-0 06	0 05	0 16
4	**Qi**	0 03	0.02	0 00	na	-0 99	0 99	0 00	-0 02	1 01	na	na	na	0 01	0.00	-0 01	0 02	0 08
5	**L**	0 02	0.01	-0 01	na	0 00	0 00	0 00	-0 03	1.01	na	na	na	0 04	0 00	-0.06	0 05	0.16
6	**r**	0 03	0.44	1 14	na	1 45	-0 45	0 18	1 79	-0 02	na	na	na	-2 83	-0.05	2 11	-1 37	-3 42
7	**P**	0 00	0 01	-0 91	na	-1 27	-0 01	0 02	-1 42	0 00	na	na	na	2 27	0 11	-4 93	3 19	7 45
8	**w**	0 00	-0 01	-0.96	na	-0 98	1.01	0.01	-1 47	0 00	na	na	na	2 40	0 11	-5 02	3 25	7 60
9	**in**	0 00	0 00	0.92	na	1.28	0 00	-0 02	1 43	0 00	na	na	na	-2 28	0.00	-0 01	0 01	0 04
10																		
11	**W**	0.02	0.00	-0 97	na	-0 98	1 01	0 01	-1 51	1 01	na	na	na	2 43	0.11	-5 08	3.30	7 76
12	**Ka**	0 02	0 01	-0 92	na	-2 27	0 99	0 02	-1 45	1 01	na	na	na	2 31	0 11	-4 99	3.24	7 61
13	**Kb**	0 02	0 00	-0 93	na	-0 95	1.00	0 02	-2 47	1 01	na	na	na	2 34	0 11	-5 05	3.29	7 71
14	**K**	0 02	0 01	-0 93	na	-1 48	1 00	0.02	-2 06	1 01	na	na	na	2 33	0 11	-5 03	3 27	7 67
15	**R**	0 05	0 45	0 21	na	-0 03	0.55	0.20	-0 28	0 99	na	na	na	-0 51	0 06	-2 93	1 90	4 22
16	**n**	0 00	0 00	0 73	na	1 02	0 00	0.18	1 14	0 00	na	na	na	-1 83	0 00	-0 01	0 01	0 04
17	**a**	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na
18	**Qs**	0 02	0.00	-0.02	na	0.33	1 01	0 00	-0 04	1.01	na	na	na	0.04	0.00	-0.08	0 07	0.18
19	**e**	0.02	0.01	-0 01	na	0 00	0 00	0 00	-0 03	0.01	na	na	na	0.04	0 00	-0.06	0 05	0.16
20	**p**	0 00	0 01	-0 91	na	-1 27	-0 01	0 02	-1 42	0 00	na	na	na	2.27	0.11	-4 93	3 19	7 45
21	**gp**	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na
22	**D**	0 02	0 00	-0.97	na	-0 98	1 01	0.01	-1 51	1 01	na	na	na	1 43	0.11	-5 08	3.30	7 76
23	**d**	0 00	0 00	-0 03	na	0 51	0.00	-0 01	0 57	0 00	na	na	na	-0 91	0 00	-0 01	0 00	0 01
24	**gd**	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na
25	**J**	0 02	0 00	-0 06	na	0.30	1 01	-0 01	-0.08	1 01	na	na	na	-0 85	0 11	-5 09	3 32	7 80
26	**Yh**	0 02	0 02	-0 92	na	-0 94	0 99	0.02	-1 46	1 01	na	na	na	2 31	0 11	-4 99	3 25	7 61
27																		
28	**Y**	0 02	0 01	-0 92	na	-1.27	0 99	0 02	-1 45	1 01	na	na	na	2.31	0.11	-4.99	3 24	7 61
29	**Rg**	0 02	0 06	-0 79	na	-2 02	0.94	0.04	-1 32	1 01	na	na	na	1 99	0.10	-4 76	3 10	7.23
30	**Rn**	0.05	0 44	0.21	na	-0 03	0.55	0 20	-0.28	0 99	na	na	na	-0.52	0 06	-2.96	1 92	4.29
31	**Yr**	0 02	0 01	-0 01	na	0 00	1 00	0.00	-0.03	1 01	na	na	na	0 04	0 00	-0 06	0.05	0.16
32	**Wr**	0 02	-0 01	-0 06	na	0 29	1 02	-0 01	-0 09	1 01	na	na	na	0.16	0.00	-0 15	0 11	0 30
33	**Rgr**	0 03	0.06	0.11	na	-0 75	0 95	0 02	0 10	1 01	na	na	na	-0 28	0.00	0 17	-0.09	-0 21
34	**Rnr**	0 06	0 43	1 12	na	1 24	0.57	0.18	1 14	1.00	na	na	na	-2 78	-0.04	1 97	-1 27	-3.14
35	**Rr**	0.06	0 44	1 12	na	1 24	0.56	0.18	1 14	0 99	na	na	na	-2 78	-0.04	2 01	-1 29	-3 20
36	**C**	0.02	0 01	-0 92	na	-0 94	0.99	0 02	-1.46	1 01	na	na	na	2 32	0 11	-5 00	3.25	7 63
37	**c**	0 00	0 00	0 00	na	0 33	0 00	0 00	0 00	0 00	na	na	na	0 01	0 00	-0 02	0 01	0 03
38	**Cr**	0 02	0 00	-0.02	na	0.33	1 01	0 00	-0 04	1 01	na	na	na	0 04	0.00	-0 08	0 07	0 18
39	**Yhr**	0 03	0 01	-0.01	na	0 33	1.00	0 00	-0 04	1.01	na	na	na	0 04	0.00	-0 06	0 06	0 16
40	**S**	0 02	0.02	-0 91	na	-2 26	0 98	0 02	-1 44	1.01	na	na	na	2.29	0.11	-4.94	3 21	7 53
41	**s**	0 00	0 01	0.01	na	-0 99	-0.01	0 00	0 01	0 00	na	na	na	-0 02	0 00	0 05	-0 03	-0 08
42	**Sr**	0.03	0 02	0 00	na	-0.99	0 99	0 00	-0.02	1 01	na	na	na	0.01	0 00	-0.01	0.02	0.08
43	**I**	0.02	0.02	-0 91	na	-2.26	0 98	0 02	-1 44	1 01	na	na	na	2 29	0 11	-4 94	3.21	7.53
44	**Ir**	0.03	0.02	0 00	na	-0.99	0 99	0 00	-0.02	1.01	na	na	na	0.01	0 00	-0 01	0 02	0 08
45	**ir**	0.01	-0 01	1.83	na	2 55	0.02	-0.04	2 86	0 00	na	na	na	-4 54	-0 11	4 94	-3 17	-7 35
46	**wr**	0.00	-0 02	-0.05	na	0 29	1.02	-0.01	-0 05	0.00	na	na	na	0 12	0.00	-0.09	0 06	0 15
47	**rr**	0 04	0 43	2 05	na	2 72	-0 44	0.16	3 21	-0.01	na	na	na	-5.09	-0.15	7.07	-4 54	-10.79
48	**ws**	0 00	-0 02	-0.05	na	0 29	0.02	-0 01	-0 05	0 00	na	na	na	0 12	0 00	-0 09	0.06	0.15
49	**rs**	0 00	0 05	0 13	na	-0 75	-0 05	0 02	0 13	0 00	na	na	na	-0 32	0.00	0 23	-0 15	-0 37
50	**k**	0 00	-0 01	-0 01	na	0 99	0 01	0 00	-0 01	0 00	na	na	na	0.02	0 00	-0 05	0 03	0 08
51	**Kr**	0 02	0.00	-0 02	na	-0 21	1.01	0 00	-0 65	1 01	na	na	na	0 05	0 00	-0.10	0 08	0 22
52	**v**	0 00	-0 01	-0 01	na	-0 21	0.01	0 00	-0 61	0 00	na	na	na	0 02	0 00	-0 04	0 02	0 06
53	**x**	0 00	-0 01	-0 01	na	-0.21	1.01	0 00	-0 61	0 00	na	na	na	0 02	0 00	-0 04	0 02	0 06
54	**gw**	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na
55	**nr**	0.01	-0 01	1.64	na	2.29	0 02	0 16	2.57	0 00	na	na	na	-4.09	-0 11	4 94	-3.17	-7.36
56	**u**	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na	na
57	**pc**	0 00	-0 01	-0 96	na	-0 98	0 01	0.01	-1 47	0.00	na	na	na	2.40	0 11	-5 02	3 25	7.60
58	**mn**	0 00	0 05	-0 78	na	-2.01	-0 06	0.04	-1 29	0.00	na	na	na	1.95	0 10	-4.70	3 04	7 07
59	**m**	0 01	0 07	0 18	na	-1 04	-0.07	0.03	0.18	0.00	na	na	na	-0 44	-0.01	0 32	-0 21	-0 52

Neither these parameter perturbations, nor those in the Misallocation Scenario, can be interpreted as part of some time-stream of random exogenous shocks, as used to perturb Neoclassical business cycle models ever since Frisch (1933) introduced the notion. Changes in these parameters during year 1 are, in fact, *permanent* and represent *fundamental revisions* of history (the initial values) or *fundamental shifts* in biology, technology or psychology (the constants) within the corn-credit economy. Such changes have manifold hysteretic and path-dependent consequences that may take many decades of simulated historical time for the economy to digest.

After quoting Abram Bergson ("Time is a device to prevent everything from happening at once") on the title page of *Essays in the Theory of Economic Growth*, Robinson (1962, p 6) points out that the time taken to approach equilibrium from an arbitrary initial starting point may be long and "... in some circumstances *indefinitely long*." [Italics added]. After noting that "Walras guarded himself by supposing that the equilibrium position is discovered before any trade takes place", Robinson concludes: "Thus, when changes in the conditions are liable to occur, the analysis predicts that equilibrium is not likely ever to be realised."

On the one hand, Robinson may have been too strict. The fully flexprice Model E corn-credit economy may take more than a century to attain the long-period equilibrium position of a terminal stationary state, but it *does* get there in the end. On the other hand, this result is established only for tiny (+0.1%) parameter perturbations in the sensitivity analysis of this section.

During year 1 of the reference stationary state, an initial value or a constant suddenly increases by 0.1%. This opens up a non-zero profitability gap between the realised and normal profit rates, thus altering the volume of seedcorn invested (Qi sacks pa) to a degree governed by the size of the reaction coefficient (ϕ). This event destroys farmers' long-held conventional judgements (i.e. their expectations of the future profitability of maintaining capital investment at its existing growth rate) and the economy begins its long disequilibrium traverse process of adjusting to the new realities by slowly converging on a new stationary state.

Table 6.7 above has 13 substantive parameter-columns. Focussing on the largest *absolute* elasticities (i.e. the |+x| and |-y| values caused by a +0.1% parameter-change), one discerns a definite size ranking of the parameters with respect to the variables they affect. The most striking feature is that Column R contains far and away the largest elasticities. Furthermore, the Column P contains the second-largest set of 55 elasticities. These results indicate the *power wielded by sovereign consumers* over all economic outcomes, *via* their income (γ) and own-price (β) elasticities of demand, respectively. A one percent change in γ (β) causes effects that range in size up to -10.79 (7.07) percent, whereas the next most powerful parameter (wage bill turnover, μ) causes a -5.09 percent maximum effect.

Table 6.8 below ranks the 13 parameters according to their largest absolute elasticities and shows the particular variable(s) on which they have these maximum effects. Each parameter has its greatest effect on one or two variables, except for the workforce (η), whose effect is pervasive. A one percent increase in the number of workers raises each of *two dozen* variables by 1.01 percent. It acts like a simple scaling factor for the entire corn-credit economy, a property already harnessed to generate steady states of 1% pa growth from parent stationary-state solutions in Models C, D and E.

Table 6.8 – Maximum Sensitivity Parameters

Parameter	**Elasticity**	**Variables Maximally Affected**
γ	-10.79	rr
β	7.07	rr
μ	-5.09	rr
χ	-4.54	rr
κ	3.21	rr
θ	2.72	rr
iz	2.05	rr
λ	1.02	wr, Wr
η	1.01	Q,Qi,L,W,Ka,Kb,K,Qs,D,J,Yh,Y,Rg, Yr,Wr,Rgr,C,Cr,Yhr,S,Sr,I,Ir,Kr
wz	0.45	R
φ	0.20	R, Rn
α	-0.15	rr
Qiz	0.06	Rr, Rnr

Table 6.8 also shows that nine of the 13 parameters has its *maximum* influence on variables which primarily concern the capitalist farmers. Eight have their greatest effects on the real profit rate, while another three maximally affect flows of profit and interest. This is hardly unexpected in a Post-Keynesian model where entrepreneurial investment decisions, based on comparing expected returns with known costs of capital, rule the roost.

The *ceteris paribus* results revealed by the columns of Table 6.7 include

- Higher seedcorn yield (θ) is associated with lower corn price (P) and higher real wage (wr) and real profit (rr) rates.
- Higher labour productivity (λ) is associated with lower corn price (P) and higher wages (w, wr)
- Higher risk premium (φ) is associated with higher corn price (P), markup (m), profits (R, Rr), and profit rates (r, rr).
- Higher price elasticity (β) or capital turnover (κ) is associated with a higher saving ratio (s), yet physical investment (Qi) and production (Q) both fall, as do corn price (P), gross

product (Y) and household income (Yh), confirming the "paradox of thrift" of Keynes (1936, p 111).

- Higher seedcorn yield (θ) is associated with changes in the real wage (w) and real profit rate (rr) that have the same (not opposite) signs, confirming the "paradox of costs" of Rowthorn (1981, p 18) since labour productivity (λ) has not increased.
- For all parameter changes, household income (Yh) and consumption (C) elasticities are similar.
- For all parameter changes, the wage (ws) and gross surplus (rs) share elasticities have opposite signs and their low values indicate that they remain fairly stable through time.

Overall, the *long-term* sensitivity analysis confirms certain comparative static results, accords with some empirical stylised facts and exhibits the paradoxes of thrift and costs. Of even greater interest is the corn-credit economy's *short-term* traverse behaviour during the long-duration process of adjustment, and this is addressed in the next four sections.

6.13 Generating Single Traverses

When discussing cycles, growth and distribution, economists tend to focus on real gross product (Yr) as an overall summary measure of economic activity. Therefore, the traverses generated below are discussed mainly in terms of the behaviour of real gross product over the 200-year time-span of simulated history. To select the most revealing traverse experiments, the 13-parameter sensitivity analysis of Table 6.7 is again consulted. Those parameters to which the Yr variable is most responsive when they are perturbed need to be identified. The size ranking of initial values and constants, according to their absolute elasticities with respect to real gross product, is presented in Table 6.9 below.

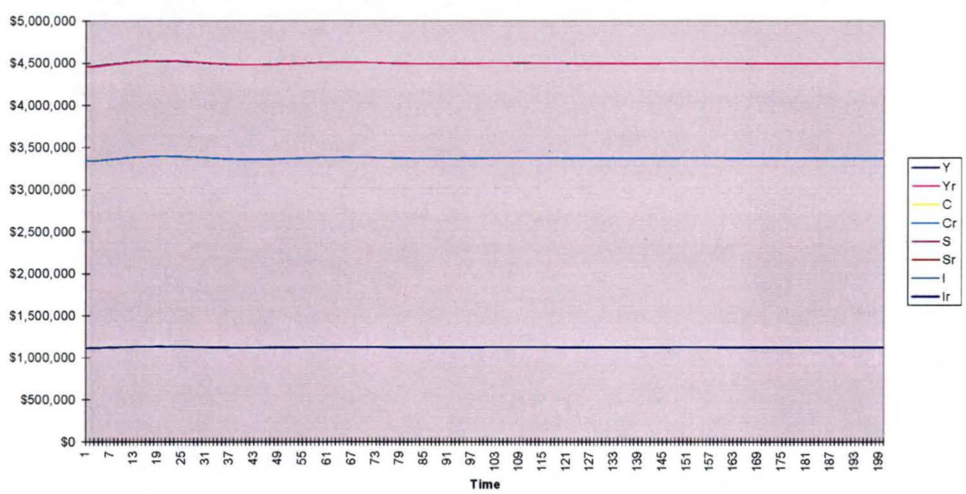
Table 6.9 – Real Gross Product Elasticities

Parameter	**Yr-Elasticity**
η	1.01
λ	1.00
γ	0.16
β	-0.06
χ	0.05
μ	0.04
κ	-0.03
Qiz	0.02
wz	0.01
iz	-0.01
θ,φ,α	0.00

From the top end of this ranking are selected η, λ, γ, β, χ, and μ as the six most highly-elastic parameters to perturb, so as to generate the real gross product (Yr) traverse plots displayed in Figures 6.4a through 6.4f below.

The parameter perturbations selected to generate the set of six observed traverses must, as always, operate *via* the central profitability gap mechanism of these corn-credit models. The aim is to initiate all traverses by a sudden change in a single constant (Figures 6.4a through 6.4d) or initial condition (Figure 6.4e and 6.4f) during year 1. This maintains the *ceteris paribus* assumption, guaranteeing that every movement away from the flatline time path characterising the opening stationary state must be due to that parameter change and to nothing else.

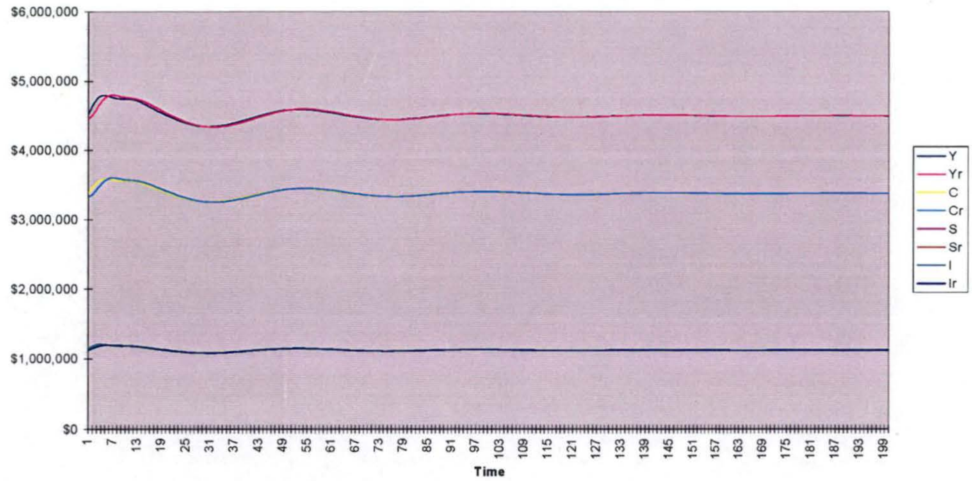
Figure 6.4a - Expenditures [η*1.01]



Despite the constant η having the largest *long-term* effect on real gross product, Figure 6.4a above shows that its *short-term* traverse behaviour is almost non-existent.

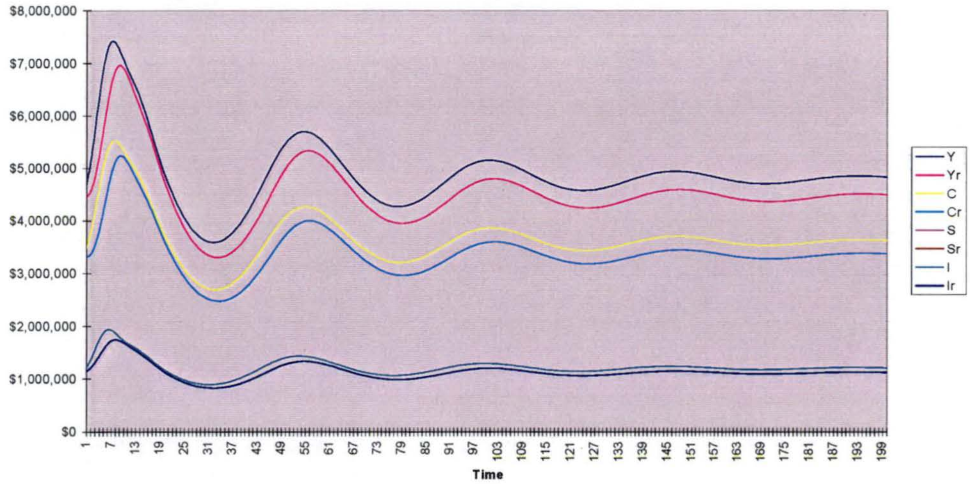
The same is not true of Figure 6.4b below, which shows that perturbing the second-ranked constant λ initiates converging cycles of real gross product.

Figure 6.4b - Expenditures [λ*1.01]



Violent traverse behaviour sets the γ apart from the others, with Figure 6.4c below showing that it has the largest short-term effect of all six constants.

Figure 6.4c - Expenditures [γ*1.01]



The second-largest short-term effect is generated by the traverse of β, as displayed in Figure 6.4d below.

Figure 6.4d - Expenditures [β*1.01]

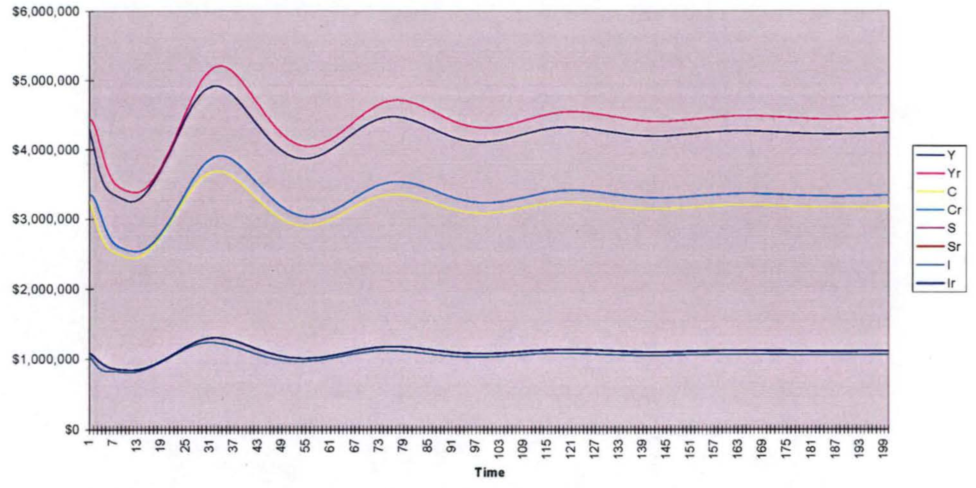


Figure 6.4e below tracks the traverse initiated by perturbing χ, the cross elasticity between foodcorn and bank deposits. It is not as violent as the traverses of γ or β above.

Figure 6.4e - Expenditures [χ*1.01]

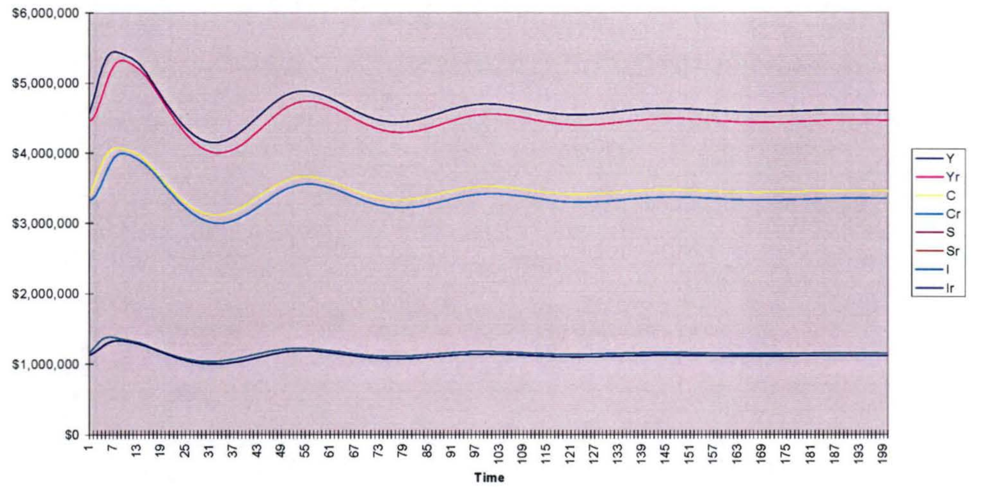
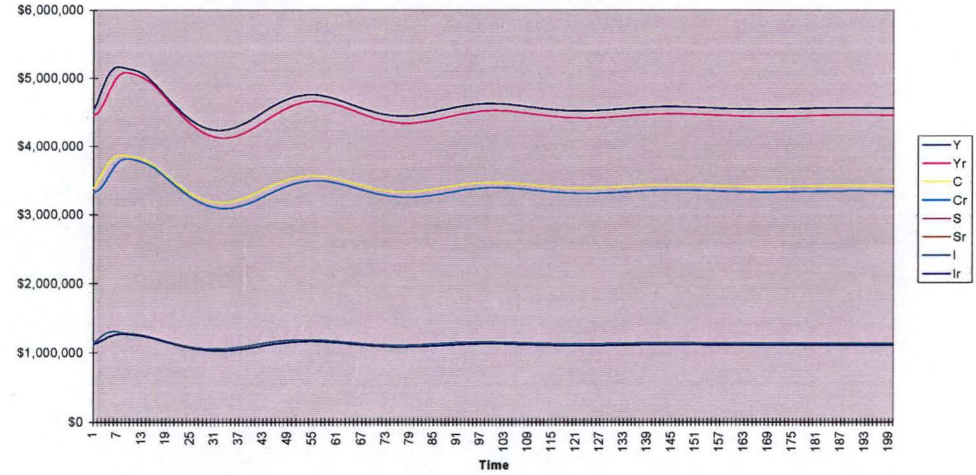


Figure 6.4f below shows that a perturbation of μ causes significant short-term cyclical behaviour of real gross product, even though its has the smallest long-term effect of all six constants.

Figure 6.4f - Expenditures [μ*1.01]



These six graphs reveal that

- there are some *four* 40-year cycles of activity in real gross product and its components;
- the first cycle initially peaks for all constants except β, which has a negative sign;
- all cycles are convergent (i.e. damped), so a terminal stationary state must eventuate;
- however, for constants γ and β the system is still cycling in year 200; and
- for the other four constants, a terminal stationary state does occur within 200 years.

The long-term ranking of constants (from Table 6.9 above) is

Largest η λ γ β χ μ Smallest

whereas their short-term ranking (from Figures 6.4a through 6.4f above) is

Largest γ β χ μ λ η Smallest

This indicates that there is no correlation between the long-term Yr-elasticities and the short-term Yr-traverses. The conclusion of section 6.12 above that the sensitivity analysis confirms certain comparative static results *cannot* be used to justify the use of "economic intuition" (derived as it is from comparative static exercises) for short-period economic analyses.

6.14 Generating Multiple Traverses

The stationary state of Model E is kicked into convergent cycles when even one parameter undergoes a slight change. Yet long-period *constancy* of parameters is merely a convenient assumption, made for purposes of modelling and *ceteris paribus* experimentation. Three parameters are historically-given "initial values", but the other 14 are classified as "constants" because (a) they are thought to change less frequently than those quantities classified as "variables" or (b) their own composition and/or dynamic evolution has yet to be modelled.

Neoclassical theory follows Frisch (1933) and Slutsky (1937) in relying on exogenous "shocks" (such as sudden parameter-changes) to create and propagate business cycles. Post-Keynesian theory, while recognising the endogeneity of business cycles, respects the Marx-Kalecki-Robinson doctrine that the economy is always in traverse. As Richard Goodwin (1997, p 162) notes, the Keynesian multiplier – as originally formulated by Richard Kahn (1931) – is not a single-valued parameter, but a dynamic, temporal sequence that eventually asymptotes to a final equilibrium, so that

> ... at any one time the economy is subject to a large number of different stimuli in various stages of decay. The sum of all these coexisting, diminishing effects will be, for any particular historical stretch, a highly complicated, irregular time series. This aspect of the theory becomes crucial in trade cycle and policy analysis.

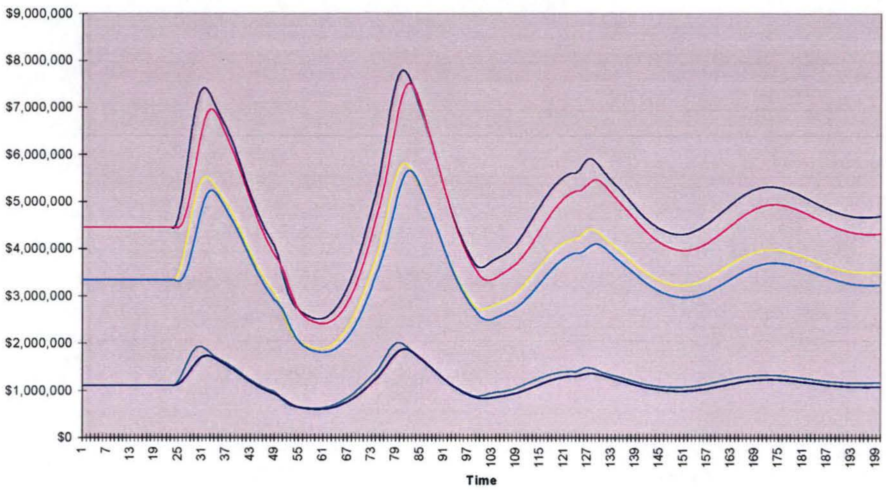
Quite apart from these intrinsic dynamics, however, the economy may experience parameter-changes sparking off a new traverse before one or more earlier traverses have run their courses. No school of thought accepts the "long-period constancy of parameters" assumption deployed to generate the 100- and 200-year traverses analysed in this thesis. This assumption is entertained solely to prevent experimental results from becoming *contaminated* by more than one parameter-change at a time.

To demonstrate some effects of relaxing the "long-period constancy of parameters" assumption in the present research work, a 200-year "multiple traverse" is generated by making a new traverse grow out of the economy's incomplete prior traverse(s) at 25-year intervals. The short-term ranking of effects on real gross product (from Figures 6.4a through 6.4f above) is used, viz.

Largest γ β χ μ λ η Smallest

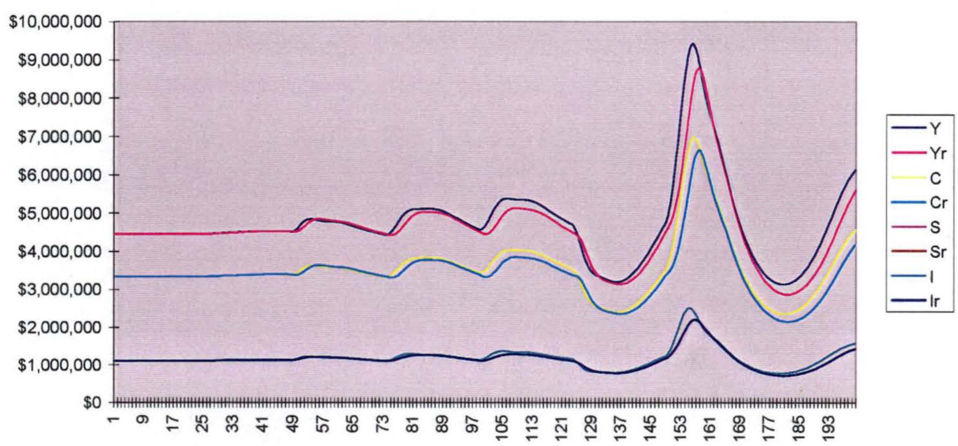
Every 25 years, the relevant parameter changes by +1% and this generates the multiple traverse economic growth paths plotted in Figure 6.5 below for Yr and its expenditure components. The parameters are perturbed in rank-order starting with the largest, the income elasticity of demand (γ).

Figure 6.5 - Expenditures



The time-series plots still show four damped cycles in real gross product and its expenditure components. Convergence is assisted by the diminishing short-term effects of the parameters being altered, but the corn-credit economy does not traverse to a terminal stationary state by year 200. This is despite the sixth traverse being ignited in year 150 by the *least* sensitive parameter (η) and having *50 years* (not 25) in which to attain a new fully-adjusted state.

Figure 6.6 - Expenditures



In Figure 6.6 above, the opposite (i.e. increasing) size-order of ranked parameter-changes is used, one being applied every 25 years as before.

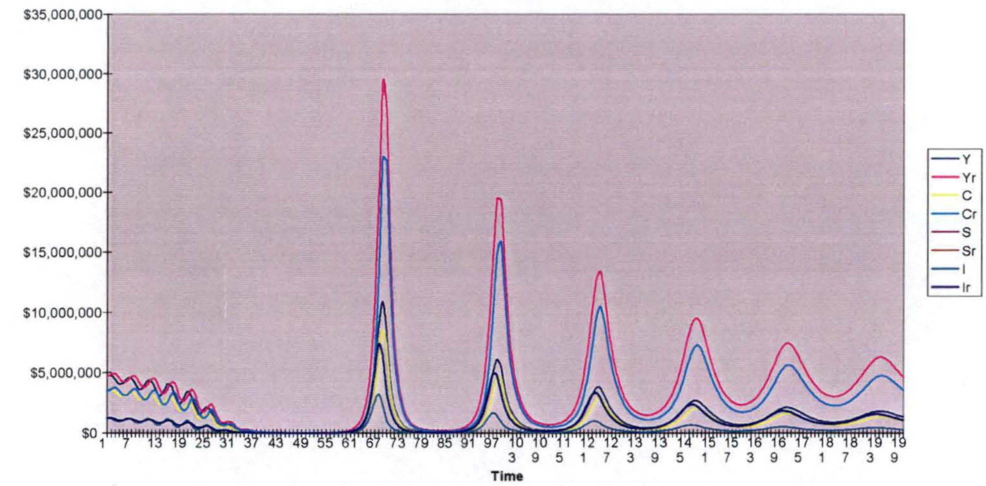
This time, the sixth and final traverse is ignited by the parameter of greatest short-term sensitivity (γ), so it is not surprising that this initiates a final cycle of large amplitude in real gross product and its expenditure components.

6.15 Viability and Convergence

An economy experiencing divergent cycles is not viable because, like the white "killer waves" in the world's oceans, a cycle of unsustainably large amplitude must eventually develop and bring on a catastrophic collapse. Even if the cycles are convergent, one or more crucial parameters might take on extreme values, causing the economy to collapse during its first high-amplitude cycle following the perturbation. This *range of viability* encloses a smaller *range of convergence*, i.e. one in which all cyclical fluctuations are damped, for otherwise the economy could not be classified as dynamically *stable*. Real-world capitalist economies – although notoriously unstable in the short term – seem to be both convergent and viable over the long term. In Model E, experimentation shows that viability and convergence are controlled by one key parameter, viz. the reaction coefficient governing seedcorn investment by farmers.

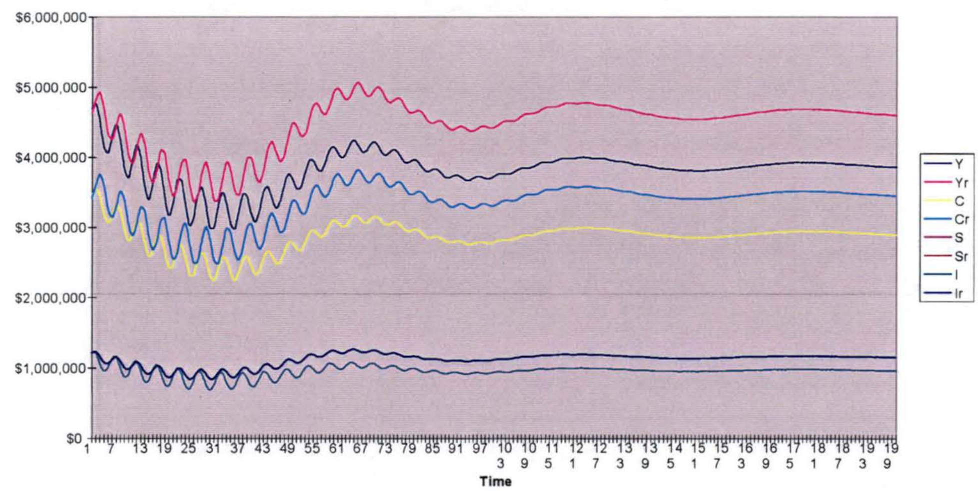
Figures 6.7a through 6.7f below are time-series plots of Model E's dynamic behaviour following a one percent increase in the income elasticity constant (γ). This is done to generate cycles in real gross product (Yr), which the experiment requires to assess the viability and convergence of Model E. Each graph results from specifying a different value for the crucial reaction coefficient (ϕ = 0.4432 is the default value), which fixes the proportion of this year's profitability gap (a = [r – n]% pa) that will be "passed through" as a percentage change in seedcorn invested relative to last year's volume, in accordance with structural-form equation (B) in Table 6.1 above.

Figure 6.7a - Expenditures [ϕ = 0.8218]



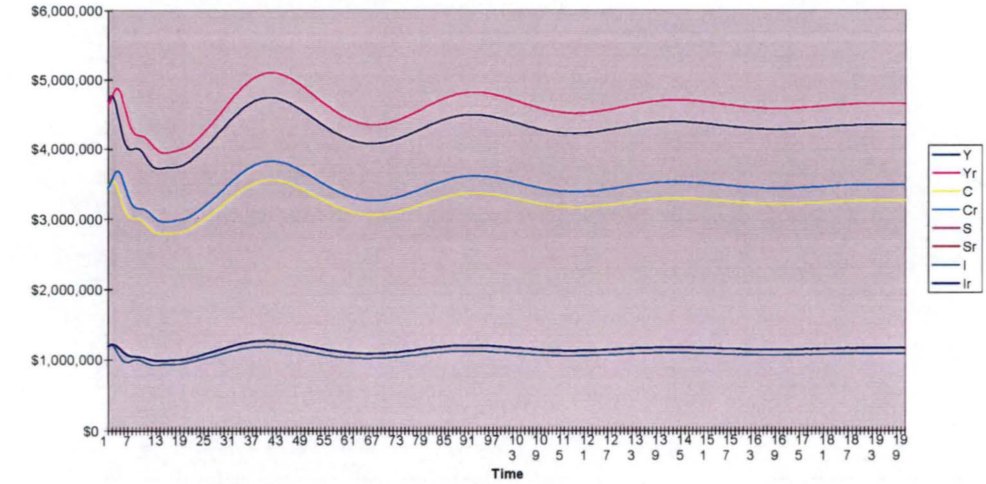
In Figure 6.7a above, six converging cycles emerge in year 60 after 35 years of high-frequency cycles and 20 years of depression.

Figure 6.7b - Expenditures [ϕ = 0.7465]



Three converging cycles with high-frequency epicycles characterise Figure 6.7b above.

Figure 6.7c - Expenditures [ϕ = 0.5147]



Four converging cycles are evident in Figure 6.7c above.

Figure 6.7d - Expenditures [ϕ = 0.4432]

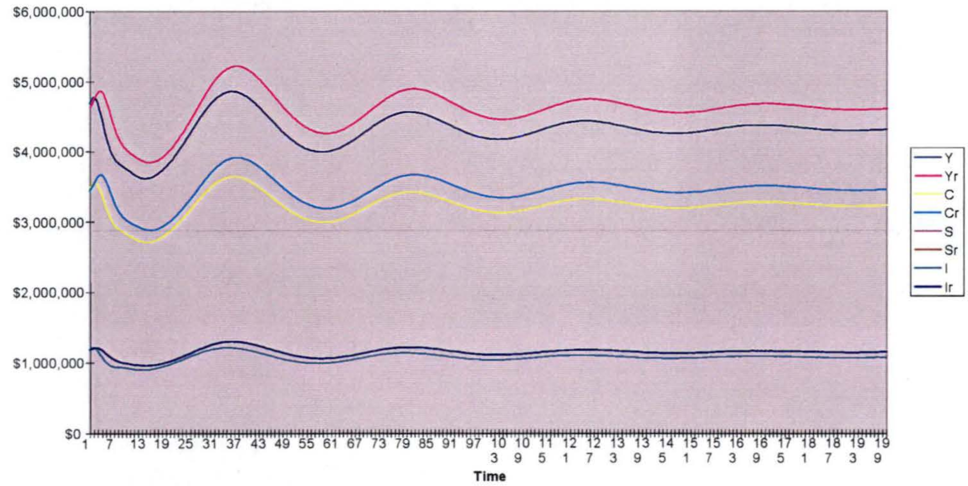
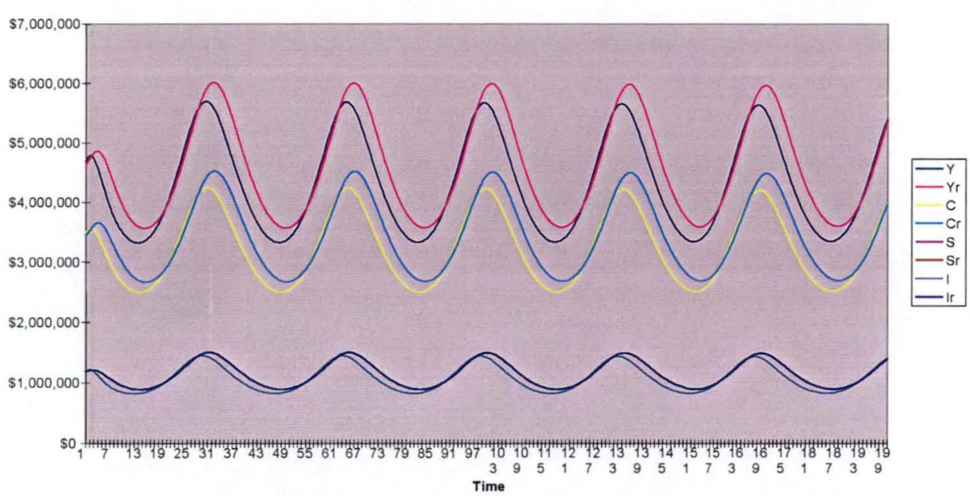


Figure 6.7d above displays five converging cycles.

Figure 6.7e - Expenditures [ϕ = 0.3751]



Six limit cycles are generated when ϕ = 0.3751 in Figure 6.7e above.

Figure 6.7f - Expenditures [ϕ = 0.3724]

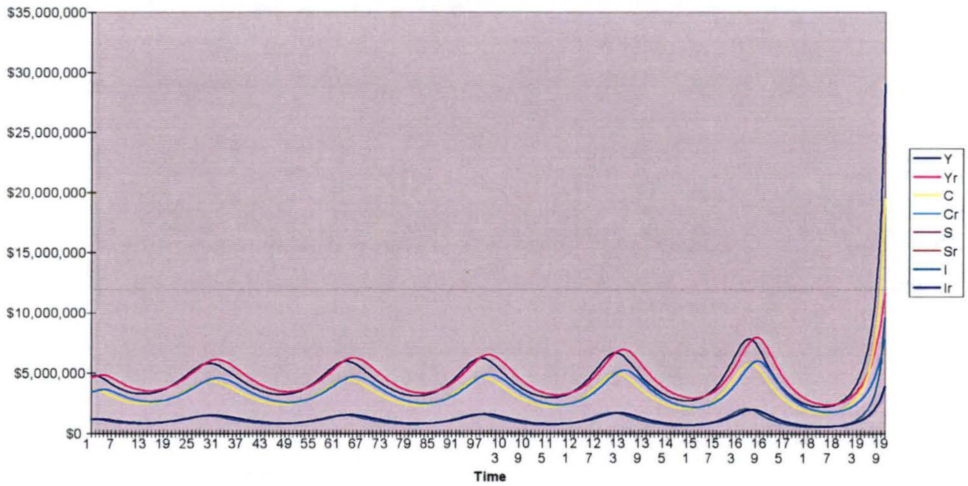


Figure 6.7f above shows six diverging cycles, with the final upswing suggesting an extreme amplitude for this particular cycle.

Table 6.10 below summarises these effects of raising and lowering the reaction coefficient from its default value of ϕ = 0.4432.

Table 6.10 – Effects of Changing the Reaction Coefficient

Figure	Reaction Coefficient	Dynamic Behaviour
Not shown	**0.8219**	System collapses in year 76 after an unsustainably high peak in its first cycle
6.7a	**0.8218**	Six converging cycles emerge in year 60 after 35 years of high-frequency cycles and 20 years of depression
6.7b	**0.7465**	Three converging cycles with high-frequency epicycles
6.7c	**0.5147**	Four converging cycles
6.7d	**0.4432**	Five converging cycles
6.7e	**0.3751**	Six limit cycles
6.7f	**0.3724**	Six diverging cycles
Not shown	**0.3723**	System collapses in year 200 after an unsustainably high peak in its sixth cycle

Clearly the corn-credit economy has a large degree of dynamic stability, as shown by the wide "range of convergence". Within the range ϕ = 0.3751 to ϕ = 0.8218, traverses *always* converge towards a fully-adjusted state. Beyond this, the economy's "range of viability" (from ϕ = 0.3724 to ϕ = 0.8218) is slightly wider and this small piece of extra territory is bordered by limit cycles and populated with diverging cycles. This is consistent with the stylised fact that real-world economies only rarely experience divergent cycles and system collapse.

In the final Chapter 8 of this thesis it is conjectured that incorporating one specific aspect of investor decision-making ("susceptibility") into Model E will guarantee that the abstract corn-credit economy *never* loses viability. Converting the exogenous reaction coefficient (ϕ) into an endogenous reaction function – should serve to keep the economy within its range of viability, regardless of the size and direction of any parameter-change.

The pure flexprice corn-credit economy has no unique Harrodian warranted rate of growth, hence no need for such conventional dynamic stability mechanisms as a capital-output ratio which changes with "factor prices" (Neoclassicals) or a saving ratio that flexes with the income shares going to labour and capital (Post-Keynesians). As with the real interest rate, both these ratios are mere derived aggregates providing no feedback into the structural form.

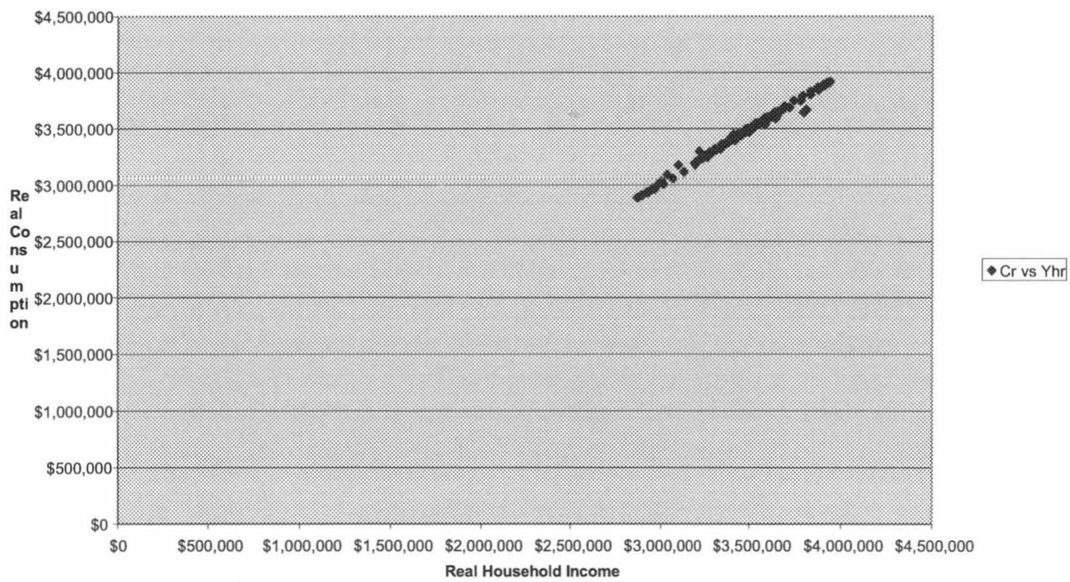
Model E can cycle, grow and distribute income within a wide range of operation, without relying on ceilings, floors or specific adjustment mechanisms. A broad "range of convergence" is one of its inherent properties.

The existence of ranges (not upper limits) for both convergence and viability recalls the analyses of Duménil & Lévy (1993, 1999). They argued that reaction coefficients must be neither too large *nor too small* for the economy to remain within its corridor of stability.

6.16 Some Theoretical Implications

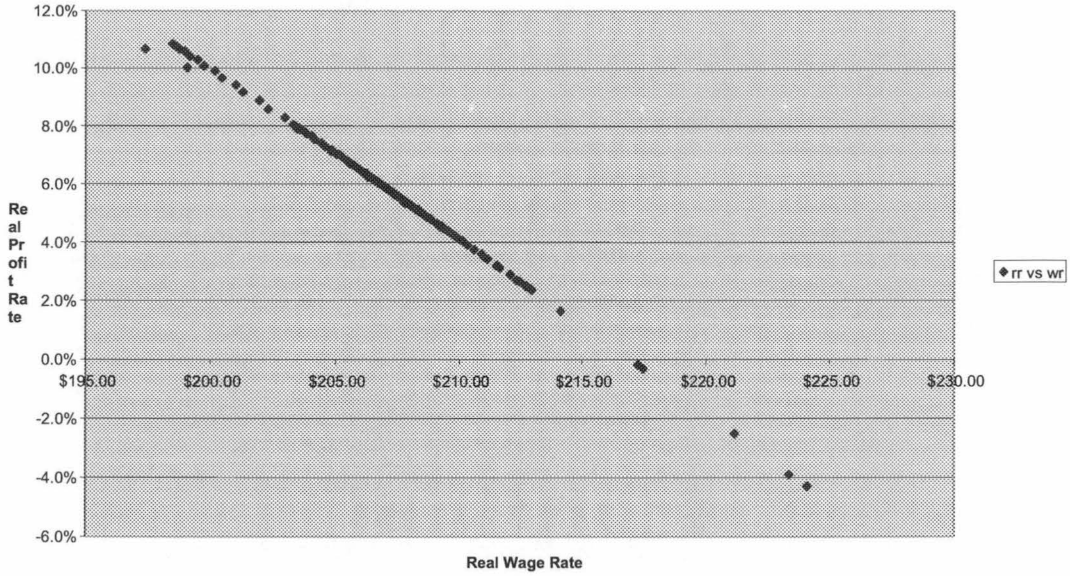
Figures 6.8a through 6.8f below are 2-D phase diagrams for six variable-pairs that are familiar to mainstream economists. Each pair of variables is scatter-plotted for the 200-year dynamic process portrayed in Figure 6.7d above, which depicts a situation when the default reaction coefficient (ϕ = 0.4432) governs seedcorn investment and the cycles are converging.

Figure 6.8a - Real Consumption vs Real Household Income



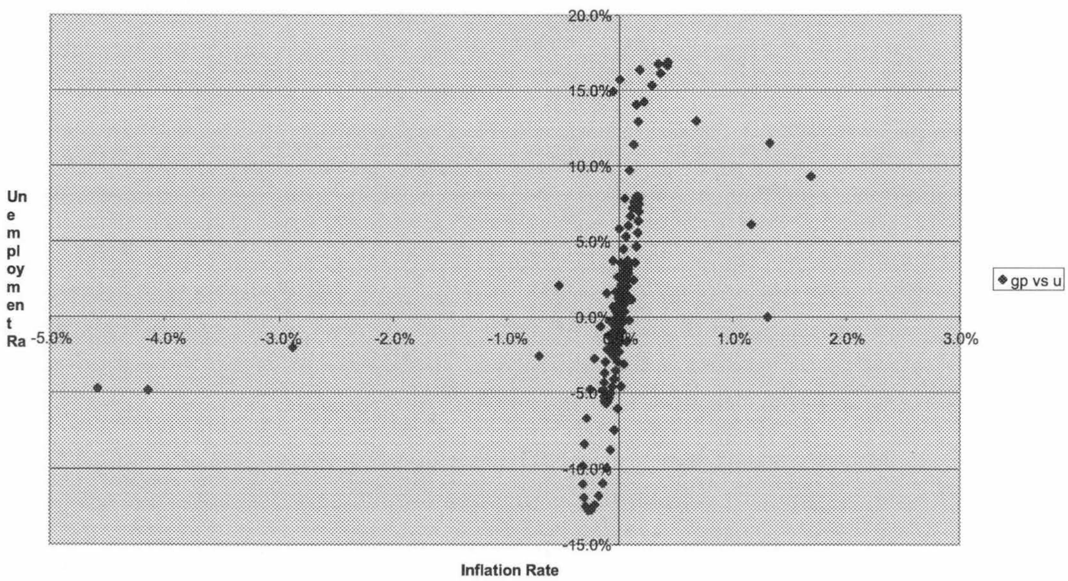
The linear object in Figure 6.8a above *resembles* a consumption function.

Figure 6.8b - Real Profit Rate vs Real Wage Rate



The linear object in Figure 6.8b above *resembles* a factor price frontier.

Figure 6.8c - Inflation Rate vs Unemployment Rate



However, it is obvious that the object in Figure 6.8c above is *not* a Phillips curve.

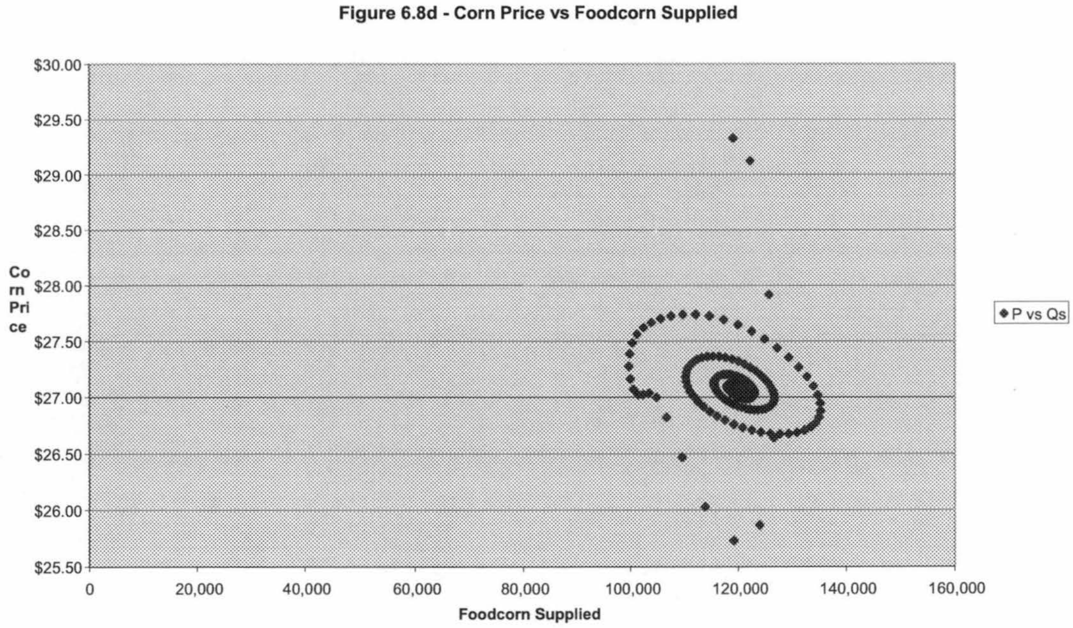


Figure 6.8d above shows neither a demand nor a supply curve from the market for foodcorn.

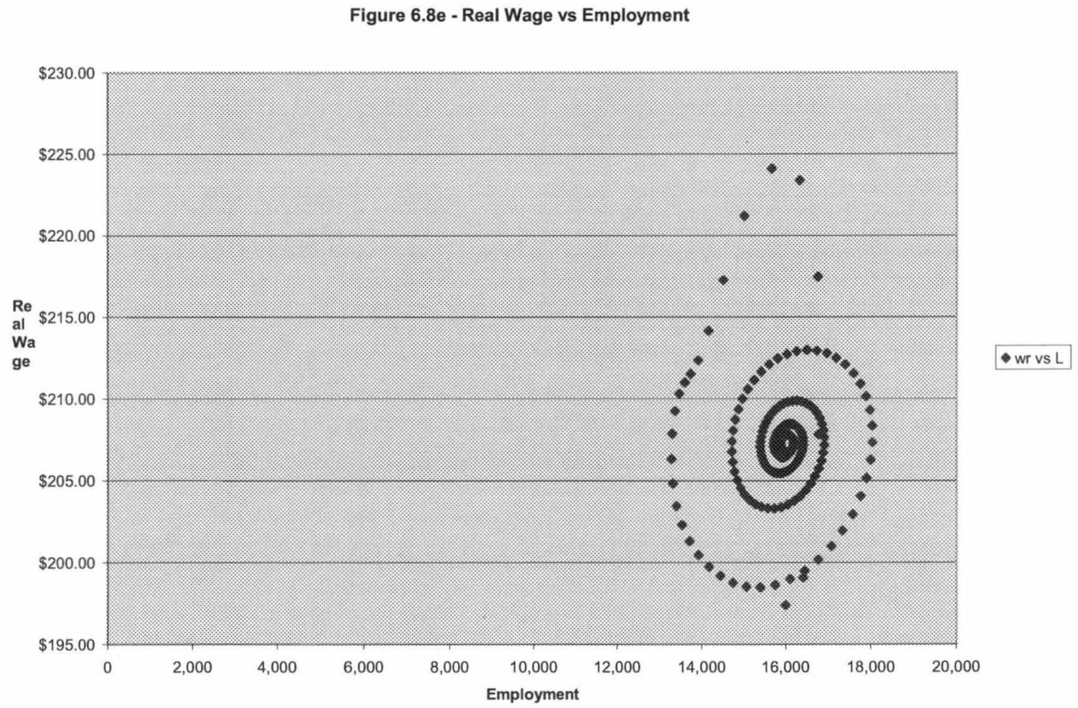


Figure 6.8e above shows neither a demand nor a supply curve from the market for labour.

Figure 6.8f - Real Profit Rate vs Real Capital Stock

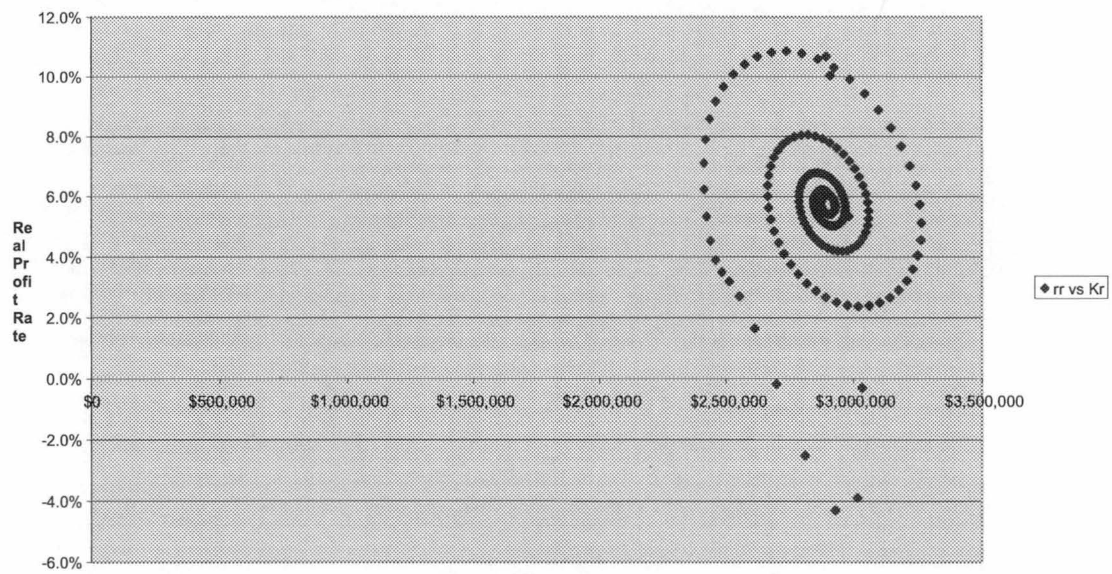
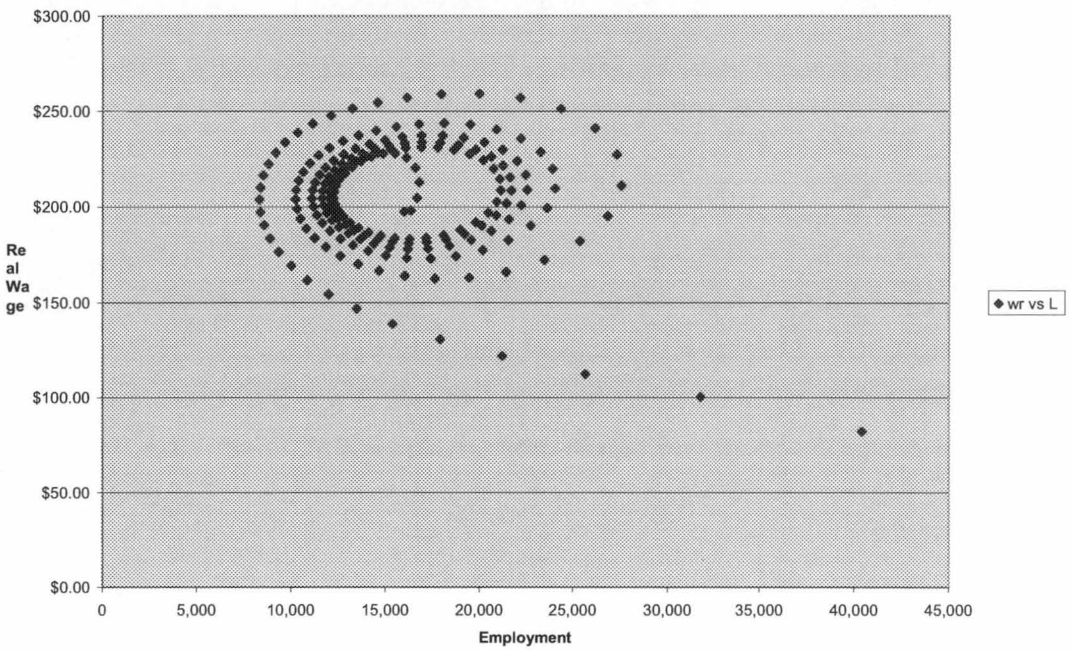


Figure 6.8f above shows neither a demand nor a supply curve from the market for "aggregate real capital" – assuming such an institution *could* exist. Given that the last four of these figures contain spiral-shaped objects, it may be that Figures 6.8a and 6.8b above are not in fact linear objects, but comprise two more spirals, each being viewed from its edge.

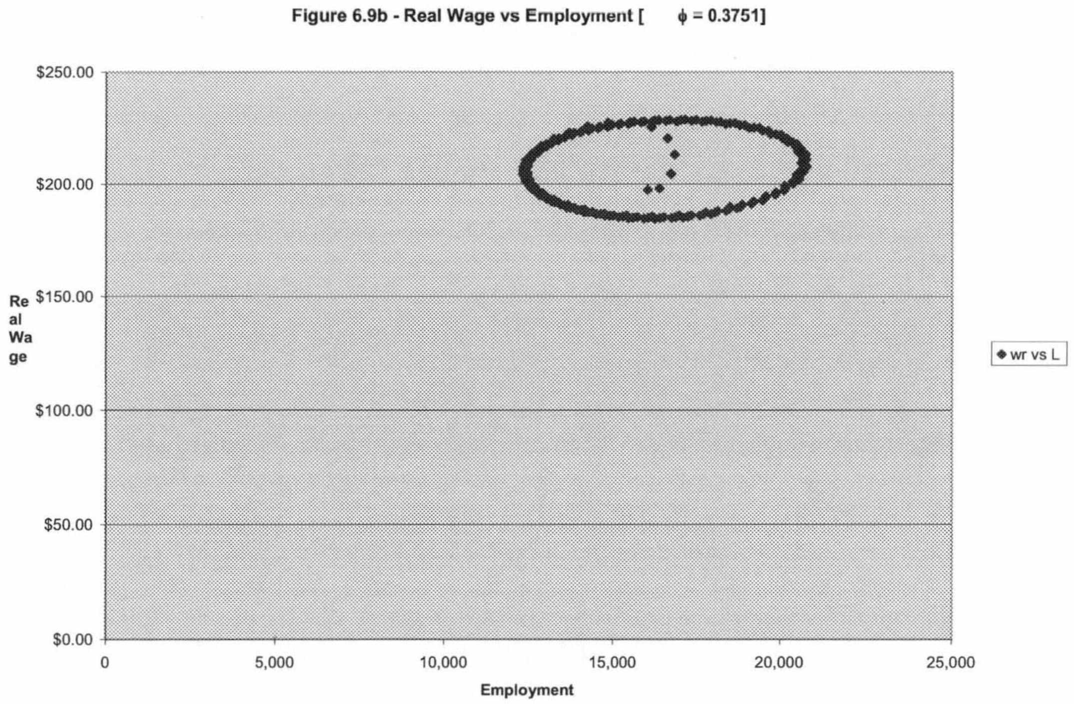
Figures 6.8c through 6.8f all exhibit well-defined and tightly-wound spiral structures. The difficult "identification problem" of deciding whether to fit a demand or a supply curve to the "cloud of data points" on an empirical price *versus* quantity scatter-plot chart would pale into insignificance for an econometrician confronted by such a "spiral of data points". The fact that at least four out of six graphs have this structure most probably indicates that the cycles generated by a "master oscillator" (most likely the profitability gap investment function) have entrained the cycles of several "slave oscillators" (such as the equations for the corn price and money wage) in the structural form of Model E. This would be consistent with the paramount role of the investment equation and its reaction coefficient (ϕ), as demonstrated in the reduced forms of Models A through D.

The single phase plot of Real Wage vs Employment in Figure 6.8e above is chosen as typical of Figures 6.8c through 6.8f for performing the following analysis. The 2-D phase diagrams of Figures 6.9a through 6.9f below typify the *evolution* of the spiral objects for all relationships except the alleged "consumption function" and "factor price frontier". These two objects retain their linear *appearance* as ϕ rises in value from ϕ = 0.3724 to ϕ = 0.8218, possibly in the same way a revolving spiral galaxy does when viewed from its edge.

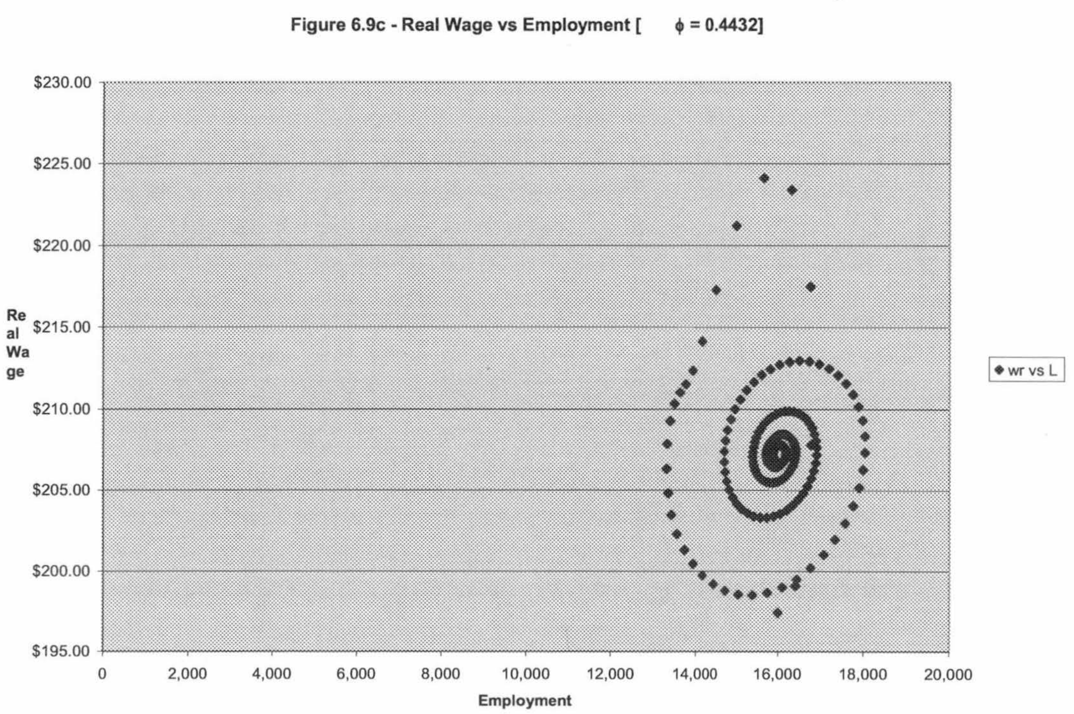
Figure 6.9a - Real Wage vs Employment [ϕ = 0.3724]



The object in Figure 6.9a above is a fairly open or loose spiral.

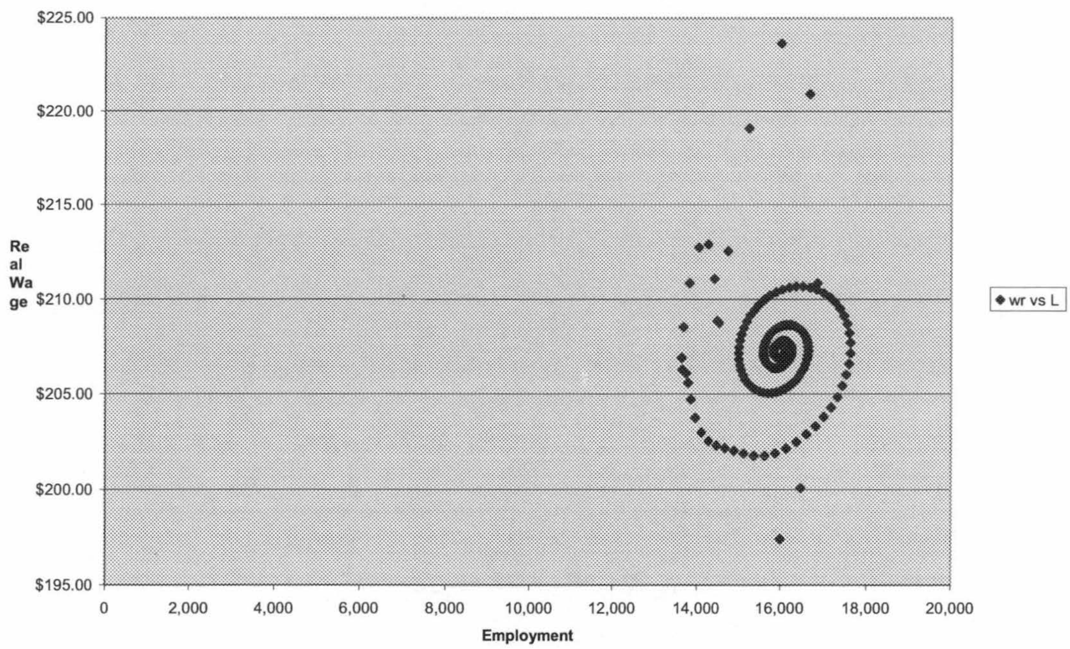


The object in Figure 6.9b above is an ellipse or closed spiral, indicating limit cycles.



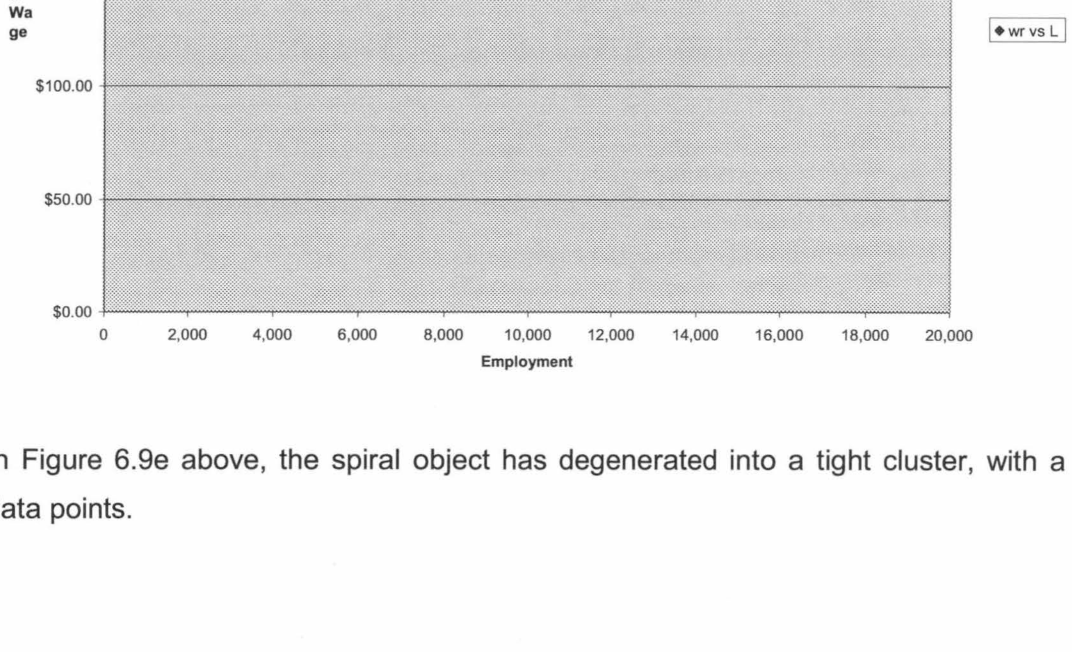
The spiral in Figure 6.9c above is tightly-wound at its centre, unlike the object in Figure 6.9a.

Figure 6.9d - Real Wage vs Employment [$\phi = 0.5147$]

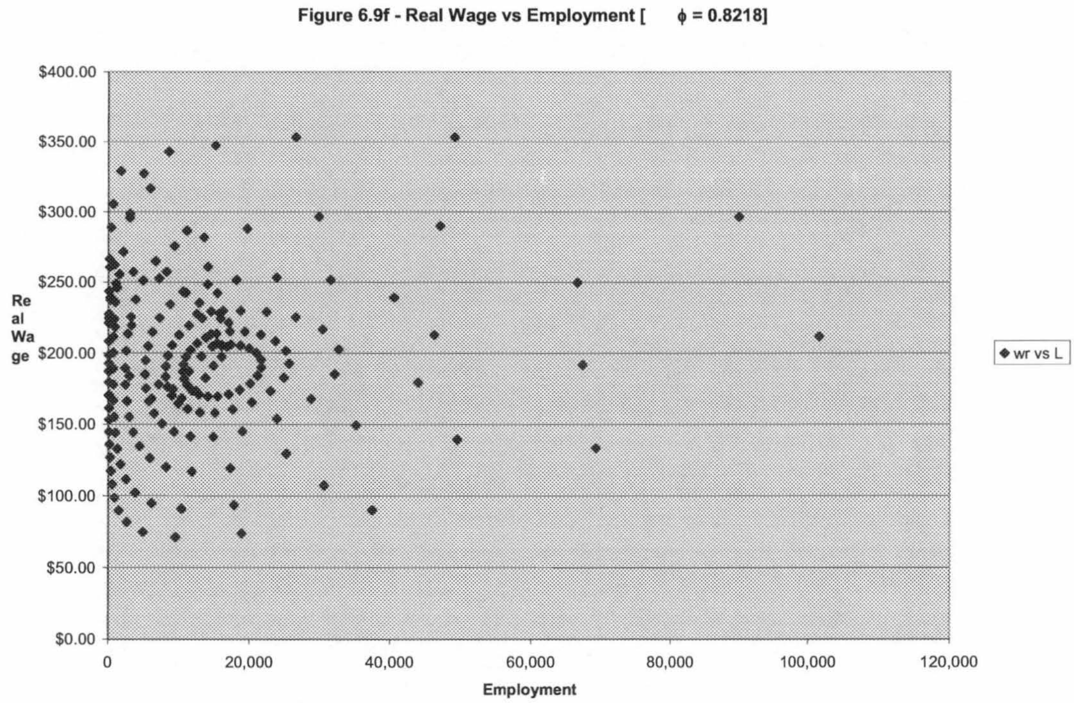


In Figure 6.9d above, the tail of the tight spiral is breaking up.

Figure 6.9e - Real Wage vs Employment [$\phi = 0.7465$]



In Figure 6.9e above, the spiral object has degenerated into a tight cluster, with a halo of data points.



In Figure 6.9f above, the degenerate object has reconstituted itself into a loose spiral with rotor arms.

Table 6.11 below describes how the morphology of the above spiral-shaped object evolves in a definite sequence as the reaction coefficient rises in value from $\phi = 0.3724$ to $\phi = 0.8218$.

Table 6.11 – Evolution of Spiral Object as Reaction Coefficient Changes

Figure	Reaction Coefficient	Shape of Scatter-Plot Object (Real Wage vs Employment Relationship)
6.9a	**0.3724**	Loose Spiral
6.9b	**0.3751**	Ellipse
6.9c	**0.4432**	Tight Spiral
6.9d	**0.5147**	Tight Spiral, with tail breaking up
6.9e	**0.7465**	Tight Cluster, with halo of points
6.9f	**0.8218**	Loose Spiral, with rotor arms

The sequence of Figures 6.9a through 6.9f is eerily reminiscent of how galaxies evolve. These phase plots suggest that there are deep and complex structural forces at work in even the simplest of agrarian economies.

Given that the reaction coefficient governs convergence to long-period equilibrium, clearly these phase diagrams say a lot about the dynamic stability of Model E. In the words of Hicks (1975, p 316), "Convergence to equilibrium has been shown to be dubious, but it has also been shown to be unimportant. Even at the best, it will take a long time ... and before the time has elapsed something new will surely have occurred." Hicks may have had economic policy in mind when he continued, "It is therefore of the first importance that something can be said ... about the short-run and medium-run effects of an exogenous disturbance."

The presence of spirals[33] in the diagrams above suggests that policy-makers would be well-advised to consult a well-specified nonlinear dynamic model, rather than trusting that long-period equilibrium results derived from static economic analysis also hold in the dynamic short-period context of cycles, distribution and growth through historical time. A policy-driven "instrumental traverse" is created in the following chapter, consistent with this principle.

The presence of spirals in 2-D "phase space" also begs a "mathematical conjecture" which is elaborated in the final chapter of this thesis. In brief, the corn-credit economy of Model E can be viewed as a complex mathematical object in 7-D "state space" with 17 degrees of freedom, i.e. one per parameter. This object changes its "hypershape" each time a constant or initial value is varied. The conjecture is that the general model for n coupled nonlinear oscillators in dissipative dynamical systems might provide the *mathematics* needed to establish as *general* the results produced by this specific numerical model and to probe more deeply the dynamic behaviour of this Post-Keynesian corn-credit economy.

If Model E captures even *some* of the essence of real-world agricultural commodity markets, this chapter's finding that consumer sovereignty has increased the pure flexprice corn-credit economy's volatility and markedly lengthened its observed traverses may help explain why "government interference" (in the shape of price stabilisation schemes and suchlike) is so common in the primary industries. Having no means to convert volatile "sovereign consumer" price-makers into passive price-takers, it is not surprising that primary producers make political demands for the kind of market power they know is wielded by large firms in the secondary and tertiary sectors.

Perfect competition, consumer sovereignty and strict *laisser faire* may massively disrupt economies, precisely *because* the free market forces of uncoordinated decision-making by investors and consumers are so powerful. Perhaps the long historical evolution of private and public sector institutions for stabilising flexible product prices in all three sectors

[33] Spirals and ellipses are the equivalent in phase space of cycles in the time domain.

constitutes a necessary *survival mechanism* for capitalism. If these institutions were to be dismantled (as recommended by many Neoclassical and Austrian economists), product flow prices could fluctuate like asset stock prices do in the financial economy, sending the physical economy onto a dangerous roller-coaster ride that threatens the very provisioning of the population.

Finally, the ultimate dependence of 55 endogenous variables on just six independent equations and one equilibrium condition calls into question (in these abstract corn-credit models, at least) the conventional two-fold division of economic theory. Before 1936, the Theory of Value and Distribution was divorced from the Theory of Money and Prices, while the modern distinction between Microeconomic Theory and Macroeconomic Theory dates from after the publication of Keynes's *General Theory*. On the one hand, the concepts of Value, Distribution, Money, and Prices all make an appearance in this nested sequence of CDG models. Yet, on the other hand, such "Macroeconomic" concepts as the saving ratio, investment multiplier and capital-output ratio are derived aggregates, as are all "Microeconomic" *relative* prices. (The latter are deflated nominal prices or secondary ratios between the *money* prices that are primary.)

A preferable term for Model E – this Post-Keynesian representation of a fully-flexprice monetary production economy exhibiting cycles, distribution and growth – would be a "mesoeconomic" or, more simply, an "economic" model. The Neoclassical mainstream's ongoing search for the elusive "microfoundations of macroeconomics" has achieved little more than convert Neo-Keynesians (who cannot explain sticky wages and prices) into New Keynesians (who can). Already, David Colander (1996) and other Post-Walrasians claim to be uncovering the "macrofoundations of microeconomics".

Post-Keynesians most likely will find that there exist certain binding "macroconstraints" governing and conditioning all microeconomic behaviour. For instance, the total realisable money profit (consistent with the aggregate volume of investment) that firms compete to capture may constitute the chief macroconstraint within which they all necessarily operate. Economics truly is an integrated whole; it has no "split personality" and top-down theorising is just as important as the bottom-up approach.

6.17 Conclusion

Model E completes the developmental sequence for modelling an abstract corn-credit economy and studying its observed traverses. Like the outermost layer of an onion or the biggest babushka doll, it has nested within it all the previous standalone Models D, C, B, and

A. The sequence starts with the fixprice Model A, then the corn price (B), money wage (C) and interest rate (D) are endogenised. It finishes with the flexprice Model E, in which the corn price is determined by a more conventional – and far more flexible – method than that used for Models B through D.

In Model E the consuming and saving householders' decisions combine with those of the investing farmers to determine the economy's path of development through time. Investor reactions (ϕ) to the time-stream of market events are now intertwined with consumer reactions (β, χ, γ) to these same events. To use an analogy from improvisational music, the investing firms establish a "bass line" of recurring cycles, while the households play contrapuntal "melody lines" of consumption and saving. Soaring and diving, these melodies feed back onto the investors' initially stolid performance, causing them to increase both the period and the amplitude of the cycles that constitute their bass line. Thus two key groups of economic agents (together with the workers and the bankers) *co-create* the puzzling market phenomena which confront them as historical time passes, ensuring their world is *nonergodic* and its future uncertain, not merely risky.

All five models solve for the stationary state, but the highly-sensitive consumption and saving behaviour of households keeps only Model E in an "unsteady state" of traverse adjustment for almost 190 years. The passive consumers of Models A through D permit convergence to a steady state in some 30 years. Like them, Model E is dynamically stable in that the reaction coefficient's critical value for limit cycles (ϕ = 0.3751) and its critical value for system collapse (ϕ = 0.8219) bracket a large region of cyclical convergence. Results consistent with "economic intuition" emerge only after convergent traverse-adjustment processes of extremely long duration occur.

Finally, several phase diagrams confirm that many of the standard "curves" in the mainstream economist's "toolbox" cannot be relied upon by policy makers seeking to stabilise a corn-credit economy like that represented by Model E. The typical phase plot reveals a galaxy-like spiral structure, which evolves as the reaction coefficient is varied, rather than showing a scatter of data points to which the standard curves can be fitted.

It seems there may be a role for *public policy* in "regularising" economies subject to traverses that are violent and of extremely long duration. Therefore, in the following chapter, the anarchy assumption is dropped and "government" begins to "interfere" with *laisser faire* by implementing policies aimed at shortening the traverse and converging rapidly to a near-full employment steady state.

CHAPTER SEVEN

AN INSTRUMENTAL TRAVERSE

7.1 Introduction

The disequilibrium time paths of Models A through E in Chapters 4 through 6 were analysed using Peter Kriesler's "observed traverse" concept. This chapter reports on a specific application of Adolph Lowe's "instrumental traverse" concept within the abstract corn-credit economy of Model E, characterised as it is by anarchy and *laisser faire*. Lowe (1959, pp 164-5) contains his first writings on *instrumental analysis*: "Suppose that, rather than studying the causes of today's state of affairs and predicting therefrom the state of tomorrow, we were to postulate a *goal* for the economic system to attain, and then to investigate the requirements for the realization of that goal ... Because this procedure studies the means appropriate to postulated ends, I call it 'instrumental'."

To postulate a preferred goal for the corn-credit economy, the anarchy assumption must be dropped. For this chapter's Model E* (i.e. Model E with a "government sector"), it is assumed that political power and responsibility is arrogated by a committee of farmers and farmer-bankers. This occurs because the society is about to transit from the zero population and workforce growth of the tranquil stationary state to an exponential growth rate of one percent pa. The new oligarchs, fearing a profit squeeze, economic instability and social unrest due to widespread unemployment with falling living standards, decide to craft and implement a set of *rational* (rather than ideological) economic policies, with the goal of smoothly and efficiently accommodating the looming demographic transition.

The members of this "government" wish to traverse from the initial stationary state of zero economic growth at full employment (which characterised the previous century) to a terminal steady state of positive growth in the coming century, one that is consistent with a more stable economy attaining near-full employment and no diminution in realised profitability, while maintaining workforce morale in the face of inevitable unemployment during the traverse. They realise this could be difficult, since analysis of Model E has shown that 188 years will elapse before a steady state is reached under a "Do Nothing Scenario".

The government does not want to practise "fine-tuning" or to legislate for "stop-go" economic policies, due to the difficulty of "getting the timing right" in a nonergodic world. Consistent with effectiveness, they want their policy package to be simple, certain, acceptable, and self-liquidating with respect to the government's indebtedness to bankers. For simplicity, no more than three policy instruments are to be deployed. For certainty, the policy settings are to remain in place for a century. For acceptibility, any necessary tax or subsidy is to be as low as possible. For fiscal responsibility, the public sector must be debt-free within the century.

As a trusted representation of their corn-credit economy, Model E is the government's principal tool for testing alternative economic policy proposals before actually implementing them. They decide to first simulate such policy packages with a view to fixing a simple theoretical problem, the Misallocation Scenario. The results of this counterfactual policy experiment gives them confidence to address the practical conundrum they actually face: stabilising the "Unsteady State" which they know develops whenever Model E (hence, also, their own *laisser faire* agrarian economy) attempts to deal with positive workforce growth. In both cases, the starting point is the tranquil stationary state that has characterised the past century of their economy's history.

7.2 Adolph Lowe and Political Economics

Together with Kalecki, Robinson and Hicks, Adolph Lowe (1893-1995) is identified as one of the pioneers of traverse analysis in Chapter 2. The roots of his life's work on "political economics" (with its techniques of structure, force and instrumental analysis) – and on the traverse – principally lie in Lowe (1952, 1959, and 1976). These roots can be traced back to Lowe (1926), in his famous essay *How is Business Cycle Theory Possible at All?* This challenge soon was followed by two seminal dynamic models of the trade cycle, built by Kalecki (1933) and Frisch (1933).

Fifty years later, in *The Path of Economic Growth*, Lowe (1976, p 3) posed another challenging question: "Is economic growth a subject at all fit for theorizing?" Following a short survey of the history of economic thought on this matter, Lowe concluded "... we feel compelled to doubt the fruitfulness of any 'positive' version of growth theory. On the other hand, the empiricist approach does not appear as a more efficient technique for obtaining practically useful generalizations. The solution must, therefore, be sought in the pursuit of prescriptive analysis ..." (p 11).

He bases this finding on a perceived loss of market "order", following an easing of the strong constraints within which early capitalism worked: "... modern regimes [are] characterized by rising affluence, imperfect competition and growing social security. In conjunction with the incessant revolutions in technology, there is an ever-widening spectrum of observed behavior that reduces the truth value of economic predictions in proportion with the length of the prediction interval." (p 7).

First, Lowe notes the multiplicity of possible social objectives: maximising the flow of consumer goods or the terminal capital stock, minimising change (even at the cost of static output), assuring a traditional standard of living, mobilising for war, or pursuing a 'scorched earth' policy during wartime. "What is at stake is quite generally *the search for the economic means suitable for the attainment of any stipulated end.* To this procedure I have assigned the label of *instrumental analysis.*" (pp 11-12)

Lowe introduces his heuristic logic of "retroduction" by stating that

> ... instrumental analysis takes as *given* not only the initial but also the *terminal* state – the latter being 'known' through explicit stipulation of a macrogoal toward which the system is to move. The unknowns to be determined are (a) suitable *paths* over which the system can move toward the macrogoal, (b) *behavioral and motivational patterns* that set the system on such paths and keep it to them, and, possibly, (c) public controls suitable to elicit the appropriate motivations ...
>
> Thus, in contrast to the deductive procedure of positive analysis that argues 'forward' from behavioral premises to terminal states, instrumental analysis resembles induction by searching 'backward' for the determinants of given states. However, it differs from induction by taking the terminal states and processes as given, not by observation, but by stipulation. (pp 12-13)

Therefore, Lowe's "political economics" involves the deliberate "social engineering" of successful traverses that attain particular goal-states, which he recommends should be selected *via* the democratic political process. But whatever (and by whichever means) society decides, it is the task of his instrumental controls to achieve these aims with the help of deliberate measures of economic policy – a contrived system taking the place of the nowadays "disorderly" (i.e. no longer self-regulating) free market. These goal-adequate "carrot and stick" control measures should be designed to complement, not override, the behavioural-motivational patterns of economic agents.

The task of "motorial" or "force analysis" is to discern such patterns of behaviour and this is where economics, according to Lowe, needs to adopt a thoroughgoing interdisciplinary approach. Instrumental controls effective for a capitalist democracy could not be used to guide behaviour in a collectivist state like the then-USSR, for instance. By contrast, "structure analysis" and the problem of the traverse are common to both systems, when viewed as what they really are, viz. "engines of provisioning" for the material needs and wants of any given population of human beings. To supplement the skills of the economist, insights from the humanities and other social sciences are needed for force analysis, while the "rules of engineering" and knowledge won by the natural sciences are relevant to structure analysis.

7.3 Structure and Force Analysis of the Corn-Credit Economy

Structure analysis of the abstract Model E economy involves two of its six independent equations. Corn production (Q) merely requires a previously-accumulated stock of seedcorn, together with a workforce to plant, tend and harvest the crop. Seedcorn yield then determines output, which fixes the level of employment (L) for given labour productivity.

Adding a lag of one year between sowing and harvesting completes the picture of this agrarian economy as an engine of provisioning. It contrasts with the complexity of Lowe's (1976, p 32) illustrative "three-sector schema of industrial production", with its outputs of dresses (Department II); sewing machines, etc. (Department Ib); and machine tools, etc. (Department Ia), together with all necessary working capital inputs of cotton, yarn and cloth (for Department II) plus ore, pig iron and steel (for Department I).

Force analysis of the Model E corn-credit economy involves the remaining four of its six independent equations, plus the r = n% pa stationary-state equilibrium condition. These represent the "motorial" forces governing human economic behaviour by farmers (Qi), bankers (i), workers (w), and by all social classes in their combined capacity as generic consumers (P). Apart from this, three initial values are given by history and all remaining relationships are definitional, such as the "identities" and the national accounting type "aggregates".

7.4 The Goals of a Rational Oligarchy

In the previous chapter, Table 6.4 (Model E Stationary State) portrayed how the corn-credit economy simply replicated itself every year over the past century. There were no disruptive business cycles, none of the 16,000-strong workforce was unemployed, the money wage stood at $200 pa, corn was priced at $27.85 per sack, and farmers were content to make 5 percent pa on the capital they had committed to produce and sell foodcorn. Now, this tranquil past is about to be transformed into the problematic future shown in Table 6.5 (Model E "Unsteady State"), whose time series are plotted as Figures 6.2a through 6.2f. This is the *laisser faire* or Do Nothing Scenario of fluctuating prices and profits and cycles of under- and over-employment, the first of which tops out at 44.7 percent of the (now growing) worktorce unemployed and receiving no income – apart from the charity of relatives and friends.

There is widespread concern about these forecast consequences of the looming demographic shift, so a committee of farmers and farmer-bankers unilaterally declare that, for the greater social good, they have decided to form a "government". Fortunately, this self-appointed oligarchy also is rational. Subscribing to no particular ideology, they are completely pragmatic when it comes to their own long-term survival and prosperity. They know that this requires a well-functioning economy to properly provision the population. The new government's primary goal, therefore, is to neutralise any future threat of "revolution from below" by ensuring that economic growth is more stable and sufficiently fast to guarantee (a) absorption of the growing workforce into employment at (b) close to its customary standard of living, and with (c) no significant reduction in their own real interest and profit rates, relative to the *laisser faire* situation.

Model E is the government's trusted apparatus for running scenarios, although the rational oligarchs do recognise that its behavioural constants (ϕ, κ, β, etc.) represent averages drawn from some wide-ranging frequency distributions, in this society of competing individual agents. Feeding in the projected workforce growth rate, they obtain the alarming Do Nothing Scenario discussed above and described as an Unsteady State in Chapter 6. Suddenly, any continuation of *laisser faire* is off the agenda because the continued prosperity (perhaps even survival) of the class of proprietors is at stake.

7.5 Goal-Adequate Economic Policies

Accordingly, the government decides to experiment with Model E, searching for a suite of economic policies which (a) will reduce these cyclically high unemployment rates and (b) make dole payments to those unemployed while (c) flattening out the irritating swings in business activity and the realised profit rate and (d) completing the instrumental traverse to a steady state of growth as soon as possible. However, they need to walk before they can run, so the government first experiments with stabilising the theoretical Misallocation Scenario before simulating policies designed to tame the unsteady state that characterises their Do Nothing Scenario.

The rational oligarchy correctly perceives that their constituency of farmers, long accustomed to earning (on average) the normal profit rate – with the co-operation of a docile workforce – will be broadly in favour of avoiding labour unrest by paying any unemployed workers some fraction of the going money wage. The cost of this new Unemployment Benefit is to be recovered by a flat-rate Income Tax on all households, including those of dole recipients. There is no objection to the Budget swinging into deficit, provided the resultant Government Debt (a) pays interest at the market rate and (b) is fully retired by year 100.

The government therefore creates Model E* by adding to Model E a set of eight Government Sector identities and policy equations, together with two policy instruments, viz. the Dole Wage Fraction (df) and the Income Tax Rate (ty). This fiscal policy requires that the Model E identity for Household Income be modified as in Table 7.1 below.

Table 7.1 – Adding a Model E* Government Sector to Model E

<u>Government Sector</u>				
Policy Switch	ts	= **0** = **1**	disables policy enables policy	(i)
Income Tax Revenue	T	= ts ty Yho	dollars pa	(ii)
Unemployment Benefit	Gu	= ts df w (η – L) for η > L = 0 for $\eta \leq$ L	dollars pa	(iii)
Government Debt Interest	Gi	= ts i Dgo	dollars pa	(iv)
Budget Balance	B	= T – Gu – Gi	dollars pa	(v)
Government Debt	Dg	= Dgo – B	dollars pa	(vi)
Dole Wage Fraction	df	= **30**	percent	(vii)
Income Tax Rate	ty	= **4.1**	percent	(viii)
<u>Identity</u>				
Household Income	Yh	= W + J + Ro + Gu + Gi – T	dollars pa	(16)

The Policy Switch allows the government to toggle Model E* between *laisser faire* (ts = 0) and the policy-driven or instrumental outcome (ts = 1), for purposes of comparison. When policy is disabled, the Income Tax Revenue (ii), Unemployment Benefit (iii) and Government Debt Interest (iv) policy equation values are multiplied by ts = 0, effectively turning Model E* back into Model E.

When the government's budget is in surplus, Government Debt (and Interest) may be negative. At such times the government's bank account is in credit and earning interest at the market rate. With Gi being negative in identity (16) of Table 7.1, Household Income (Yh dollars pa) is lower and this will have feedback effects in the following year, when the government taxes this reduced income, now tagged Yho dollars pa. In accordance with Equation (ii) of Table 7.1, the revenue side of the government's budget (T dollars pa) will be lower than in the previous year. Secondary effects from lower Yh implying less aggregate demand, reduced employment, higher payments of unemployment benefit, etc. also will feed back onto the government's budget on the expenditure side.

7.6 Spreadsheet Realisation

The spreadsheet formulae corresponding to the algebraic formulae of Table 7.1 above are melded in with those of Table 6.3 in the previous chapter to create Table 7.2 below. The structural form of Model E* was programmed into a Microsoft Excel spreadsheet and given the computer filename Fsted (see Appendix D, with enclosed CD-ROM).

Table 7.2 – Model E* Spreadsheet Formulae

	A	B	C	D	E	F
1	**E* - INSTRUMENTAL TRAVERSE**	**SN**	**rg**	**ag**	**0**	**=+E1+1**
2	**Equations**					
3	Corn Produced	Q				=+F38*E4
4	Seedcorn Invested	Qi			**40000**	=+E4*(1+F37*F26)
5	Employment	L				=+F3/F39
6	Profit Rate	r				=+F24/F23
7	Corn Price	P				=+EXP((LN(E27)-F47-F49*LN(1/F9)-F50*LN(F35))/F48)
8	Money Wage	w			**200**	=+E8+F44*E30+F43*(F28-1)
9	Interest Rate	ii			**0.04**	=+E9+F45*F33
10	**Government Sector**					
11	Policy Toggle Switch	ts			**1**	=+E11
12	Income Tax Revenue	T			**0**	=+F18*E35*F11
13	Unemployment Benefit	Gu			**0**	=+IF(F42>F5,(F42-F5)*F8*F17,0)*F11
14	Government Debt Interest	Gi			**0**	=+E16*F9*F11
15	Budget Balance	B			**0**	=+F12-F13-F14
16	Government Debt	Dg			**0**	=+E16-F15
17	Dole Wage Fraction	wd			**0.3**	=+E17
18	Income Tax Rate	ty			**0.041**	=+E18
19	**Identities**					
20	Wage Bill	W				=+F8*F5
21	Seedcorn Capital	Ka				=+F7*E4
22	Foodcorn Capital	Kb				=+F7*E27/F41
23	Capital Stock	K				=+F21+F22
24	Profit	R				=+F7*F3-F20-F21-F34
25	Normal Profit Rate	n				=+F9+F40
26	Profitability Gap	a				=+F6-F25
27	Foodcorn Supplied	Qs				=+F3-F4
28	Employment Ratio	e				=+F5/F42
29	Price Level	p				=+F7/$E7
30	Inflation Rate	gp				=+F29/E29-1
31	Average Debt	D				=+F20/F46
32	Debt Assets Ratio	d				=+F31/E23
33	D:A Ratio Growth Rate	gd				=+F32/E32-1
34	Interest Bill	J				=+F9*F31
35	Household Income	Yh				=+F20+F34+E24+F13-F12+F14
36	**Constants**					
37	Reaction Coefficient	ϕ			**0.4432**	=+E37
38	Seedcorn Yield	θ			**4**	=+E38
39	Labour Productivity	λ			**10**	=+E39*0 9941
40	Risk Premium	φ			**0.01**	=+E40
41	Capital Turnover	κ			**2.00000000000007**	=+E41
42	Workforce	η			**16000**	=+E42*1 01
43	Employment Wage Coefficient	ε			**4**	=+E43
44	Inflation Wage Coefficient	ρ			**12**	=+E44
45	D A Ratio Growth Coefficient	δ			**0.1**	=+E45
46	Wage Bill Turnover	μ			**52**	=+E46
47	Intercept Constant	α			**0.21573414123849**	=+E47
48	Price Elasticity of Demand	β			**=-E49-E50**	=+E48
49	Cross Elasticity of Demand	χ			**2**	=+E49
50	Income Elasticity of Demand	γ			**1**	=+E50

The rows of Table 7.2 are numbered 1 through 50 and the columns are tagged A through F. Columns A and B list the long and short names, respectively, of all variables and constants in the model. The formulae contained in columns C and D are suppressed for clarity. Column E (year zero) holds the three initial values and all 14 constants. All its year-zero formulae also are suppressed for clarity, but they are based on those shown in the next column. Column F displays formulae for the model's equations, identities, policy instruments, and constants for year one. The (missing) columns for years two through 100 simply continue the pattern established in column F.

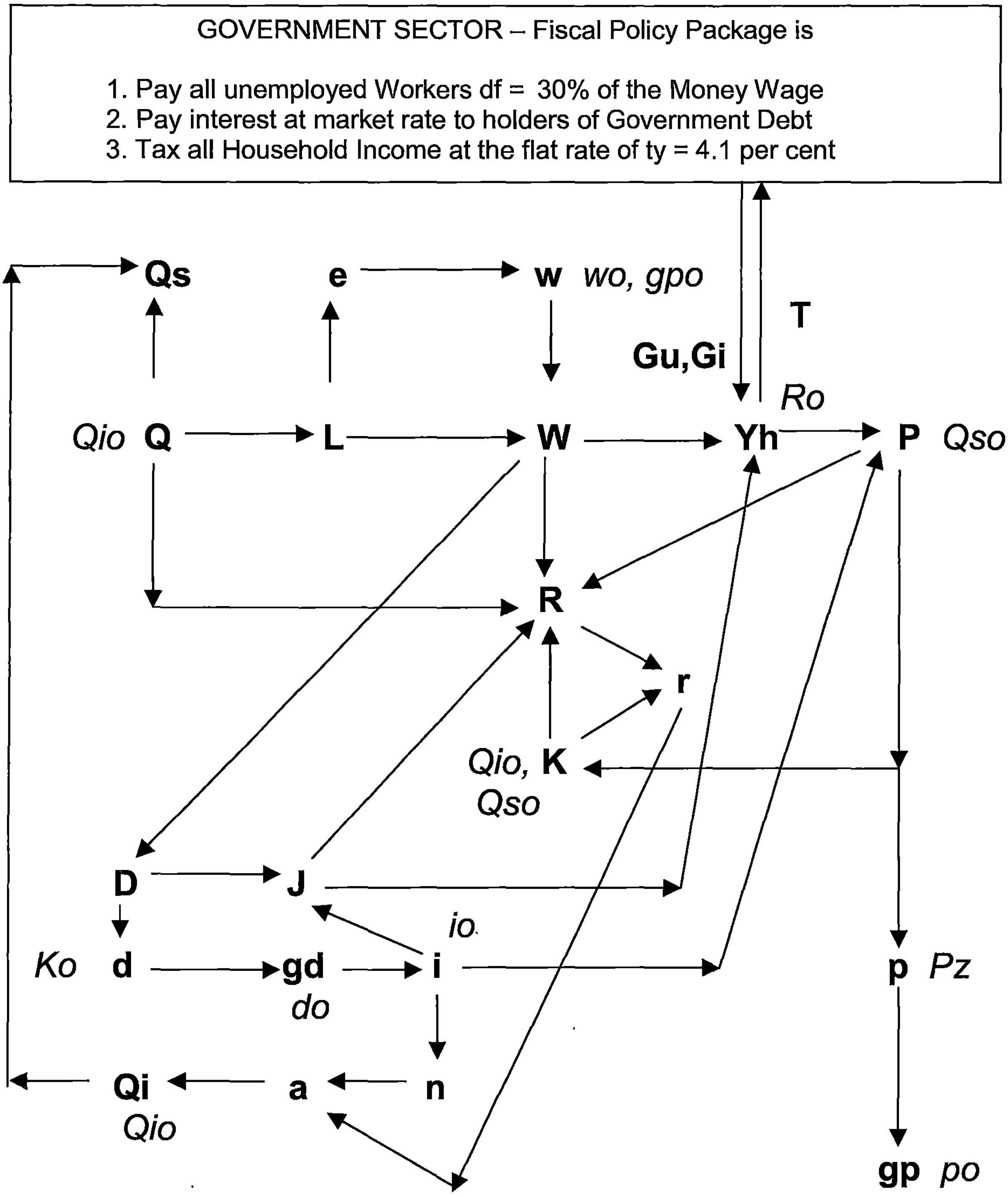
As before, the three initial values (**bold** type) in year zero are given by history as 40,000 sacks pa of seedcorn invested, a $200/worker pa money wage and a 4% pa interest rate. Apart from the 14 constants (also **bold** type), all other year-zero values are *computed* rather than specified. *Prima facie*, the whole of column E should have been filled with known historical base-period data. Yet, as this is not an empirical model, there are no historical data. So, reliance is placed on the assumption that, in any given year, history cannot be internally inconsistent. That is why the model's standard set of equations and identities (not shown in column E) is used to compute all remaining year-zero values.

7.7 Corn-Credit Economy Description

The flowchart in Figure 7.1 below shows that the Model E* economy is *identical* with the Model E economy of Figure 6.1 in the previous chapter, except for the addition of a Government Sector, as modelled in Table 7.1 above. The Household Income identity of that table is the *only* point of contact between the public and private sectors. Fiscal policy is directed solely at the Household Income variable, previously Yh = W + J + Ro dollars pa, but now modified to Yh = W + J + Ro + Gu + Gi – T dollars pa. Income tax revenue (T) is taken from households and unemployment benefit (Gu) plus government debt interest (Gi) is returned to them.

The fiscal policy package favoured by the rational oligarchy also happens to be popular in the real world of nation-state Treasuries and Ministries of Finance. As will be seen in the instrumental traverse analyses below, the Gu-Gi-T policy combination acts as a powerful "automatic stabiliser" for the *laisser faire* corn-credit economy of Model E.

Figure 7.1 – Key Fiscal Policy Intervention Point in the Corn-Credit Economy



7.8 Taming the "Misallocation Scenario"

The government conducts its initial experiments with the Model E Misallocation Scenario of Chapter 6, i.e. in the context of an opening stationary state. In the "Constants" section of

Table 7.2, they remove from Model E* the forecast decline in labour productivity (so that now $\lambda = 10$ for the full century) and the forecast growth in the workforce (so that now $\eta = 16{,}000$ for the century). These actions are taken to return the Fsted model to its stationary-state solution, which has characterised the past 100 years of their corn-credit economy's history. The standard 4% seedcorn-foodcorn misallocation event is made to occur in year 30 of this new Fstat spreadsheet, with the same results as in section 6.11 (Specimen Traverse) of the previous chapter, including unemployment rates as high as u = 10.7% of the workforce.

Although the Misallocation Scenario is only a theoretical experiment, the government can see the dire consequences of *laisser faire* economic instability. They realise that unemployed workers will need to be paid some level of Unemployment Benefit, in order to subsist without continual recourse to the charity of those still in employment. Fearing social unrest among the unemployed (and the still-employed relatives and friends on whom they would depend for their means of subsistence), the government settles on a Dole Wage Fraction (vii) of 30% of the money wage. The rational oligarchs decide to finance this policy by setting an Income Tax Rate (viii) that they see as "equitable": a *flat* rate applied to *all* Household Income, i.e. including the new dole payments to unemployed workers.

Given the Dole Wage Fraction, the government varies the Income Tax Rate while closely monitoring its effects on the Budget Balance (v), on the behaviour of the stock of Government Debt (vi) and on their liability to pay Government Debt Interest (iv). Household Income (16) is boosted by these Government Debt Interest payments and by Unemployment Benefit (iii), but simultaneously is reduced by the Income Tax Revenue (ii) yield.

By a process of trial and error, the government soon finds that a terminal stationary state can be achieved (with zero government debt by year 100) if the Dole Wage Fraction is set at df = 30% and a flat Income Tax Rate of ty = 0.266% is applied. For the standard 4% seedcorn-foodcorn misallocation event in year 30, this two-part policy package tames the resultant instability by year 93. The policy-driven outcomes shown in Table 7.3 below are directly comparable with the *laisser faire* outcomes of Chapter 6.

For reference, the rd column in Table 7.3 shows percentage differences between the basecase and traverse time paths when ts = 0, i.e. the *laisser faire* solution of Table 6.6 in Chapter 6. The ad column provides the same information for the instrumental or policy-driven solution when ts = 1. On most measures, the latter outcomes are superior, e.g. Q, Qi, L, W, K, Qs, etc. all fall by lesser percentages, relative to the same opening stationary state. In addition, the distribution of income changes in favour of profits, e.g. R and r both rise by larger percentages than under the Do Nothing Scenario.

Table 7.3 – Model E* Tames the Misallocation Scenario

	A	B	C	D	E	F	G
1	**E* - INSTRUMENTAL TRAVERSE**	**SN**	**rd**	**ad**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	-0.50%	-0.32%	160,000	153,105	159,302
4	Seedcorn Invested	Qi	-0.49%	-0.26%	**40,000**	39,393	39,836
5	Employment	L	-0.50%	-0.32%	16,000	15,311	15,930
6	Profit Rate	r	1.34%	3.31%	5.0%	11.2%	5.1%
7	Corn Price	P	-0.07%	0.01%	$27.85	$29.43	$27.69
8	Money Wage	w	-0.19%	-0.20%	**$200.00**	$199.36	$198.76
9	Interest Rate	ii	0.10%	0.13%	**4.0%**	3.6%	4.0%
10	**Government Sector**						
11	Policy Toggle Switch	ts	na	na	**1**	1	1
12	Income Tax Revenue	T	na	na	$0	$8,794	$8,787
13	Unemployment Benefit	Gu	na	na	$0	$41,235	$4,163
14	Government Debt Interest	Gi	na	na	$0	-$10,806	$144
15	Budget Balance	B	na	na	$0	-$21,635	$4,481
16	Government Debt	Dg	na	na	$0	-$279,198	-$908
17	Dole Wage Fraction	wd	na	na	**30.0%**	30.0%	30.0%
18	Income Tax Rate	ty	na	na	**0.3%**	0.3%	0.3%
19	**Identities**						
20	Wage Bill	W	-0.68%	-0.51%	$3,200,000	$3,052,292	$3,166,247
21	Seedcorn Capital	Ka	-0.59%	-0 32%	$1,113,900	$1,126,565	$1,102,885
22	Foodcorn Capital	Kb	-0.58%	-0.38%	$1,670,849	$1,783,941	$1,653,863
23	Capital Stock	K	-0.58%	-0.36%	$2,784,749	$2,910,507	$2,756,748
24	Profit	R	1.60%	3.93%	$139,237	$325,296	$139,959
25	Normal Profit Rate	n	0.08%	0.11%	5.0%	4.6%	5.0%
26	Profitability Gap	a	na	na	0.0%	6.6%	0 1%
27	Foodcorn Supplied	Qs	-0.50%	-0.33%	120,000	113,712	119,466
28	Employment Ratio	e	-0.50%	-0.32%	1.000	0.957	0.996
29	Price Level	p	-0.07%	0.01%	1.000	1.057	0 994
30	Inflation Rate	gp	na	na	0.0%	6.0%	0.0%
31	Average Debt	D	-0.68%	-0.51%	$61,538	$58,698	$60,889
32	Debt:Assets Ratio	d	-0.17%	-0.17%	0.022	0.021	0.022
33	D:A Ratio Growth Rate	gd	na	na	0.0%	-4.1%	0.0%
34	Interest Bill	J	-0.52%	-0.33%	$2,462	$2,108	$2,448
35	Household Income	Yh	-0.59%	-0.20%	$3,341,699	$3,213,919	$3,303,889
36	**Constants**						
37	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4432**	0.4432	0.4432
38	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
39	Labour Productivity	λ	0.00%	0.00%	**10.00**	10.00	10.00
40	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
41	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
42	Workforce	η	0.00%	0.00%	**16,000**	16,000	16,000
43	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
44	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
45	D A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0 1	0.1
46	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
47	Intercept Constant	α	0.00%	0.00%	**0.2157**	0.2157	0.2157
48	Price Elasticity of Demand	β	na	na	**-3.0**	-3.0	-3.0
49	Cross Elasticity of Demand	χ	0.00%	0.00%	**2.0**	2.0	2.0
50	Income Elasticity of Demand	γ	0.00%	0.00%	**1.0**	1.0	1.0

The policy-driven outcomes shown in Table 7.3 and Figures 7.2a through 7.2c below are directly comparable with the *laisser faire* outcomes of Chapter 6. In comparing Figures 7.2a through 7.2c with Figures 6.3a through 6.3c, the most striking change is that (from year 50 onwards) the single large cycle generated under *laisser faire* has been ironed out into a smooth continuous approach to a stationary-state flatline. The unemployment benefit/income taxation policies have worked in concert as "automatic stabilisers" of the unregulated capitalist economy.

Figure 7.2a - High Values

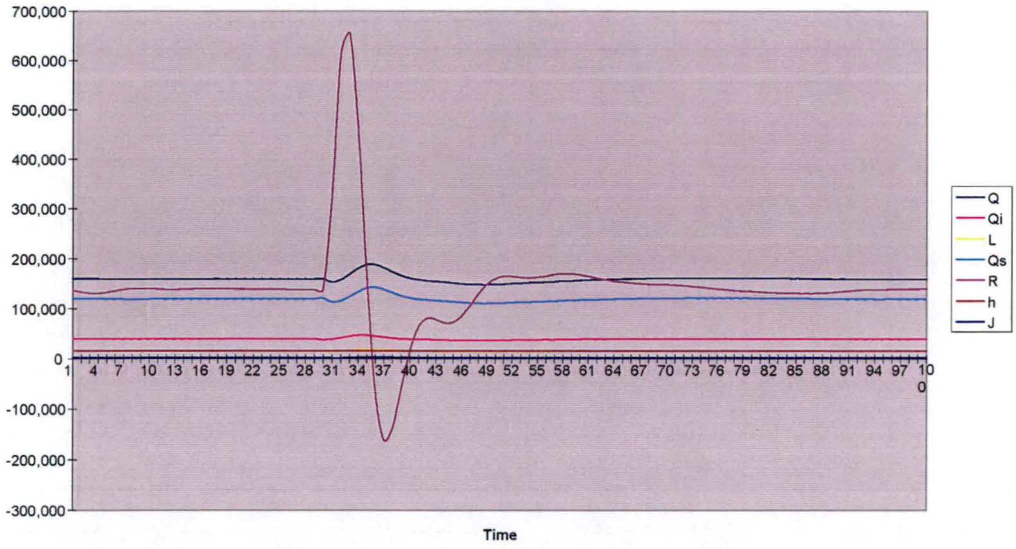


Figure 7.2a above shows that, although the initial cycle in profit is just as violent as in Figure 6.3a in the previous chapter, the subsequent time path of R is approaching flatline.

Figure 7.2b - Percentages

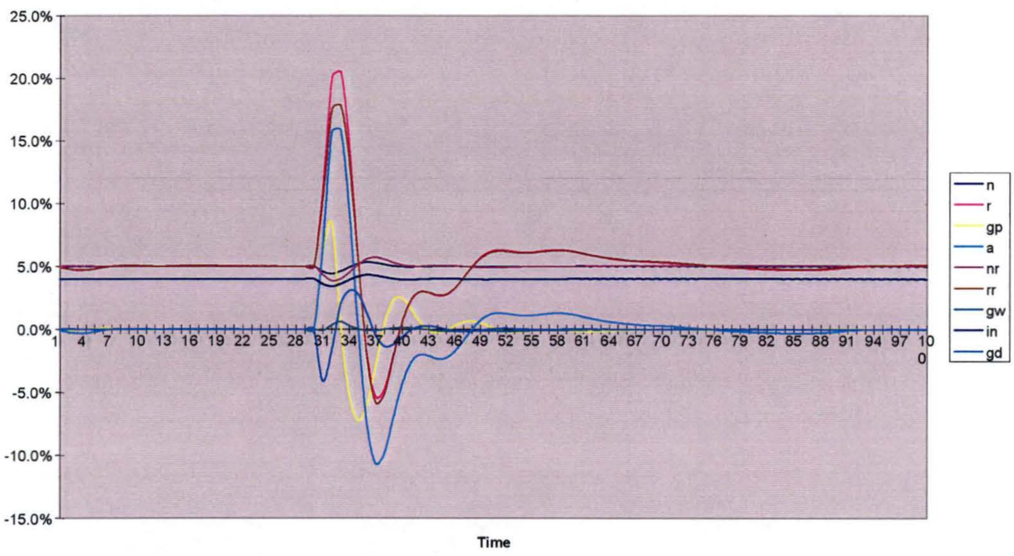


Figure 7.2b above confirms that the same is true of the various profit rate measures, as compared with their pattern in Figure 6.3b of the previous chapter.

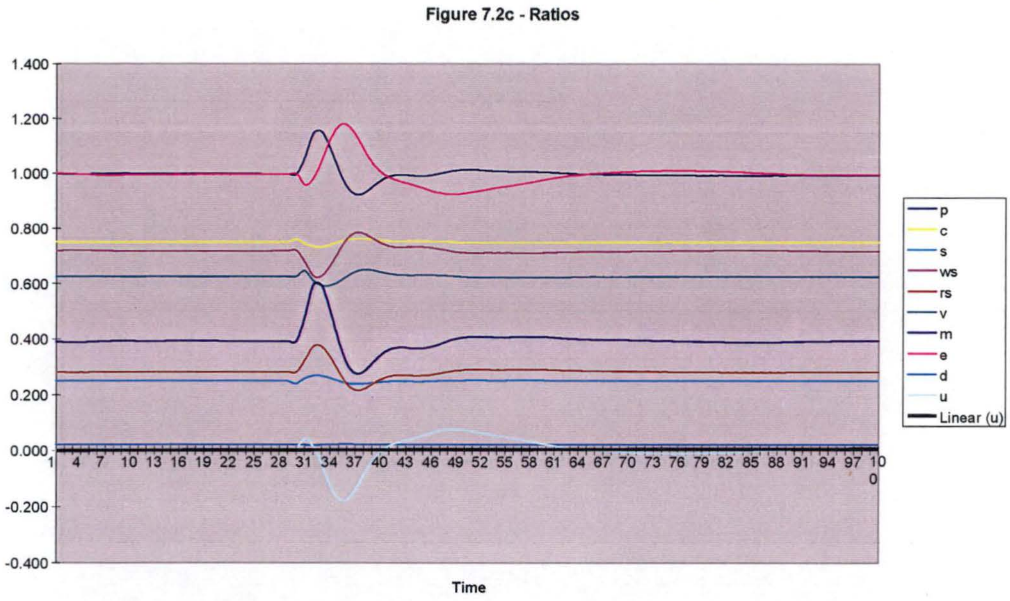


Figure 7.2c above shows that the maximum unemployment rate is now 7.6% of the workforce, as compared with 10.7% in Figure 6.3c of the previous chapter.

Three further graphs show how the government accounts move during 100 years of fiscal policy implementation.

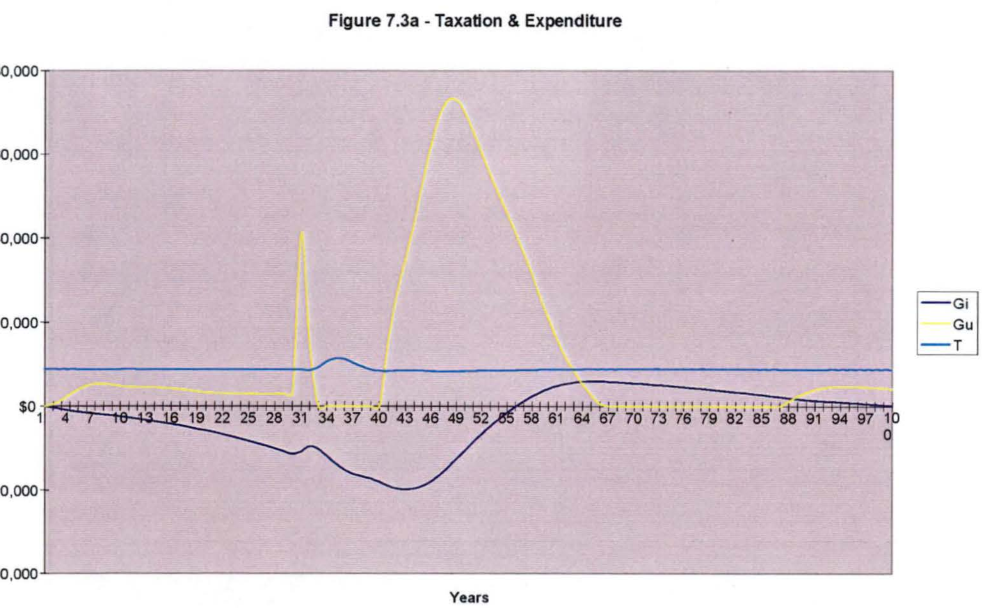
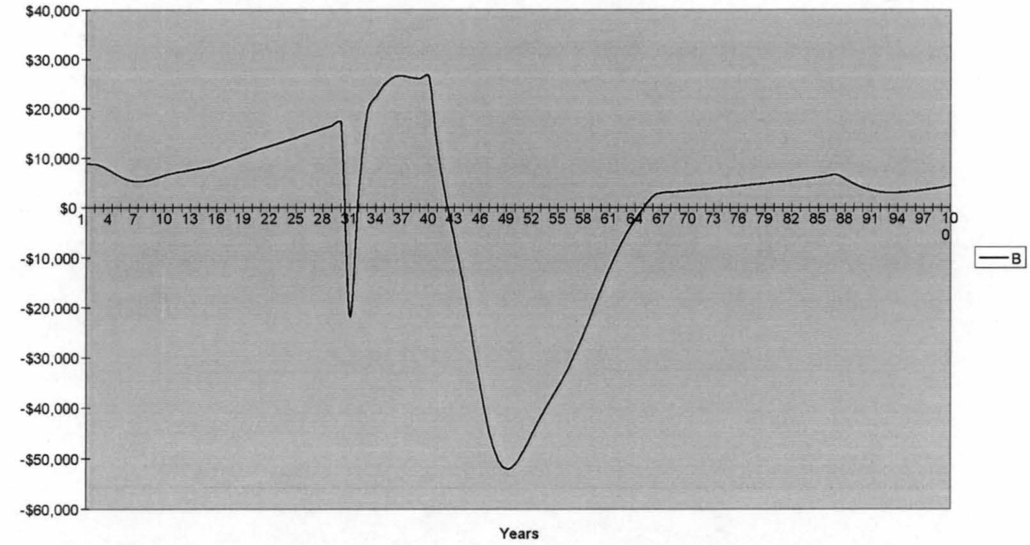


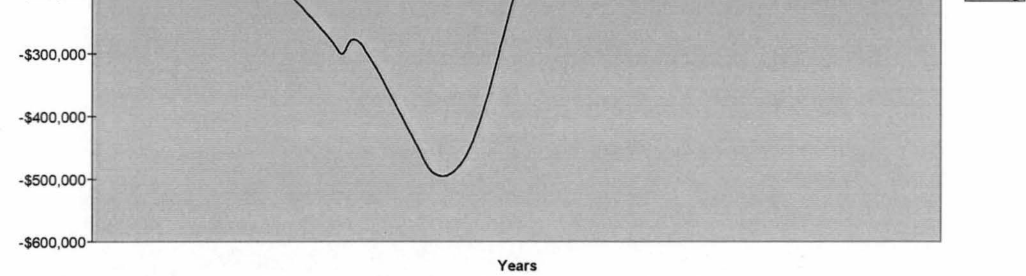
Figure 7.3a above displays (a) fairly stable Income Tax Revenue (T); (b) Government Debt Interest (Gi) *receipts* until year 56 and payments thereafter that fall away to zero by year 100; and four peaks in Unemployment Benefit (Gu) payments, the main one rising to $73,000 pa then falling to zero over the ensuing 26 years.

Figure 7.3b - Budget Balance



These three influences come together in the Budget Balance (B) of Figure 7.3b above, which goes from surplus to deficit twice, reflecting the two highest peaks in Gu payments.

Figure 7.3c - Government Debt



Government Debt (Dg) is the running total of budget deficits (less surpluses) and Figure 7.3c above shows how it mimics the time path of Gi payments – being in negative territory (hence a *Government Asset*, in fact) for 55 years – and is totally retired by year 100.

The harmful *economic* instability associated with *laisser faire* has been transferred to the government's budget by its new policy package. There it has been transmuted into harmless *fiscal* instability, to the benefit of society as a whole. This has been achieved by "setting and forgetting" a pair of simple automatic stabilisers. Economic agents enjoy a century of *certainty* about the fiscal policy settings, budget deficits and surpluses offset each other over the long term, government debt starts and ends at zero, and the disruptive "stop-go" discretionary fiscal policy changes associated with "fine tuning" are neatly avoided.[34]

7.9 Taming the "Unsteady State"

Having confirmed that a combination of unemployment benefits and income taxation *can* stabilise the consequences of a minor misallocation event, the government gains confidence that counter-cyclical automatic stabilisers could ameliorate the economy's major problem, *viz.* the Model E Unsteady State of Chapter 6. The oligarchs know that this is the socially dangerous situation that will prevail if *laisser faire* is not modified by rational economic policies.

In order to simulate the looming demographic transition, the government reinstates (in Table 7.2) the labour productivity decline and workforce growth forecasts temporarily deleted to run the Misallocation Scenario experiment. By trial and error, they soon find that the growth rate of Employment attains near-equality with that of the Workforce between years 31 and 100 ($gL = 0.98\%$ pa $\approx 1.00\%$ pa $= g\eta$), when a Dole Wage Fraction is specified as $df = 30\%$ of the Money Wage and an Income Tax Rate is set at $ty = 4.1\%$ of Household Income.

This brace of fiscal policy settings also ensures that Government Debt reaches its maximum after 40 years, then is progressively retired and extinguished before the century ends, as shown in Table 7.4 below. The growth-enhancing effects of these fiscal policy initiatives by the new government are displayed in column D. For comparison, the growth rates associated with the anarchy and *laisser faire* of Model E appear in column C.

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[34] Discretionary fiscal policies can have perverse effects because of their associated recognition, administrative and operational time lags. By contrast, automatic stabilisers typically apply a known and certain *rate* to an appropriate *base*, whose size fluctuates with the volume of economic activity. Economic agents primarily respond to the rate, not to the base or to the (rate x base) yield or cost.

Table 7.4 – Model E* Tames the "Unsteady State"

	A	B	C	D	E	F	G
1	**E* - INSTRUMENTAL TRAVERSE**	**SN**	**rg**	**ag**	**0**	**31**	**100**
2	**Equations**						
3	Corn Produced	Q	0.78%	0.98%	160,000	177,010	376,166
4	Seedcorn Invested	Qi	0.76%	0.97%	**40,000**	45,436	94,926
5	Employment	L	0.78%	0.98%	16,000	19,925	42,343
6	Profit Rate	r	na	-0.29%	5.0%	11.0%	7.1%
7	Corn Price	P	-0.03%	-0.04%	$27.85	$29.04	$27.62
8	Money Wage	w	0.03%	-0.02%	**$200.00**	$176.03	$173.08
9	Interest Rate	ii	0.00%	0.00%	**4.0%**	4.0%	4.0%
10	**Government Sector**						
11	Policy Toggle Switch	ts	na	na	**1**	1	1
12	Income Tax Revenue	T	na	na	$0	$159,274	$305,831
13	Unemployment Benefit	Gu	na	na	$0	$98,030	$48,519
14	Government Debt Interest	Gi	na	na	$0	$167,143	-$14,712
15	Budget Balance	B	na	na	$0	-$105,899	$272,023
16	Government Debt	Dg	na	na	$0	$4,335,148	-$641,142
17	Dole Wage Fraction	wd	na	na	**30.0%**	30.0%	30.0%
18	Income Tax Rate	ty	na	na	**4.1%**	4.1%	4.1%
19	**Identities**						
20	Wage Bill	W	0.81%	0.96%	$3,200,000	$3,507,349	$7,328,847
21	Seedcorn Capital	Ka	0.76%	0.94%	$1,113,900	$1,285,217	$2,597,509
22	Foodcorn Capital	Kb	0.81%	0.95%	$1,670,849	$1,861,187	$3,847,835
23	Capital Stock	K	0.79%	0.95%	$2,784,749	$3,146,405	$6,445,344
24	Profit	R	na	$0	$139,237	$345,638	$458,063
25	Normal Profit Rate	n	0.00%	0.00%	5.0%	5.0%	5.0%
26	Profitability Gap	a	na	-0.66%	0.0%	6.0%	2.1%
27	Foodcorn Supplied	Qs	0.79%	0.98%	120,000	131,574	281,240
28	Employment Ratio	e	-0.21%	-0.02%	1.000	0.915	0.978
29	Price Level	p	-0.03%	-0.04%	1.000	1.043	0.992
30	Inflation Rate	gp	na	na	0.0%	-0.2%	-0.1%
31	Average Debt	D	0.81%	0.96%	$61,538	$67,449	$140,939
32	Debt.Assets Ratio	d	0.00%	0.00%	0.022	0.022	0.022
33	D:A Ratio Growth Rate	gd	na	na	0.0%	-0.1%	0.0%
34	Interest Bill	J	0.81%	0.96%	$2,462	$2,666	$5,617
35	Household Income	Yh	0.75%	0.87%	$3,341,699	$3,952,339	$7,516,611
36	**Constants**						
37	Reaction Coefficient	ϕ	0.00%	0.00%	**0.4432**	0.4432	0.4432
38	Seedcorn Yield	θ	0.00%	0.00%	**4**	4	4
39	Labour Productivity	λ	0.00%	0.00%	**10.00**	8.88	8.88
40	Risk Premium	φ	0.00%	0.00%	**1.0%**	1.0%	1.0%
41	Capital Turnover	κ	0.00%	0.00%	**2**	2	2
42	Workforce	η	1.00%	1.00%	**16,000**	21,781	43,277
43	Employment Wage Coefficient	ε	0.00%	0.00%	**4**	4	4
44	Inflation Wage Coefficient	ρ	0.00%	0.00%	**12**	12	12
45	D:A Ratio Growth Coefficient	δ	0.00%	0.00%	**0.1**	0.1	0.1
46	Wage Bill Turnover	μ	0.00%	0.00%	**52**	52	52
47	Intercept Constant	α	0.00%	0.00%	**0.2157**	0.2157	0.2157
48	Price Elasticity of Demand	β	na	na	**-3.0**	-3.0	-3.0
49	Cross Elasticity of Demand	χ	0.00%	0.00%	**2.0**	2.0	2.0
50	Income Elasticity of Demand	γ	0.00%	0.00%	**1.0**	1.0	1.0

It can be seen that all this so-called "government interference" stimulates higher growth rates of output (Q), accumulation (Qi, Kr) and employment (L), which now rise at rates ranging from 0.97 to 0.99% pa (Model E*) instead of 0.76 to 0.81% pa (Model E). Real consumption (Cr) grows at 0.98% pa in place of 0.79% pa, although the real wage (wr) improves by 0.02% pa instead of by 0.06% pa. Most significantly, the employment ratio (e) now falls by only 0.02% pa rather than by 0.21% pa. For technical reasons, the Fsted spreadsheet program is unable to compute growth rates for most measures of profit and profitability in Model E, although Model E* is unaffected. However, there are comparable growth rates for margins (mn) and for markups (m), both of which fall in Model E, but more slowly in Model E*.

Favourable growth rate comparisons cannot really convey the impressive effectiveness of the new instrumental fiscal policies in taming the economic instability associated with Model E. For this task a set of time series plots of the relevant Model E* variables is required, particularly for the unemployment rate and for measures of profitability.

7.10 The Instrumental Traverse

The Model E* economy's instrumental traverse and its terminal steady state of constant positive exponential growth is portrayed in three graphs below. They are directly comparable with three Model E graphs, Figures 6.2a through 6.2c above. In terms of Model E*, these Chapter 6 graphs portray the Do Nothing Scenario or *laisser faire* outcome, the one which also results from setting ts = 0 in Model E*, thus effectively reinstating Model E. When ts = 1 is set in Model E*, this generates the "Instrumental Traverse Scenario" or policy-driven outcome.

Figure 7.4a displays only one initial cycle of economic activity, in contrast to the instability exhibited throughout all 100 years under the Do Nothing Scenario.

Figure 7.4a - Expenditures

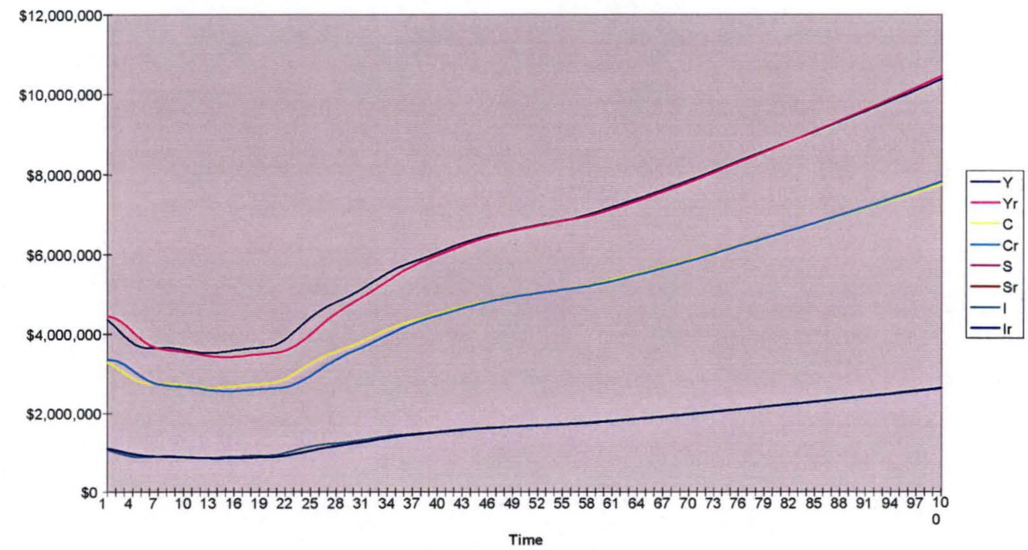


Figure 7.4a above shows that the adoption of a rational fiscal policy *has* successfully stabilised the corn-credit economy, not just raised its rate of growth.

Figure 7.4b - Percentages

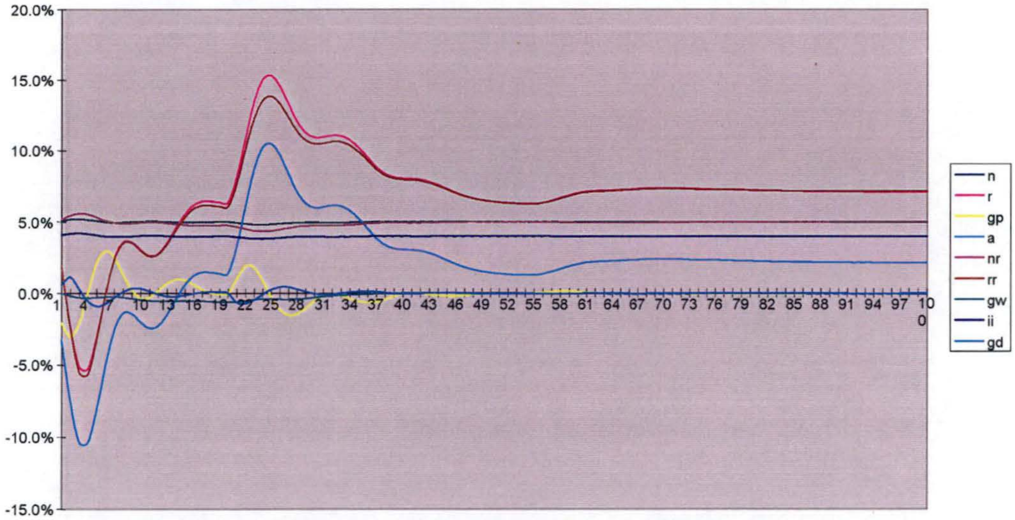


Figure 7.4b above shows that the corn-credit economy has shed most of its instability by year 60, although another 30 years pass before the profitability gap attains its precise steady-state value of a = 2.1% pa. (Under the Do Nothing Scenario of Figure 6.2b, such a tranquil state of constant positive growth is *never* attained.)

Figure 7.4c - Ratios

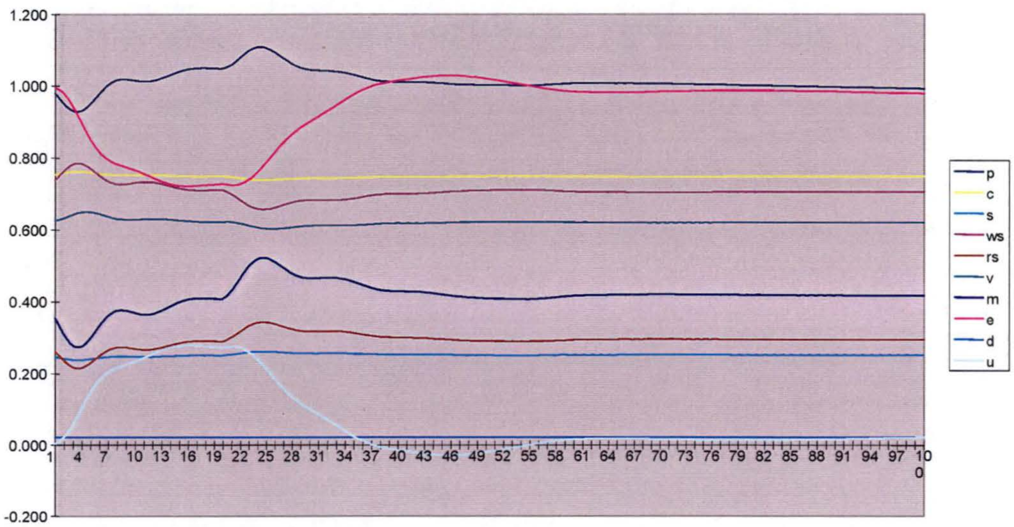


Figure 7.4c above shows that involuntary unemployment is all but wiped out after year 37, following an instrumental traverse during which the unemployment rate rises to a maximum

of almost 28 percent of the workforce. (The comparable figure during the observed traverse of Figure 6.2c in the previous chapter is nearly 45 percent.) Not only are fewer workers unemployed when the government "interferes" with *laisser faire*, but those who *do* lose their jobs no longer live in the abject poverty that could breed "revolution from below".

In the world of Model E*, capitalist farmers responsible for determining the annual volume of seedcorn invested (Qi) confidently base their decision-making on a sequence of profitability gaps that has been stabilised by the "visible hand" of enlightened government intervention.

Three further graphs show how the government accounts move during 100 years of fiscal policy implementation.

Figure 7.5a - Taxation & Expenditure

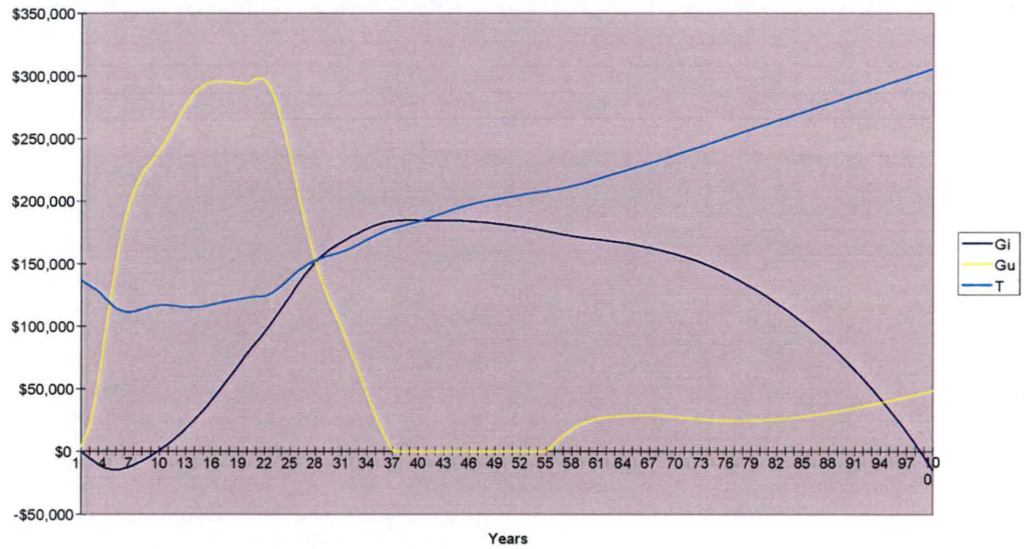
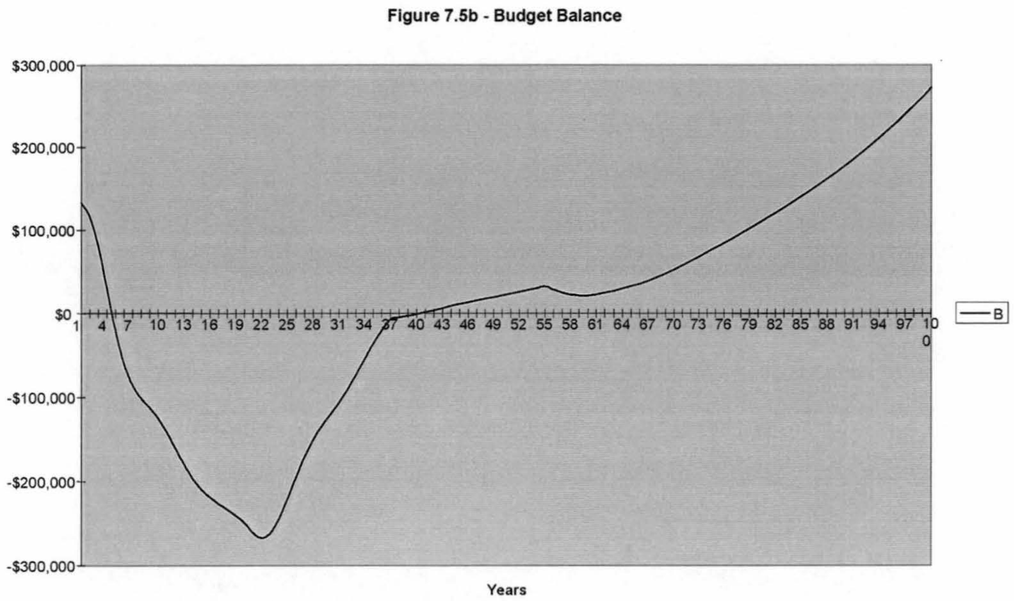
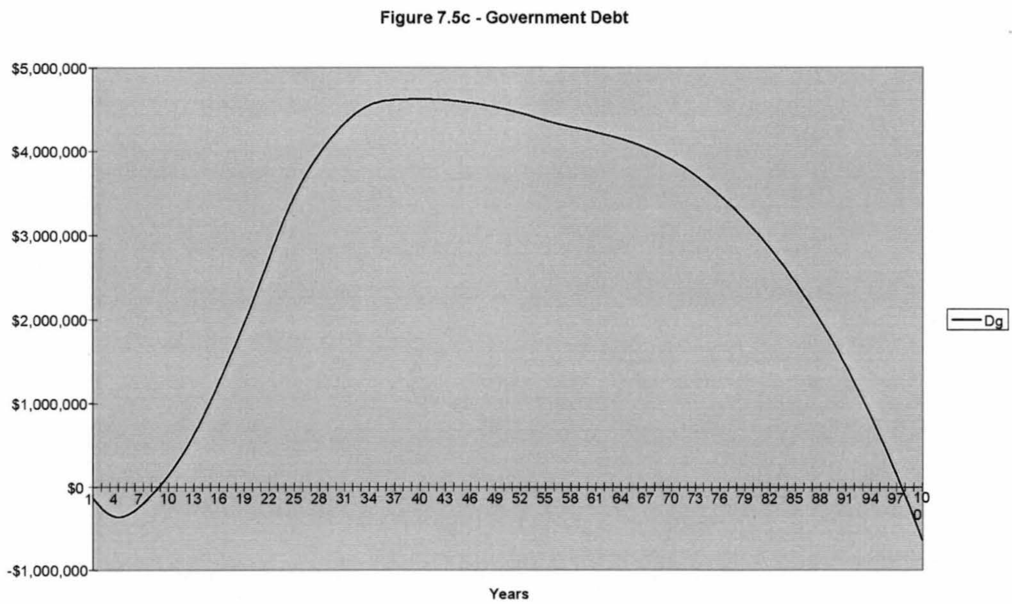


Figure 7.5a above displays (a) a initial dip in Income Tax Revenue (T) which recovers by year 25, with continued growth thereafter; (b) Government Debt Interest (Gi) payments that peak in year 39 and thereafter fall away to zero before year 100; and Unemployment Benefit (Gu) payments which rise to almost $300,000 pa then fall to zero by year 37, thereafter going no higher than $50,000 pa.



These three influences come together in the Budget Balance (B) of Figure 7.5b above, which goes from surplus to deficit by year 5, then swings back into continuous surplus from year 41 onwards.



Government Debt (Dg) is the running total of budget deficits (less surpluses) and Figure 7.5c above shows how it peaks in year 40 then falls to zero by year 97.

Under the Instrumental Traverse Scenario, the extreme economic instability of the Do Nothing Scenario is transferred to the government's budget, where it is coralled, contained and rendered harmless to those who live and work and lend and invest in the corn-credit economy.

This is a remarkable result, achieved by legislating that workers suffering periods of unemployment must be sustained by lightly taxing all citizens in receipt of money incomes – a humane policy that also promotes social cohesion and good industrial relations. Transfer payments in the form of Unemployment Benefit and Government Debt Interest shore up Household Income during downswings in economic activity and work in concert with Income Tax Revenue as "automatic stabilisers" of the corn-credit economy.

Halevi & Kriesler (1992, p 233) state that both Robinson and Lowe show that this kind of beneficial result "... cannot be reached by unguided individualistic behaviour ... Without a visible hand, the invisible hand is likely to guide us onto the wrong path; this is perhaps the most important conclusion from the analysis of the traverse."

7.11 Some Theoretical Implications

Adolph Lowe's retroductive "instrumental traverse" concept can be of great assistance to policy makers who possess a dynamic nonlinear computer simulation model of the economy they wish to influence. Provided it is a fairly accurate representation of the production/financial structure, and of the behavioural forces at work within the economy, policy makers can run instrumental traverse experiments to determine which stable policy settings are likely to deliver the economic outcomes that society has, somehow, indicated to be preferred above the alternatives.

In particular, with sufficient research, it should be possible to identify policy instruments and settings that only need to be changed infrequently. They then would become "part of the furniture" of the economy, so that policy-uncertainty is not added to the raft of other uncertainties faced by economic agents in the nonergodic real world of investment, production, employment, and exchange.

7.12 Conclusion

A government sector can be added to Model E, thus creating Model E* as an experimental testbed for simulating the results of applying Adolph Lowe's "instrumental traverse" concept within the abstract corn-credit economy. It is shown that (i) a misallocation of the harvest between seedcorn and foodcorn and (ii) a looming demographic shift (with its attendant productivity-sapping implications) *can* be accommodated successfully, provided an enlightened fiscal policy featuring automatic stabilisers is crafted by a non-ideological, pragmatic and rational government, then implemented.

After some initial instability due to the demographic transition, a purposively-designed instrumental traverse carries the corn-credit economy from its original stationary state of zero economic growth to a terminal steady state, one that is consistent with constant positive economic growth and near-full employment, while maintaining living standards and promoting social cohesion. This is achieved by "setting-and-forgetting" three fiscal policy instruments: an Unemployment Benefit which is financed by an Income Tax and paying Government Debt Interest to bondholders at market rates. A more benign and less intrusive successful government intervention in the economy is difficult to imagine.

In the "Discretion *versus* Rules" policy debate within the economics profession, the instrumental traverse results of Model E* favour the use of Rules. The approach of long-term automatic stabilisation has advantages in terms of avoiding the time-lags, uncertainties and political business cycles associated with total policy discretion. There is a downside, of course. Because Lowe's instrumental approach (in the context of Model E*) is so powerful, it could be used to "steer" the traverse towards a wide range of end-state goals. An irrational and unscrupulous government could institute damaging Rules: fiscal policy settings that produce a good fit with their own ideology, rather than harnessing the economy's full potential to function as an engine of provision for all its citizens.

CHAPTER EIGHT

PRINCIPAL FINDINGS, LIMITATIONS AND FUTURE DIRECTIONS

8.1 Introduction

In this chapter, the four Principal Findings of the thesis are discussed first. Then the seven main Limitations which presently preclude most of these findings from claiming applicability beyond the abstract corn-credit economy (let alone claiming *general* applicability) are identified. Finally, a set of Future Directions for research aimed at investigating certain conjectures, reducing the limitations and generalising the findings is discussed and two specific Recommendations are made.

8.2 Principal Findings

This research effort has established that

- Post-Keynesian theory can be used to create an original model of a *laisser faire* flexprice corn-credit economy developing through historical time in a nonergodic world;
- violent, long-duration "observed traverses" – which typically are convergent, albeit in the very long term – can be initiated by small changes in consumer behaviour, with their time-shapes being governed by investor reactions;
- a rational and pragmatic government can use a well-targetted fiscal policy to engender an "instrumental traverse" which overcomes the shortcomings of such *laisser faire* traverses; and
- certain Post-Keynesian theoretical propositions are confirmed by this research.

These four principal findings are expanded and discussed below.

8.2.1 Modelling a Flexprice Corn-Credit Economy

From an analytical survey of the traverse literature, it was determined that a traverse model having the following coded characteristics could be viewed as an original contribution:

> **AT**, NER, HT, ATED, PAR, AIB, MONY, CLRE, NVI, WK, FIX, CORN, RN
> [Robinson, Kalecki, Lowe]

Six of these characteristics are ones which appear most frequently in the literature, so the claim of originality cannot lie with them. Except for seven Neoclassical models inspired by the First Traverse Analysis of Hicks, all models exhibit the nonergodic (NER) world view and their traverse paths evolve through historical time (HT). Mathematical tractability probably explains why most traverse modellers assume fixed coefficient technology (FIX) in a one-commodity world (CORN). The majority of traverse models also feature Classical circular reproduction (CLRE) and are non-vertically-integrated (NVI) with respect to labour as the primary factor of production.

The remaining eight characteristics are what make Model E an *original* contribution. It is the only traverse model inspired by Robinson, Kalecki and Lowe, and this mandates modelling a monetised production economy (MONY) that can be always in traverse (AT). In the literature, almost all models feature *fixed* capital, inviting the (false) conclusion that working capital (WK) is a passive force; the corn-credit economy exhibits significant disequilibrium traverse phenomena due to the accumulation of circulating capital only. To provide a good testbed for running traverse experiments, all the model's parameters (PAR) – not just the usual one or two – are perturbed, with the resulting traverse paths being initiated from stationary- and/or steady-state (ATED) basecase comparators.

From the viewpoint of human agency, each of the model's parameters is an average of individual behaviours (AIB), a characteristic shared with only six other contributions to the traverse literature. Finally, the traverses are propelled by endogenous forces striving for equality between two different earning rates on capital (RN), as in only one other model surveyed; a gap between the realised and required profit rates determines the accumulation of working capital, which drives the rest of the economy.

Model E features a theoretically sound "profitability gap accelerator mechanism" driving cycles, distribution and growth. The conventional accelerator has positive feedback onto real investment occuring *via* differences in the current and lagged values of real output, consumption, profits, or capacity utilisation. However, in the corn-credit model, such positive feedback occurs *via* a succession of changing "profitability gaps" between the expected and the normal rates of profit. Appendix B finds that all four differences appearing in conventional accelerator mechanisms are in fact *proxies* for the fundamental profitability gap that underlies and explains each of them. The profitability gap concept is demonstrated to be implicit in most theories of investment behaviour currently deployed by economists; in the history of economic thought, this seminal concept is traced back through Keynes, Fisher and Wicksell to Henry Thornton (1802).

Only the classic *stationary state* of zero growth can be described as an "equilibrium solution" of any historical-time dynamic model wherein real investment responds to the profitability gap. Outside the stationary state (in which zero pure profits are earned), all time paths are afflicted by "profitability disequilibrium", which causes traverses, fluctuations, cycles, and crises. Surprisingly, this same profitability disequilibrium also is responsible for keeping the classic *steady state* of constant positive exponential growth in existence. It is more accurate to describe this regime as "fully-adjusted", rather than exhibiting long-period equilibrium.

Along the entire steady-state growth path, entrepreneurs are experiencing a constant positive difference between their expected and normal profit rates, so the economy *cannot* be in equilibrium. In such historical-time models, stationary states can be *solved for* but steady states of growth must be *generated.* This is in contrast with logical time models, in which the fixed *ratios* between positive-growth variables in a steady state are conflated with the fixed *levels* of zero-growth variables in a stationary state, inviting the (false) conclusion that *both* are equilibrium states. Thus it is that many growth models, alleged to be in "dynamic equilibrium", are in fact in static equilibrium – but with respect to their proportions, not their levels.

Mathematically, historical-time representations of the level variables of a dynamic economy must be *nonlinear models* because the profitability gap is a difference variable and there are multiplicative variables and time lags throughout the model's structure. The *expectation* by entrepreneurs that a certain profit rate will be realised on the current replacement cost of their capital stocks can only be based on past (i.e. lagged) economic outcomes; projected economic outcomes necessarily are based on historical data. Furthermore, analytical solutions to nonlinear models containing more than two difference equations are not available, so computer simulation necessarily is used to solve Models A through E for their stationary-state basecases.

The fixprice Model A exhibits no traverse behaviour, only *instantaneous* adjustment to perturbations of its initial values and constants. The reduced form of Model A shows that the dynamic behaviour of the entire physical/real side of the abstract corn-credit economy is governed by the investment function, with its profitability gap and reaction coefficient. This crucial investment function is common to all five models in the nested series. For Model A, the only effect of raising (lowering) the reaction coefficient is to increase (decrease) all physical/real magnitudes instantaneously.

Model B, with its flexible corn price and classical assumption of zero saving by workers, is a true cycles, distribution and growth (CDG) model exhibiting stationary- and steady-state growth paths, from which traverse paths (which typically are cyclical) may depart. The value of its reaction coefficient determines which particular regime it will traverse towards, once an initial fully-adjusted state has been perturbed by changing some parameter. There is a critical value of the reaction coefficient at which regular cycles (limit cycles) occur. Below it there is smooth or cyclical convergence to a terminal stationary or steady state. Above it there is cyclical divergence which ultimately leads to system collapse. These findings also are true of Models C and D, but not of Model E, which dispenses with the classical saving assumption. The reduced form of Model B shows that the dynamic evolution of both the physical/real *and* the monetary/nominal sides of the abstract corn economy are governed by the profitability gap investment function.

Potentially, Model B (with only four independent equations and one equilibrium condition) can be used to teach economics undergraduates the rudiments of economic dynamics. The responses of entrepreneurs to the time-sequence of profitability gaps that ultimately drive all prices and quantities plus all cycles, distribution and growth in this corn-credit model can be modified by parameter-changes to demonstrate many dynamic regimes, paths and propositions.

Model C, which also has a flexible money wage, requires the same reaction coefficient as Model B, in order to traverse to a *regularly cycling* (yet fully-adjusted) disequilibrium regime. Analysis of Model C shows that the money wage is a kind of "sheet anchor", which determines how low or high all money prices and nominal values will float within the corn-credit economy. This finding also is true of both subsequent CDG models in the nested series.

Model D, which also has a flexible interest rate, requires a slightly larger reaction coefficient than the two preceding models, in order to traverse to a *regularly cycling* disequilibrium regime. Analysis of Model D shows that the nominal and real interest rates are far less volatile than the nominal and real profit rates, a finding which accords with the stylised facts of real-world CDG phenomena and also is true of Model E.

Model E, in which corn price flexibility is achieved *via* a conventional demand function for foodcorn, requires a much smaller reaction coefficient than does Model D, in order to traverse to a *regularly cycling* (yet fully-adjusted) disequilibrium regime. Above that critical value there is cyclical convergence to a terminal stationary or steady state. Below it there is cyclical divergence which ultimately leads to system collapse. These directions are the *reverse* of those in Models B, C and D, which feature the classical saving assumption.

With no choice between spending and saving, households in receipt of wages (and interest, in Model D) outlay *all* such income on foodcorn. Clearly this is a stabilising force, since convergent traverses in Models B, C and D typically last around 30 years, compared with 188 years in Model E, where consumers are permitted wide variations in the proportion of household income they choose to spend on foodcorn. This strongly suggests that extra price flexibility *promotes economic instability.* This finding is completely at variance with the Neoclassical mainstream's presumption that coordination of economic activity always can be improved if the degree of consumer choice, competition and price flexibility is increased by legislating for "microeconomic reform", "national competition policy" and similar attempts at forcing the real world into conformity with the Walrasian general equilibrium axioms.

The majority of Post-Keynesians have allocated their impressive theory-building resources to models of mark-up pricing in the context of an oligopolised industrial economy. This thesis shows – admittedly in the context of an agrarian economy only – that Post-Keynesian results *can* be derived from models of flexible pricing under pure competition. Neo-Keynesians ("It's just a regrettable fact that real-world prices are *sticky*") and New Keynesians ("Here's *why* real-world prices are sticky: asymmetric information, menu costs, efficiency wages, ...") have ignored the fact that the formal analysis in Keynes (1936) actually is built upon Marshallian short-period foundations of pure competition and *flexible* prices, money wages and interest rates.

If Joan Robinson's ambitious project of generalising *The General Theory* is still taken seriously by her successors, more work will have to be done within the Post-Keynesian paradigm to explore the implications of open competition between price-takers rather than concentrating on the analysis of implicit collusion between price-makers.

8.2.2 Observed Traverses

The profitability gap, when multiplied by the reaction coefficient, determines the percentage change in the previous year's real investment aggregate. A traverse can be initiated from the steady state by *investors* altering their reaction coefficient, because the profitability gap has a positive value in a growing economy. This is not true of the stationary state, in which the profitability gap remains on zero along the economy's entire flatline time path. However, *consumers* can initiate such a traverse, by altering the elasticity constants of their demand function. All other parameter changes have far less power to disrupt a corn-credit economy.

Significantly, it is the average value of all investors' individual reaction coefficients that governs whether the traverse converges, diverges or cycles regularly. This same parameter also sets the *extrema* of the economy's "range of viability", governing how soon it will crash on either its first convergent (but too large) cycle or its last divergent (and too large) cycle.

This research has shown that much of the dynamic behaviour expressed in the time series of Model E accords with the stylised facts of real-world economies, e.g. stability of the wage and profit income shares; volatility of the profit rate exceeding that of the interest rate; and cyclical growth occurring in a sawtooth pattern. Furthermore, this is achieved using a flexprice model in a regime of pure competition, without the rigidities introduced by monopoly structures and/or the mark-up pricing rules that characterise most Post-Keynesian models.

In addition, although it takes around 30 years to generate steady states from stationary states in Models B, C and D, this is not true of the final model. Due to consumer sovereignty, the highly-sensitive Model E propagates endogenous cycles *whenever* it traverses away from the stationary-state solution time path, no matter what impulse is responsible for initiating the disequilibrium situation. For workforce growth rising from zero to 1% pa, the steady state is not attained until 188 years have elapsed.

8.2.3 Using Fiscal Policy to Engender an Instrumental Traverse

A fully-flexible model of Post-Keynesian dynamics (the "instrumental" Model E*, with its simple Government Sector) has been created. It was used to identify a package of effective, efficient and equitable "economic policies", one that a newly-formed non-ideological "government" of farmers and bankers (a Rational Oligarchy) might enact to generate an Instrumental Traverse to a steady state that can stabilise and grow the Model E* corn-credit economy. These policies were designed in light of the sensitivity, dynamic stability and traverse analysis results of the *laisser faire* Model E, as reported in Chapter 6.

The main policy implications of Model E* are that automatic stabilisation can occur *via* a fiscal policy package featuring unemployment benefits, flat-rate income taxation and the receipt/payment of interest at market rates on government debt. Policy-makers need to have a dynamic nonlinear recursive model at their disposal and cannot rely on "economic intuition" developed through the intensive study of static economic models. Evidence for this proposition comes from the evolution of spiral patterns in two-variable phase diagrams as the reaction coefficient is varied.

The ability to simulate an economy's "alternative futures" allows policy makers to distance themselves from the "fine-tuning" and "stop-go" policies whose uncertain effects have contributed to the ousting of Keynesian economics from the mainstreams of theory and policy. Instead, experimentation with different fiscal policy settings, followed by careful analysis of the instrumental traverses they ignite, is likely to reveal a "set-and-forget" policy package that gives certainty to economic agents and may remain in force for decades. The "visible hand" of government intervention *can* have a feather-light touch.

8.2.4 Confirmation of Post-Keynesian Results

The findings of Model E are consistent with many results from those streams of Classical, Post-Classical (and especially Post-Keynesian) economic thought flowing from Petty through Quesnay, Smith, Ricardo, Marx, Marshall, Lowe, Kalecki, Keynes, Myrdal, Robinson, Weintraub, Minsky, Davidson, Harcourt, Asimakopulos, Rowthorn, Moore, Lavoie, Arestis, Courvisanos, and many others having contributions listed in the Bibliography. For instance, both the Keynesian paradox of "thrift" and the Post-Keynesian paradox of "costs" have been confirmed.

This consistency was achieved despite basing the corn-credit model on only *four* specifically Post-Keynesian axioms:

1. The world is nonergodic;
2. Historical not logical time passes;
3. Investment depends on profitability expectations; and
4. Money is credit-driven and demand-determined.

Within the Classical tradition, corn modelling began with Petty (1662) and traverse analysis with Ricardo (1821). Within the Post-Classical/Post-Keynesian tradition, the corn-credit CDG models especially draw on those streams of economic thought associated with Adolph Lowe, Michal Kalecki and Joan Robinson. Knut Wicksell's "pure credit economy" assumption has been adopted, so that money is a unit of account and a standard of value *but nothing else*. Hence, nominal prices do not depend on the quantity of money but mainly on the money wage, with the corresponding real wage being determined *outside* the labour market.

This research upholds the Keynesian doctrine that "money saving" and "real saving" always will adjust to accommodate *any* level of real investment decided upon by capitalist entrepreneurs. The capital accumulation process governs most economic outcomes *via* the reaction coefficient and the profitability gap, which is a difference between two *nominal* rates of profit. The money wage is the "sheet anchor" of money prices and the pattern of *nominal* values. All real magnitudes are derived from these nominal rates and values *via* division by the inflation rate; real variables have no independent influence on decision-making.

For corn-credit models, a theoretically correct solution has been provided to a long-standing puzzle in capital theory. Do realised money profits ultimately stem from compensation for capitalists voluntarily practising "frugality" (Smith), "abstinence" (Senior) or "waiting" (Marshall), or for suffering "lacking" (Robertson)? Do profits come from "the" marginal productivity of "capital"? the "natural rate of interest"? the duration of the "period of production"? or the "surplus value" extracted from labour? From natural growth/maturation processes? rewarding entrepreneurial "risk-bearing"? or from none of the above? What this research has shown is how realised money profits plausibly can be determined by the Kalecki/Robinson mechanism: expected profitability determines investment, which determines realised profitability, which determines expected profitability afresh, and so on in an endless sequence of Myrdalian/Kaldorian circular and cumulative causation.

The research has demonstrated that Keynes's refutation of Say's Law need not stand or fall on the practice of "hoarding liquidity" in an economy where money is a *medium of exchange* and a *store of value*. Say's Law of Markets turns out to be false, even in an economy where money is merely a *unit of account* and a *standard of value*. In this Wicksellian "pure credit economy", bankers create (destroy) money by extending (recalling) loans, in response to the demand by farmers to finance the money wage bill at current market interest rates.

8.3 Limitations

In the current state of mathematical knowledge, beyond a small degree of nonlinearity it is *impossible* to obtain analytical solutions for such dynamic recursive models. Therefore a numerical analysis technique such as computer simulation of spreadsheets must be used to investigate their properties. The final, fully-flexprice corn-credit model is highly nonlinear and recursive. It creates plausible endogenous cycles, distribution and growth endogenously, but no reduced form can be obtained from its structural form.

There are seven principal theoretical and policy limitations of the computer simulation model used to generate the traverses of this thesis.

8.3.1 Use of Numerical Analysis

Numerical analysis models necessarily have *specific* functional forms and parameter values. Such models cannot be used to mathematically establish *general* results.

8.3.2 Average of Individual Behaviours Assumption

The AIB assumption concerns parameters such as the demand function elasticities and the investors' reaction coefficient. There is a risk of such behavioural coefficients changing due to different groups becoming dominant within the population from time to time.

8.3.3 Over-Simplified Economy

A one-commodity closed economy with no outside money, no fixed capital equipment and no choice of techniques is too sparse and unrealistic for serious policy analysis purposes. Some models can be used for teaching the principles of economic dynamics, but the final Model E* is no more than a starting point for building a realistic applied CDG model that will find acceptance among economic policy-makers.

8.3.4 Profitability Expectation Function

Naïve, myopic or static expectations ($re_{t+1} = r_t$% pa) is merely one of several possible expectation functions economists have proposed, including the $re_{t+1} = r_{t+1}$% pa ("rational expectations") function that would be appropriate if the world was ergodic. Other nonergodic specifications such as "adaptive" and "least squares" expectations should be assessed. It is likely that these will mute the instability displayed by Model E to some extent.

8.3.5 Catastrophic Collapse

The corn-credit economy collapses if the reaction coefficient constant is set outside its own (albeit wide) "range of viability". Model E presently contains no endogenous mechanism to maintain ϕ within the bounds of this range.

8.3.6 Four Post-Keynesian Axioms

It is accepted that the world is nonergodic; that historical not logical time passes; that investment depends on profitability expectations; and that money is credit-driven and demand-determined. All other axioms are Neoclassical, e.g. those underpinning the consumer demand function. However, rival axioms from the Post-Keynesian (or other heterodox) traditions could well be superior.

8.3.7 Fiscal Policy Package

It is not enough to test only *one* particular suite of fiscal policies, viz. Unemployment Benefits financed by Income Taxation at a flat rate, with Government Debt Interest paid at market rates. No other fiscal policies were assessed and no monetary or commercial policies *could* be tested in these models of a closed economy having no outside money or central bank.

8.4 Future Directions

Most of the limitations identified above can be overcome or ameliorated by significantly expanding the structural form of Model E* before using advanced econometric techniques to identify the parameter values it takes on within one particular mixed capitalist economy. The model's real-world relevance has not been (indeed, could not be) assessed – let alone tested – within the scope of this thesis. Furthermore, there remain many opportunities for simulating the full range of responses to traverse-inducing parameter changes. Other limitations will be overcome if the four conjectures made below are confirmed by further investigation and the two recommendations which flow from this research are implemented.

8.4.1 Complexity Conjecture

That Model E is a complex systems model, amenable to further investigation using the same methodologies as have been developed to deal with the Post-Walrasian and other models analysed by the Santa Fe Institute and similar Complexity Economics research centres.

It is significant that no reduced form could be derived from the model's (apparently simple) structural form: six independent equations and a single equilibrium condition. The reduced form of an even simpler system (viz. Model D) is byzantine. It was found to have 46 instances of several known quantities determining its unknown flexible corn price, with the right-hand side of this equation including four squared parameters and one raised to the fourth power.

8.4.2 Mathematical Conjecture

That the Model E Post-Keynesian dynamic corn-credit economy can be viewed as a mathematical object in seven-dimensional "state space" with 17 degrees of freedom, one per parameter. This object changes its "hypershape" from year to simulated year in accordance with certain changing forces, captured in the model's independent equations and its equilibrium condition.

These forces are generated by a set of coupled nonlinear oscillators for (at least) Qi, w, i, and P, which are implicit in the six independent equations (plus one equilibrium condition) that drive the corn model's seven endogenous variables, whose state-space coordinates define how the 7-D hypershape changes as simulated historical time passes.

Matthews, Mirollo & Strogatz (1991) developed the general mathematical model for an indefinite number of coupled nonlinear oscillators in dissipative dynamical systems, so this (or a later and/or similar) model might provide the mathematics needed to reveal the envelope of dynamic behaviour for a Model E corn-credit economy – and to make many other mathematically precise statements having *general* applicability, statements whose *economic* implications can be probed in future research.

8.4.3 Susceptibility Conjecture

That the constant reaction coefficient can be made into a *variable* using an equation whose right-hand side determinants will keep the coefficient within known limits for maintaining the economy's viability. This equation can be developed using the concept of "susceptibility" developed by Courvisanos (1996), in order to explain turning points in the capitalist economy's cycle of real investment orders.

At present the corn-credit economy suffers a catastrophe if the reaction coefficient constant is set outside its own range of viability. At one end of this spectrum, the economy collapses at the high-amplitude peak of its first convergent cycle; at the other end, this occurs at the peak of its last divergent cycle.

8.4.4 Literature Conjecture

That further research into the history of economic thought will uncover a trove of "hidden literature" produced by progenitors other than Ricardo and Marx and containing descriptions (or misrecognitions) of disequilibrium traverse phenomena. Before this can happen, economists generally and historians of economic thought in particular, will need to have their "traverse consciousness" raised. At present, the analysis of traverse processes is a minority endeavour within the discipline of economics.

8.4.5 Recommendations

It is recommended that the complexity, mathematical, susceptibility, and literature conjectures be investigated with a view to determining whether or not they have merit. In particular, if the mathematical conjecture proves correct, it may become possible to assess whether the specific numerical Model E possesses algebraic generality.

Provided the problem of "viability" can be solved (e.g. by validating, then implementing, the Susceptibility Conjecture), it is further recommended that Model E* be converted into an applied multisectoral CDG model having

a land classes and **k** capital outfits
distributed among
d industries[35]
which utilise
s labour skills
to produce
n products (where **n** > **d**)
absorbed by the
d industries (intermediate sales)
and
h household types (final sales).

Over time, the model could be expanded by increasing **a**, **k**, **d**, **s**, **n**, and **h**. With international trade added, pricing would be exogenous[36] for the **m** < **n** products subject to import competition and for the **x** < **n** products sold on export markets. Within the trade-sheltered sector, Post-Keynesian mark-up pricing would apply to products of the concentrated secondary and tertiary industries. Products of the primary, the residual secondary and the tertiary industries would be competitively priced, as in the corn-credit model of this thesis.

Already Model E* possesses a rudimentary Treasury or Ministry of Finance (fiscal policy). For the model to acquire a Central Bank (monetary policy), it is necessary to supplement the unit-of-account and standard-of-value functions of inside money with the medium-of-exchange and store-of-value functions of outside money. This new monetary base would be linked with credit creation and support a superstructure of progressively less liquid financial

[35] Incuding various land development, labour skilling and capital formation industries.
[36] Based on world prices in USD and the AUD/USD exchange rate.

instruments priced in competitive markets. Numerous accounting identities would enforce the logical relationships that bind together the stocks and flows within and between the model economy's physical/real and monetary/nominal sides.[37]

The **d** profitability gap investment functions and the (**h** x **n**) consumer demand functions, together with workforce growth, technical progress and fiscal/monetary/commercial policy settings, are expected to be the main determinants of cycles, distribution and growth.[38] The economy's industrial structure – the relative sizes of its **d** industries – will change as investment flows into high-profitability industries and capital stocks depreciate in low-profitability industries.

The parameters of this proposed multisectoral model would be econometrically identified, with a view to commissioning it as a Post-Keynesian CDG alternative to Neoclassical CGE models in formulating Tasmanian and Australian economic policy. Model E* can be used as the initial *core* for constructing nonlinear and recursive CDG models of real-world mixed capitalist economies, models that ultimately will approach the same large scale as linearised CGE models such as FEDERAL-F, a descendant of MONASH, itself derived from ORANI.[39]

8.5 Conclusion

When J R Hicks (1965) named the Traverse and made it into a respectable object of mainstream research, most economists thought this adjustment path would turn out to be smooth and dynamically stable, carrying the economy (in short order and with a minimum of fuss) from one long-run equilibrium steady-state growth trajectory to another. The semi-log graphs of income and consumption against "time" in Solow (1970, pp 24-6) are typical of this Neoclassical belief, derived from the First Traverse Analysis of J R Hicks. Yet, even in that same year, John Hicks (1970, 1973) shifted his attention to the more problematic, but dynamically far richer, Neo-Austrian approach of his Second Traverse Analysis. Much has changed since then.

Nowadays, all heterodox schools of economic thought seem to be gradually coalescing around the Krieslerian "observed traverse" concept utilised in this research. In the particular (Post-Keynesian) models of this thesis, the traverse typically is found to comprise a sequence of short-period situations, with this time path of adjustment jagged rather than

[37] *Vide* Lavoie & Godley (2000) for an excellent treatment of the necessity for, and the beneficial results of enforcing, these accounting relationships.
[38] Workforce growth (and technical progress) can be endogenised by building a Demographic (and an Innovations) Module which accepts economic feedback from the CDG Model itself.
[39] *Vide* Giesecke (1997) for information on the FEDERAL-F Model and its lineage.

smooth, of longer duration in more flexible economies, and even misnamed – on account of its evolving into, rather than connecting with, some final dynamic path that may (or may not) constitute a fully-adjusted stationary, steady or cyclical state.

There are four Principal Findings and seven theoretical and policy Limitations of this research. It would seem worthwhile to follow up the Future Directions as recommended in this chapter. At present, the task of constructing Post-Keynesian CDG monetary production economy models is being undertaken at only a handful of research centres worldwide. By contrast, the creation and operation of Neoclassical CGE barter exchange economy models has been underway for some four decades and is now a mature "industry", one in which Australian economists are at the forefront of research.

There exists an obvious opportunity for an Australasian university or economics institute to gain a "first in the field" advantage by founding a regional Centre for Dynamic Economics dedicated to constructing complex multisectoral CDG models for real-world application and for performing economic research and undertaking consultancies in the numerous associated fields. The Centre's principal role would *not* be forecasting but policy analysis, viz. the identification of stable settings for policy instruments that promote long-term sustainable growth by minimising the disruptions that accompany economic development through decades of historical time. In short, the Centre would practice "policy discovery" of various means for automatically stabilising the *observed* traverses that afflict real-world economies. Relatively smooth *instrumental* traverses would result from choosing policy instruments whose settings need not change as frequently as they do at present.

APPENDIX A

TRAVERSE MODELS AFTER 1973[40]

Bhaduri (1975)

IP, ERG, LT, STED, LAB, SIC, REAL, CLRE, NVI, FK, FIX, COTR, SD

[Hicks FTA] Hicks simply *assumed* that saving would equilibrate to investment along his Neoclassical quantity-traverse. The Capital Intensity Theorem is proved for this Q-traverse, but also for its analogous price-traverse. This symmetrical P-traverse "dual" is derived from the disequilibrium process of dynamic price-adjustment, assuming no saving out of wages and a $0 < s_c < 1$ saving propensity out of profits. It is shown that more adjustment time is required along the P- than along the Q-traverse, and that these times will only be equal when all profits are saved ($s_c = 1$). Finally, both adjustment times will be faster, the closer the ratio of C-sector mechanisation to T-sector mechanisation approaches 1.0 from above.

Lowe (1976)

CP, NER, HT, STED, LAB/INN, AIB, REAL, CLLN, NVI, FW, FIX, CTLA, EP

[Lowe] To absorb faster workforce growth, the M-sector must be made to expand its capacity at the expense of the T-sector, thus temporarily denying tractors to the C-sector and reducing its output of corn. This lowers the real wage (*via* forced saving) for the duration of the traverse. This is paradoxical: In order ultimately to *increase* the output of corn, such output must, to begin with, be *reduced*. Lowe's principal finding is unchanged, but is proved for the steady state using a complex model having four intermediate production stages within both Departments, I and II.

[40] Those contributions marked with an asterisk are written in Italian or French, so the characteristics and findings were gleaned from comments and quotations made by authors of English-language contributions.

Craven (1977)

IP, ERG, LT, STED, LAB, SIC, REAL, CLRE, NVI, FK, FIX, COTR, SD

[Hicks FTA] Bhaduri (1975) found that adjustment time along the P-traverse is greater than along the Q-traverse, yet this result depends upon an implicit assumption that no capital gains are saved. Saving out of capital gains modifies Bhaduri's result to the extent that adjustment times are *equal* under certain conditions. Bhaduri also assumed no saving out of wage income ($s_w = 0$), but Craven finds that the Capital Intensity Theorem of Hicks is sufficient for stability of the P-traverse in all cases, except for when $s_w > s_c$.

O'Connell (1978)

IP, ERG, LT, DSEQ, LAB, SIC, REAL, CLRE, NVI, FK, FIX, COTR, SD

[Hicks FTA] A disequilibrium growth rate of accumulation (g) above (below) that of the workforce (n) implies the overall saving ratio (s) is above (below) its equilibrium value. Given fixprices and a more mechanised corn sector, s moves towards (away from) equilibrium according as g moves towards (away from) n. Given flexprices, $s_w < s_c$ and a more mechanised corn sector, the price ratio of tractors to corn tends towards its equilibrium value. The case can be made that the required savings are more likely to be generated under a flexprice regime.

***Belloc (1980)**

MP, NER, HT, STED, INN, SIC, REAL, LNPR, NVI, FW, FIX, CORN, SD

[Hicks STA] A non-vertically-integrated Neo-Austrian model avoids the restriction that only economies producing "non-basic" consumption goods can be analysed. That is, none of the output of one process may comprise an input to another process, so that not even such "basic" commodities as energy or lubricants may exist. This model overcomes this restrictive assumption of Hicks in his STA.

***Magnan de Bornier (1980)**	**MP**, NER, HT, STED, INN, SIC, REAL, LNPR, VIN, FW, FIX, CORN, EP [Hicks STA, Lowe] This paper implicitly adopts Lowe's instrumental approach when analysing a "planned traverse path" in a Neo-Austrian model featuring vertically-integrated production. This makes the author realise that Hicks's treatment of the traverse as a sequence of temporary equilibria is inappropriate for describing non-steady-state growth paths.
***Gozzi & Zamagni (1982)**	**MP**, NER, HT, STED, INN, SIC, REAL, LNPR, VIN, FW, FIX, CORN, SD [Hicks STA] The late phase of the traverse ends once the final steady state (compatible with the innovation) has been attained. But only under very special conditions is such "convergence" assured, e.g. when the profile of the new process is of the point input-continuous output type. This suggests that the Hicksian "traversibility property" of GE models is substantially irrelevant, as in the Kaleckian observed traverse concept. Such traverses are worthy of study, but an equilibrium not approached by any traverse can hold no interest for the analyst.
Violi (1983)	**MP**, NER, HT, STED, INN, SIC, REAL, LNPR, NVI, FW, FIX, CORN, SD [Hicks STA, Schumpeter] Analyses a neo-Schumpeterian traverse, in which entrepreneurs are dichotomised into innovators and imitators. Only the former are able to introduce new processes immediately, while the latter follow with a certain time lag.
***Baldone (1984)**	**MP**, NER, HT, STED, INN, SIC, REAL, LNPR, NVI, FW, FIX, CORN, SD [Hicks STA] Shows that the Hicksian methodology, when applied to more general models of production (which can either be of the non-integrated Neo-Austrian variety or of the interindustry type), leads to an *indeterminateness* of the traverse paths. This is due to the fact that Hicks's Full Performance hypothesis necessarily

will have to be modified in the context of these more general productive structures.

Halevi (1984)

AT, NER, HT, STAT, CAP, SIC, REAL, CLRE, NVI, FK, FIX, COTR, EP

[Lowe, Kalecki, Keynes] There exists no feasible market-economy traverse making capital so abundant that the profit rate falls to zero. Capitalists in the tractor sector may indeed lower their rate of investment as capital accumulates, but this entails lower profits on corn production. Barring Lange's investment planning, nothing can prevent the emergence of excess capacity in the corn sector (which faces constant money wages) except rising prices of capital equipment. Lowe's specificity and complementarity of tractor production and operation will thwart Wicksell's cumulative process on the way to the *ultimate* stationary state of zero profitability.

***Violi (1984a, 1984b)**

MP, NER, HT, STED, INN, SIC, REAL, LNPR, VIN, FW, FIX, CORN, SD

[Hicks STA] The crucial condition for technological unemployment to occur is not the specific form of the innovation, but the effect on the development of the "gross produce", as in the Ricardian traverse. The Neo-Austrian profile having the highest probability of leading to temporary unemployment combines a strongly forward-biased innovation with a lengthening of the construction period. Asymptotic convergence of the Fixwage traverse path is demonstrated, but it is meaningless to prove convergence for the late phase if the traverse path, during its early phase, exhibits a negative rate of process starts.

Zamagni (1984)

MP, NER, HT, STED, INN, SIC, REAL, LNPR, VIN, FW, FIX, CORN, SD

[Hicks STA] Along a Fixwage traverse, the workforce poses no constraint, so the rate of new process starts depends on saving behaviour, supplemented by resources released *via* the truncation of old processes. A Hayek Effect occurs if capitalist consumption *increases* in response to capital gains during the traverse, creating a resource bottleneck that causes a *negative* rate of starts. Hayek (1931) noted that the current demand for consumer goods might be too urgent to permit investment of current productive services in long processes. During the early phase, a Ricardo Effect always occurs because lower employment due to truncation of old processes outweighs higher employment due to new process starts. This holds true even if the new technique is less mechanised (i.e. more labour-intensive) than the old.

Amadeo (1987)	**IP**, NER, HT, STED, R/Y, SIC, REAL, CLRE, NVI, FK, FIX, CORN, CU [Kalecki] Long-period equilibrium is defined by equality between the realised (u) and expected (u^e) degrees of capacity utilisation; only by coincidence will these equal the planned or normal ($u^n < 1$) degree. If there is an exogenous fall in the share of profits in output, consumption will increase and first u, then u^e, will start to rise at *different* rates. The traverse will be complete as soon as $u = u^e$ once more, regardless of the value of u^n. This will occur despite the differing rates of increase being caused by u^n appearing on the right-hand sides of the equations determining both u and u^e.
Amendola & Gaffard (1988)	**MP**, NER, HT, STED, INN, SIC, MONY, LNPR, VIN, FW, FIX, CSEC, SD [Hicks STA] Intertemporal complementarity affects capacity constraints (construction lag) and decision processes (information lag), so traverse paths embody the sequence "constraints-decisions-constraints". Criticises the use of multisectoral production models for traverse analysis because these assume that capital equipment is non-specific, hence easily shiftable among different lines of production. Creation of both new and upgraded labour skills is made to depend on the number of innovative (as distinct from routine) production processes actually carried on.

Nardini (1990)

MP, NER, HT, STED, INN, SIC, REAL, LNPR, VIN, FW, FIX, CORN, SD

[Hicks STA] If capitalist consumption *does* respond to capital gains and losses (*contra* Hicks) along the Fixwage traverse, the immediate and temporary effect is that extrawage consumption falls, allowing a substantial number of new processes to start. This discontinuity in the age-distribution of the capital stock is propagated into the future. Under two different truncation assumptions, during the late phase of the traverse, the system runs into a resource bottleneck, overcome *via* a medium- to long-term "pure reinvestment cycle". In time, this oscillation peters out, allowing a growth trend to emerge. The growth rate increases as the IRR on new processes rises, and decreases as the propensity for extrawage consumption rises. There is slow convergence to the final steady state.

Halevi (1992)

AT, NER, HT, DSEQ, CAP, AIB, REAL, CLRE, NVI, FK, FIX, COTR, UR

[Hicks FTA, Lowe, Marx] Harrod (1939) noted that if the warranted exceeds the natural growth rate, there will be "chronic unemployment" and not just a Keynesian recession. It is shown how special and unlikely an assumption is the Capital Intensity Theorem, uncovering the structural (not just behavioural) reasons for persistence of unused capacity. Prices are led by quantity relations so, as Hicks (1985, p 142) pointed out, they cannot "give much guidance about the planning of production, about the choice of the path to equilibrium." Allocating capital goods between sectors on the theory that prices reflect relative scarcities will lead market-oriented decision-makers to raise the rate of accumulation when they should be lowering it. Policies will have to be based more on sectoral planning and less on the management of aggregate demand.

Nardini (1993)

MP, NER, HT, STED, WAG, SIC, REAL, LNPR, VIN, FW, FIX, CORN, SD

[Hicks STA] The consequences of an unexpected rise in the real wage rate are more sudden than for an invention, because they affect both old and new processes in the population mix. Two assumptions on the dynamics of the "take-out" (excess of output over real wage bill) are made: either it is unaffected by any changes, or it depends on an index of wealth. Under the first assumption, it is shown that the labour market will struggle towards its new equilibrium in a series of jerks and that eventually output cannot keep up with raised consumption. Under the second assumption, a convergent cyclical oscillation occurs, while in the long run the economy asymptotically approaches its new steady-state path as entrepreneurs reduce their take-out in the face of their declining (expected) wealth.

Desai & Redfern (1994a) **IP**, NER, HT, STED, SAV, REP, MONY, LNPR, VIN, WK, FIX, CORN, SD

[Hayek] The first part of this analytical reconstruction of the trade cycle model in Hayek (1931) shows a 5-year *equilibrium* traverse between two vertically-integrated systems as the initial ("short", 2-sector) transforms itself into a final ("long", 3-sector) economy, following a voluntary increase in saving. As entrepreneurs use these resources to insert a new intermediate sector into the process, labour is diverted and corn output is reduced until increasing work-in-progress emerges as final output. The new economy demands more money, but this is not inflationary because there are more goods as well.

Desai & Redfern (1994b) **MP**, NER, HT, STED, CRC, REP, MONY, LNPR, VIN, WK, FIX, CORN, SD

[Hayek] The second part of this paper shows the first three years of a *frustrated* traverse from the short to the long economy. Commencing with creation of credit by the banking system, entrepreneurs start longer processes of production but eventually are defeated by the banks' unwillingness to keep creating credit. The boom collapses and the newer methods are abandoned before they yield corn. Because the abandoned unfinished products of the new processes are not usable as inputs in the old processes, their output cannot be expanded rapidly, nor can employment. Hyperinflation develops and the trade cycle of Hayek (1931) goes into its downswing.

Kim (1994)

CP, NER, HT, STED, INN, REP, REAL, LNPR, VIN, FW, FIX, CORN, SD

[Hicks STA] A general equilibrium asset pricing model with neo-Austrian production technology utilising labour and land is used to explain how the benefits of innovation in the Korean economy accrued to entrepreneurs and, in particular, landowners. The early phase of a traverse to more mechanised techniques in the population of live processes creates a bubble in equity prices and land values, which will be exacerbated by foreign borrowing and rapid credit-creation. During the late phase of the traverse, the bubble bursts, with dire consequences for real economic activity.

Baldone (1996)

MP, NER, HT, STED, INN, SIC, REAL, CLLN, NVI, FW, FIX, CORN, SD

[Hicks STA] The composition of the opening stock of means of production helps determine the traverse path. Output of Neo-Austrian processes should be redefined as consumption goods *plus basic goods* (such as energy or lathes). Full Performance should be redefined as that situation where the number of starts of any process cannot be increased without reducing that of some other process. The transfer from one time period to another of "residuals" (commodities delivered by old processes and not immediately used up in starting new processes) may significantly influence the actual course of the traverse, which no longer is a unique time path. Even the vertically-integrated model has a multiplicity of trajectories if its many pure C-goods are not consumed in fixed proportions.

Belloc (1996)

MP, NER, HT, STED, WAG/INN, SIC, REAL, CLLN, NVI, FW, FIX, CORN, SD

[Hicks STA] Shows that the Neo-Austrian vertically-integrated linear production model is a special case of this "non-integrated" model, in which the inter-relationships among elementary processes started at different times are tracked. Paradoxical results are obtained, e.g. a higher wage rate can accelerate the rate of starts (hence also employment) of more mechanised processes in a particular branch of the economy because the demand for its output increases and because the negative relative profitability effect in other branches leads to their processes being truncated, causing a shift of available resources into the more mechanised branch.

Gehrke & Hagemann (1996)

CP, NER, HT, STED, LAB/INN, AIB, REAL, CLLN, NVI, FW, FIX, CTLA, EP

[Lowe] A clear exposition (and favourable evaluation) of Lowe's instrumental traverse analysis, defending it against misconceptions and criticisms by Neo-Austrians such as Amendola & Gaffard (1988) and two reviewers of Lowe (1976), viz. Metcalfe (1977) and Steedman (1977). The existence of a machine tool sector (e.g. lathes producing both tractors and lathes) establishes a hierarchy of production melding Austrian linearity with Classical circularity. This makes it possible to analyse physical bottlenecks and the speed at which traverse processes can restructure the capital stock to overcome them.

Lavoie (1996)

AT, NER, HT, STED, SAV, SIC, REAL, CLRE, NVI, FK, FIX, CORN, CR

[Kalecki] Critics of Kaleckian growth models claim that the short-period paradoxes of *thrift* (more saving → lower growth & profit rate) and *costs* (higher real wage → higher growth & profit rate) cannot survive into the long period because there is no mechanism to equate the realised to the normal capacity utilisation and profit rates. It is shown that traverses set off by changes in saving behaviour can exhibit hysteresis and move the standard Kaleckian model towards a fully-adjusted position because there is no *unique* normal capacity utilisation or profit rate. This mechanism relies on adaptive behaviour, whereby gaps between these normal and realised rates are slowly closed as entrepreneurs' pricing and investment decisions react to such differences.

Halevi (1997)	**MP**, ERG, LT, STED, LAB, SIC, MONY, CLRE, NVI, FK, FIX, COTR, SD [Hicks FTA, Kaldor] Investment is assumed to be fixed at the level that guarantees a full-employment expansion of the capital stock. But then workforce growth increases and pushes the system off the steady-state growth path appropriate to this assumption. Both money and real wages are depressed, hence aggregate demand as well. The Kaldor assumption merely keeps the tractor sector growing, but corn sector workers become unemployed and their numbers increase as this modified traverse process continues to operate. The source of the disequilibrium is structural but its evolution is Keynesian.
Henry & Lavoie (1997)	**IP**, ERG, LT, STED, LAB, SIC, REAL, CLRE, NVI, FK, FIX, COTR, SD [Hicks FTA] The traverse may be seen as a process of "reproportioning" the stocks of capital and labour so that full capacity utilisation and employment may be regained and maintained. Changes in workforce growth *force* the system to reproportion itself. The Capital Intensity Theorem is confirmed, but it is shown that if workforce growth exceeds a critical level the traverse will fail (the "unsustainable growth" case). The same is true in the two "dynamically unstable" cases of higher/lower workforce growth, when the tractor sector is more mechanised than the corn sector. In all three cases, extreme specialisation in one sector occurs and the economy eventually is unable to produce any output of consumption goods.
Lavoie & Ramírez-Gastón (1997)	**AT**, NER, HT, STED, PAR, SIC, REAL, CLRE, NVI, FK, FIX, COTR, SD [Hicks FTA, Kalecki] The Kaleckian growth model (which has oligopolistic industries, cost-plus prices, excess capacities, and endogenous capacity utilisation rates) is extended to two sectors. Traverses are initiated by changing the sectoral profit margins or autonomous investment. Kaleckian traverses do *not* require the restrictions on technology that the Hicks FTA does. Provided the model is stable in the short period, it will be dynamically stable as well. When profit margins are raised, or

	investment is lowered, the sectoral rates of accumulation fall, yet converge, until a terminal steady state is reached – not necessarily with lower sectoral rates of profit.
Amendola & Gaffard (1998)	**MP**, NER, HT, STED, PAR, SIC, MONY, LNPR, VIN, FW, FIX, CSEC, SD [Hicks STA] The time path of an economy thrown out of equilibrium by a parameter-change is determined by initial conditions, by price and wage reaction coefficients, and by two control variables: bank overdrafts by firms and capitalist consumption out of profit, the take-out. Numerical analysis is used to find a parameter-set which generates a steady state of growth over 200 periods for this "constraints-decisions-constraints" sequence model with its one-period gestation lag. Computer simulation establishes that the main problem is how to prevent catastrophic economic collapses in the face of parameter perturbations. A rules *with* discretion policy approach is recommended to keep the economy within its "viability corridor".
Gehrke (1998)	**CP**, NER, HT, ATED, LAB/INN, AIB, REAL, CLRE, NVI, FW, FIX, CTLA, EP [Lowe] The Full Performance assumption of Hicks's STA, whereby all full-capacity saving is invested in process starts, is inappropriate unless the economy's production structure is of an extremely simple nature. Belloc (1980) and Violi (1982) introduce *ad hoc* restrictions on the technology set but do not overcome this problem. Two requirements for a proper analysis of traverse processes are (a) an investment function that is independent of full-capacity saving and (b) allowing for above- and below-normal capacity utilisation. Gehrke supports the call by Hagemann (1992) for integrating Keynes's principle of effective demand in the longer run into Post-Classical traverse analysis.

Duménil & Lévy (1999)

IP, ERG, LT, STED, PAR, REP, MONY, CLRE, NVI, FK, FIX, COTR, CR

[Kalecki, Ricardo, Keynes] The traverse to a long-period classical equilibrium, with normal capacity utilisation and prices of production, is obtained as a gravitating sequence of short-period temporary equilibria (in which outputs are quickly adjusted to demands), with the real wage given. Slowly-adjusting variables are relative prices, capital stocks, the rate of inflation, and the stock of money. In this formulation, the short-period paradoxes of thrift and costs – see Lavoie (1996) above – cannot survive into the long-period.

Boehm & Punzo (2000)

AT, NER, HT, DSEQ, INN/CAP, AIB, REAL, CLRE, NVI, FW, VAR, CSEC, PI

[Schumpeter] The 1970s and 1980s were years of highly unstable growth regimes in Europe: Italy was the least stable, France somewhere in between and Germany the most stable. Traverse dynamics showing the instability of individual growth paths is a generic behaviour across both sectors and countries. The actual histories of the sectors, moreover, are made up of sets of traverses taking them from path to path, but also from regime to regime. Six growth regimes (and one rare regime, the steady state) are analysed and Europe's experiences – similar to those of the United States – are contrasted with the far more stable Japanese economy.

Saraceno (2002)

MP, NER, HT, STAT, PAR, SIC, MONY, LNPR, VIN, FW, FIX, CORN, SD

[Hicks STA] The model includes trade between two economies with money supplies and demand links. Numerical simulations show that temporary demand shocks have permanent effects on the economies, following the traverse. Some wage/price stickiness is necessary to avoid the system imploding, under both autarky and trade. Wage flexibility is unambiguously harmful. Price flexibility is harmful when monetary policy accommodates demand shocks. By itself, such accommodation softens the constraints faced by economic agents having bounded rationality.

APPENDIX B

THE INVESTMENT FUNCTION GENOME

B.1 Introduction

This Appendix demonstrates that most Keynesian *marginal efficiency*; Classical *uniform profitability*; Neo-Keynesian *multiplier-accelerator*; and Neoclassical *q-ratio* and *user cost* investment functions are individual "ontogenic" expressions of a general "phylogenic" investment function, *viz.* the one used in Models A through E of this thesis. By analogy with an organism's genes, the profitability gap is characterised as the common "genome" of most specimens within the investment equation species. By analogy with the Greek-language key to the Egyptian hieroglyphic and demotic scripts, the profitability gap is characterised as a "Rosetta Stone" which can unlock the common underlying basis of investment theories proposed by separate schools of economic thought.

Investment theories featuring differences, gaps and ratios have been proposed by economists since long before Keynes (1936) showed that investment determines both output and saving, rather than saving and investment jointly determining the real interest rate. There are five broad classes of investment functions and each is discussed in a separate section below.

B.2 Keynesian "Marginal-Efficiency" Investment Functions

The internal rate of return (IRR) or expected profit rate (re% pa) is the concept Keynes (1936, pp 135-46) uses in his *General Theory*, viz. the marginal efficiency of capital or, more accurately, of investment (MEI). In Chapter 11, Keynes argues that, if equilibrium prevails, then aggregate investment has been pushed to the point where the *economy-wide* MEI has fallen into equality with the ruling rate of interest on long-term government bonds. Here he is abstracting from risk; Keynes's interest rate is basically the opportunity cost of capital or required rate of return, n = (i + φ)% pa, but with φ = 0% pa as his risk premium. So, in the long-period equilibrium of a stationary state, re = r = n% pa. For the stationary state to be maintained, re = r = ro = n = no% pa must remain true for all entrepreneurs, year after year – where "o" indicates a one-year time-lag.

Three years before the *General Theory*, Kalecki (1933, in Polish) had published a model "... identifying aggregate investment orders as a function of both anticipated gross profitability and interest rates", according to Courvisanos (1996, p 15), who also quotes Josef Steindl

(1981, p 125) as having identified three versions of the investment function in Kalecki's writings on the business cycle. Of these, only Kalecki's (1943) Version II does not contain the profitability gap genome – although it does feature gaps in the *levels* of profits (ΔR) and capital stock (ΔK).

In Kalecki's (1933) Version I model, the investment function

$$I = f(r - i) \quad (1)$$

began life with re% pa in place of r% pa. The right-hand side is identical with the gap between the MEI and the interest rate that drives investment in Keynes (1936) and, moreover, pre-dates it by three years – as does Kalecki's independent derivation of Keynes's principle of effective demand. Kalecki substituted average realised profitability for the unobservable expected profit rate, a procedure that Malinvaud (1986, p 382) also recommends. Kalecki soon dropped the interest rate variable, on the empirical grounds that it closely follows fluctuations in profitability, though with smaller amplitude.

In Kalecki's (1968) Version III model, the investment function is

$$I = f(r' - i) \quad (2)$$ [41]

where $r' = (\Delta R / \Delta K)$% pa is the "marginal profit rate". Due to technical progress, later vintages of capital stock tend to exceed earlier ones in productivity performance, hence also in profit potential. Courvisanos (1996, p 19) says "The marginal profit rate ... replaces the average profit rate ... of version I as the expectations guide to further investment ... As the rate of investment orders slows down towards the top of the boom, the marginal profit rate declines more sharply than the average profit rate, developing negative expectations and the eventual reduction in investment orders."

Courvisanos (1996, p 20) states that "Josef Steindl is the most important Kaleckian writer on excess capacity and accumulation" and goes on to discuss how Steindl proposes the investment function

$$I = f(u - u^*) \quad (3)$$

[41] Not shown is another determinant, the positive response to cash flow (Rc). Kalecki included this to reflect the "principle of increasing risk", viz. more internal financing means less recourse to risky external borrowing.

where u is the actual, and u* the planned (normal/target/required/desired) degree of capacity utilisation. At first sight, this does not look like a Keynes-Kalecki marginal-efficiency equation. However, there is a close relationship between u% and r% pa, on the one hand, and u*% and n% pa, on the other.

At the microeconomic level, since the *profitability* associated with a given stock of fixed capital rises (falls) with every increase (decrease) in a firm's actual degree of capacity utilisation, one can see the direct analogue of the profitability gap mechanism in Steindl's vision of entrepreneurs investing more (less) as $u > u^*$ ($u < u^*$) or leaving investment unchanged ($u = u^*$).

At the macroeconomic level, one may define $u = Y / Z$ as the *aggregate* degree of capacity utilisation. If the "capacity-capital ratio" is $v = Z / K$ and the "profits share" is $rs = R / Y$, the profit rate identity $(R/K) = (R/Y)\ (Y/Z)\ (Z/K)$ can be rewritten as r% pa = (rs u v)% pa. Now the long-period constancy of macroeconomic income shares (such as rs) is accepted as a "stylised fact" and often it can be assumed that the capacity-capital ratio (v) also is constant. Thus the profit rate (r% pa) must vary directly with, and proportionally to, the degree of capacity utilisation (u%), as maintained by Steindl.

B.3 Classical "Uniform-Profitability" Investment Functions

With the exception of Karl Marx (see below) and Thomas Malthus, all Classical economists accepted that supply creates its own demand, in accordance with Say's Law of Markets. Therefore, given the Classical "iron law" that real wages tend to the subsistence level – the corollary being zero saving by workers – it is saving by capitalists out of their flow of profits (R dollars pa) that determines investment.

Prima facie, there is no room here for real net investment to be determined by the profitability gap (a% pa) genome. At best, one component of it (viz. the interest rate, i% pa) could be said to influence the amount saved out of capitalists' profit incomes. But the really interesting question is: What is it that determines this macroeconomic flow of *profits* (hence also saving and investment) in the Classical model?

At the microeconomic level, all Classical economists were aware that industries differed with respect to the risk premium (φ% pa) that capitalists had to anticipate covering, before investing part of profit-determined saving (opportunity cost = i% pa) in some particular industry. At this level, therefore, the allocation of real saving across all lines of production must have been governed by the rule re ≥ n% pa, where both sides of the inequality differed across industries. However, the right-hand side of the inequality only differs because the risk premium (φ% pa) is *specific* to each industry. (The left-hand side differs because prospects of profitability are industry-specific as well.) What is *general* across all investment opportunities is the basal opportunity cost (viz. i% pa, quoted by the rentiers) of converting foregone consumption (i.e. saving) into particular stocks of working capital and outfits of capital equipment.

Combining these facts with the insistence, by all Classical economists except Marx, upon a natural tendency for the economy to gravitate towards its "dismal" stationary state, one is left with a long-period equilibrium situation where the economy's *average* re% pa has come into equality with its *average* n% pa (underpinned by the economy-wide interest rate of i% pa). The equalities re = r = ro = n = no% pa are replicated, in the stationary state, year after year *ad infinitum*. At this set of "uniform" rates of return to physical capital and to (risk-adjusted) money loans – and with equilibrium saving out of equilibrium profits being equal to equilibrium investment – we can see that I = S = g (R) = f (re – n)% pa, as per the phylogenic investment function with its profitability gap genome.

In other words, it is *microeconomic* competition between capitalists to invest their flow of saving out of profits (in those industries they anticipate will yield the highest rates of return) that results in a particular *macroeconomic* outcome. The economy will be pushed onto its production possibilities frontier (PPF), with real income being continuously maximised in a classic stationary state. So, in this equilibrium of zero wage and price inflation, Y = Z dollars pa, where Z is the maximum flow of output that the economy can produce with all firms operating at their full capacity utilisation levels. By definition, Y = W + R dollars pa and W = w.L dollars pa. With the uniform money wage (w dollars/worker pa) being fixed at the subsistence minimum, only the stock of employment (L workers) and the flow of profits (R dollars pa) are free to adjust.

So, with fixed w, it must have been L and/or R that were the motive forces pushing Y all the way out to Z on the economy's PPF. Whereas Marx assumed an industrial "reserve army of labour" (viz. the urban unemployed), all other Classical economists relied on unlimited supplies of low-productivity rural labour. Effectively, both scenarios result in an unlimited supply of labour at the going real wage. As the level of employment (L) is therefore a purely passive variable, the *sole* active force is the microeconomic competitive struggle between capitalists to maximise differences between re% pa and n% pa, industry by industry. This process maximises the macroeconomic flow of R dollars pa that they receive as profits, so the economy ends up on its PPF, and stays there for as long as the stationary state endures.

As the last of the Classical economists, Marx begged to differ. He accepted that fierce competition between capitalists tended to make profit rates uniform throughout the economy. What he could *not* take on board was the Classical economists' creed that capitalists frugally saved, then passively invested, real resources that always were limited by whatever profit incomes the market dictated. Those whom Marx criticised do not seem to have been aware that their own Classical microeconomic investment process, which *equalised* profitability across all industries, also *maximised* the macroeconomic flows of profits, hence saving, hence investment. So, during any given year when the process was active, aggregate net investment (I dollars pa) really *was* determined by the profitability gap genome (a% pa) in the Classical model. This disequilibrium process continued until the economy attained a long-period equilibrium stationary state in which a = 0% pa, hence $I = Ig - \delta Ko = 0$.

Marx, however, was a dynamic disequilibrium theorist. To him, the economy was *always* in traverse. "Accumulate, accumulate; that is Moses and the Prophets!" Marx (1867, p 742) exclaimed, thus ruling out the Classical inevitability of the stationary state and substituting in its place a relentlessly growing and fluctuating economy, subject to intermittent crises. Positive net investment was the *norm* and Say's Law inoperative. If capitalist entrepreneurs lacked sufficient current profits to support their investment schemes, there were always plenty of capitalist rentiers on hand to extend money loans on the promise of future profits ... provided the entrepreneurs had sufficient collateral, of course.

Lack of collateral was the only thing preventing frugal workers from becoming capitalists, since Marx (like Ricardo) did permit wages to fluctuate above subsistence, thus allowing workers to save from time to time. When accumulation was strong (weak), wages rose (fell) and the reserve army of labour shrank (expanded). Whenever an investment boom carried the economy onto its PPF, this did *not* usher in a stationary state. For Marx, the PPF was forever moving outwards, due to net investment embodying the fruits of technical progress.

According to Ernest Mandel (1990), Marx showed that the "... inner logic of capitalism is ... not only to 'work for profit', but also to 'work for capital accumulation' ... Capitalists are compelled to act in that way as a result of competition. It is competition which basically fuels this terrifying snowball logic: initial value of capital - accretion of value (surplus-value) - accretion of capital - more accretion of surplus-value - more accretion of capital, etc. 'Without competition, the fire of growth would burn out', (Marx, 1894, p 368)." Obviously, it is the Classical uniform-profitability investment function (based on the competitive struggle and including the profitability gap genome) that Marx was using to explain capital accumulation.

B.4 Neo-Keynesian "Multiplier-Accelerator" Investment Functions

The Neo-Keynesian school of thought comprises those economists who developed and utilised the "Neoclassical Synthesis", according to which Walras ruled long-run analysis and Keynes was relevant only in the short run, when prices, wages and interest rates were slow to adjust, i.e. "sticky". They have been characterised as "ISLM", "hydraulic" or "bastard" Keynesians and include J R Hicks, Paul Samuelson, Alvin Hansen, Lawrence Klein, and James Tobin. Their intellectual heirs are today's New Keynesians, who differ only in having provided rigorous microeconomic foundations for the previously unexplained phenomenon of "price stickiness".

Kalecki's macroeconomic income distribution analysis is important for detecting the presence of profitability gaps, differences and ratios in the multiplier-accelerator investment theories that follow. His analysis shows how the economic activity aggregates favoured by Neo-Keynesian investment theorists (mainly consumption and income, but profits and productive capacity also have been used) all depend upon total investment outlays.

Kalecki expands the *expenditure* components of a closed economy's gross domestic product into Y = Cw + Cr + I, where the first two right-hand side terms are consumption out of wage and profit incomes. Then he uses the Classical assumption concerning propensities to save out of wages (W) and profits (R), i.e. sw = 0 < sr < 1, to forge a link with the corresponding *income* components of GDP, viz. Y = W + R. So, assuming that the wage bill W = Cw, then the profit residual R = Cr + I must follow. Finally, Kalecki proposes (Cr + I) $\rightarrow$ R as the *direction* of causation. This is plausible since capitalists, having collateral (hence preferred access to finance), can decide their own investment and consumption outlays, but not their own profits.

It is apparently an exogenous set of forces ("the market") that decrees what profits may subsequently be earned by capitalist entrepreneurs. However, what no isolated investor ever perceives is this: the aggregate of all capitalists' investment outlays (I dollars pa) principally determines what *level* of profits (R dollars pa) "the market" will generate for them all to partake of, in the form of the average *rate* of profit (r% pa) they realise on the economy's aggregate capital stock (K dollars).

Thus, Kalecki (1971, p 13) could state that "... capitalists, as a whole, determine their own profits by the extent of their investment and personal consumption", an insight he attained in the 1930s. It has since become known as *Kalecki's dictum*, which states that "Workers spend what they get, but capitalists get what they spend". Sidney Weintraub (1979, p 101) describes Kalecki's dictum as "... a penetrating light beam that speeds us close to the real situation". Independently, Keynes (1930a, p 125) had derived his equivalent "widow's cruse" explanation of how profits are generated:

> If entrepreneurs choose to spend a portion of their profit on consumption ..., the effect is to increase the profit on the sale of liquid consumption goods by an amount exactly equal to the amount of profits which have been thus expended. ... Thus, however much of their profits entrepreneurs spend on consumption, the increment of wealth belonging to entrepreneurs remains the same as before. Thus profits, as a source of capital increment for entrepreneurs, are a widow's cruse which remains undepleted however much of them may be devoted to riotous living.

Subsequently, Post-Keynesians such as Kaldor, Robinson and Pasinetti have analysed the implications of Kalecki's dictum for aggregate demand, income shares and economic growth paths.

In the Neo-Keynesian theory of Samuelson (1939), the relevant gap is a difference between the current and lagged values of consumption

$$I = f\,(C - Co) \tag{4}$$

whose right-hand side is a proxy for the profitability genome, as demonstrated below. This also is true of the "standard" output accelerator theory that Roy Harrod (1936) had pioneered, J R Hicks (1950) had extended and econometricians such as Lawrence Klein (1950) had utilised, viz.

$$I = f\,(Y - Yo) \tag{5}$$

Recall that Keynes (1936) showed how I – *via* the multiplier – determines Y, and hence also C = Y – I = Cw + Cr. Furthermore, Kalecki's dictum showed how (Cr + I) determines R, albeit by assuming that Cw = W. Yet, *regardless* of the saving behaviour of workers, it remains true that W = Y – R in any short-period analysis. Combining the insights of Keynes ($I \rightarrow Y$) and Kalecki ($I \rightarrow R$), this leaves the wage bill (W = w L) as a pure residual. The money wage (w) might be contractual, but employment (L) is not, so it would seem that investment (determining profits, output, consumption, and employment) rules the roost. Both Y and C have wage (w L) and profit (r K) components.

Now in the current (previous) short period, the capital stock K (Ko) is given, so the current realised profit rate r = (R / K)% pa must be implicit in both C and Y, while the lagged profit rate ro = (Ro / Ko)% pa must be implicit in both Co and Yo. Now, the usual starting point of a two-period analysis (indeed, any analysis) is an assumption that the economy is in equilibrium, i.e. that ro = n% pa. Thus, both Neo-Keynesian accelerator formulations – the consumption gap equation (4) and income gap equation (5) above – may be viewed as *proxies* for investment functions containing the profitability gap genome, I = f (r – n)% pa.

B.5 Neoclassical "Q-Ratio" Investment Functions

The q-ratio theory, which began with William Brainard & James Tobin (1968) and Tobin (1969), states that net investment by a business firm depends directly on the ratio of the stock-market valuation (Kd) to the replacement cost (Ks) of that firm, viewed as a collection of capital assets:

$$I = f(q) \qquad (6)$$

where q = Kd / Ks. If q > 1 (q < 1) there will be positive (negative) net investment. If q = 1 there is no incentive to change the firm's capital stock, so only replacement investments will be made. This is a firm-level analogue to the economy-wide stationary state.

When q = 1, this indicates that the firm considers it already possesses an *optimal* capital stock (K*), so that K* = Kd = Ks must represent the outcome of successful efforts by managers to maximise the equity value of the firm to its shareholders. Associated with each possible value of capital stock is some maximum capacity to produce output. Optimal capital stocks (K*) or production flow capacities are key concepts in the Neoclassical user-cost investment functions discussed below.

In microeconomic investment analysis, the firm's opportunity cost of capital (n% pa) is used to discount back the net proceeds expected over the life of (say) a factory. Call this amount Pd (the factory's Marshallian demand price), then accumulate forward the construction outlays to find Ps, its Marshallian supply price. For a firm whose only asset is such a factory, Kd = Pd and Ks = Ps, so we can see that the q-ratio investment theory involves comparing the results of forward-looking and backward-looking present value calculations.

The option which has the highest net present value (NPV) also is the one with the greatest excess of the internal rate of return (IRR) or expected profit rate (re% pa) over the normal profit rate or hurdle rate of return (n% pa). So, moving to an economy-wide focus, if all managers are striving to maximise the NPVs of all the firms they control, the q-ratio theory reduces to the profitability gap theory.[42]

Empirical Tobinesque investment functions have been estimated, but most use stock-market valuations in the numerator of the q-ratio, rather than internal company valuations based on the underlying fundamentals and made by better-informed directors and managers. One cannot assume that the opinions of those who *own* the firm (its shareholders) are identical with the views of those who *control* the investment decision (its managers). In fact, the opinions of shareholders often are based on irrational hopes and fears concerning company valuations and speculation is rife in the equity markets.

Keynes (1936, pp 156-8) contrasted "enterprise" with "speculation", noting that the former paid close attention to the underlying fundamentals while the latter was based on devoting "... our intelligences to anticipating what average opinion expects the average opinion to be" (i.e. stock-market sentiment). He saw that a 20th century phenomenon (the separation of ownership from control) encourages speculation and reduces enterprise, with rentier share-trading being comparable to a farmer who, having tapped his barometer, withdraws all his capital from agriculture during a few days of expected bad weather. Keynes (1936, p 159) also warned that "Speculators may do no harm as bubbles on a steady stream of enterprise. But the position is serious when enterprise becomes the bubble on a whirlpool of speculation. When the capital development of a country becomes a by-product of the activities of a casino, the job is likely to be ill-done".

B.6 Neoclassical "User-Cost" Investment Functions

Tobin's q-theory is the bridge linking the Neo-Keynesian multiplier-accelerator and Neoclassical user-cost investment functions with the Keynes/Kalecki marginal-efficiency approach. In the Neoclassical investment theories inspired by Dale Jorgenson (1963), the relevant gaps are those between last period's and this period's optimal capital stocks

$$I = f\,(K^* - Ko^*) \qquad (7)$$

or between the corresponding optimal output flow capacities

$$I = f\,(Z^* - Zo^*) \qquad (8)$$

42 Abel (1983) points out that the *marginal* q-ratio is a more relevant measure than the *average* q-ratio discussed above. Marginal q is defined as the ratio of the market value of an *additional* piece of capital equipment to its replacement cost. Hayashi (1982) has shown that there are cases where marginal q is *proportional* to average q.

along a steady-state growth path. Trygve Haavelmo (1961) earlier had pointed out that "the demand for investment cannot simply be derived from the demand for capital. Demand for a finite addition to the capital stock can lead to any rate of investment, from almost zero to infinity, depending on the additional hypothesis we introduce regarding the speed of reaction of capital-users". Thus, Jorgenson had to invoke various *ad hoc* "delivery lags" and "adjustment costs" (modelled by distributed lags) to explain why gap closure does not occur instantaneously.

In the Preface to Volume I of his *Collected Works*, Jorgenson (1998) reminisces that he had "... defined the *user cost* of capital as the rental price of capital services, representing this price as the product of the price of investment goods and the cost of capital ... I reserved the term 'cost of capital' for the sum of the rate of return, the rate of depreciation and the rate of capital loss, adjusted for the taxation of capital income".

Jorgenson's "user cost of capital" (c) is simply the uniform annual lease payment for renting capital assets. This time-stream can be discounted back at n% pa to find the associated capital value of the lease. A fuller statement of the investment function, based on the gap between two adjacent optimal capital stock values ($I = \Delta K^* = K^* - Ko^*$) – and shorn of its *ad hoc* distributed lags structure – would be

$$I = f\ (\Delta x, \Delta p, \Delta c) \tag{9}$$

where x is output and p is its price. So, with sales revenue = p.x dollars pa (incorporating the firm's expected profit rate, re% pa) and user-cost = c dollars pa (incorporating the firm's opportunity cost of capital, n% pa) determining the optimal capital stock, this Neoclassical investment theory already *resembles* our phylogenic investment function with its genomic profitability gap. Furthermore, the presence of these quantity and price terms shows that Jorgenson's investment function includes a "sales accelerator", comparable with the consumption (ΔC) and output (ΔY) accelerators of Neo-Keynesian theory.

But resemblance is not enough. Jorgenson's *ad hoc* adjustment-costs soon were separated by Eisner & Strotz (1963), Lucas (1967) and Gould (1968) into "intrinsic" factors (i.e. costs of installation) and "extrinsic" factors (i.e. rising Marshallian supply price), then formalised as a convex function of the firm's capital stock, to reflect *marginal* adjustment costs. Thus was the Neoclassical investment model "perfected"; it yields an entire optimal adjustment path for the scale of the firm and, on the representative agent assumption, the entire economy.

Several commentators, including Hayashi (1982) and Abel (1990), have shown that the Eisner-Strotz-Lucas-Gould Neoclassical model with marginal adjustment-costs is *identical with* Tobin's (marginal) q-ratio theory of investment under certain assumptions, e.g. that the firm's cash flows are a linear homogeneous function of its capital and labour inputs and its investment outlays.

B.7 Other Neoclassical Investment Theories

The perfected Neoclassical [user-cost ≡ q-ratio] investment function has proved to be flexible, even amoebic, readily absorbing such critiques as the influential "financial-constraint" and "option-value" approaches to explaining investment. Jensen & Meckling (1976), Stiglitz & Weiss (1981), Myers & Majluf (1984), and Chirinko (1987) initiated the *financial-constraint* investment theory by showing how easily the well-known "MM theorem" – proposed by Franco Modigliani & Merton Miller (1958, 1963) – can break down in real-world financial markets. One implication of the MM theorem (which both Jorgenson and Tobin accept7ed) is that the opportunity cost of capital for a firm is independent of both its financial structure (i.e. debt-equity ratio) and the mix of retained earnings, bond issues and share floats it chooses to finance investment projects.

The financial-constraint theories may be seen as confirming Kalecki's principle of increasing risk in that they imply a certain "pecking order" among sources of finance. At the top of this "financing heirarchy" sit retained earnings (least risky and cheapest), then come share floats (which dilute equity) and, finally, bond issues (most risky and dearest). The key assumption of the MM theorem is that firms can never increase their own capital value through purely financial operations because, if this *were* possible, rentiers could profit through arbitrage by replicating such operations in their own portfolios. But to do this, the rentiers need to possess precisely the same data as the managers of corporations.

Unfortunately, just as in George Akerlof's (1970) used-car markets, access to key information in the financial markets is "asymmetric". To compensate for their lack or mistrust of what information *is* available on the real investment opportunities confronting firms, rentiers tend to raise the price of external finance above the opportunity cost to managers of using cash flows generated within their own firms. Basically, rentiers cannot know the full range of risk-classes (possible "adverse selection"), what action the firm's managers will take (possible "moral hazard with hidden action") or what outcomes are revealed by the firm's monitoring of its own investment projects (possible "moral hazard with hidden information"), so they add a "lemons premium" to the normal market-clearing borrowing rate.

In hindsight, it was Kenneth Arrow (1968) who initiated the *option-value* investment theory by introducing the concept of "irreversibility", whereby capital goods either cannot subsequently be resold to other firms or can be resold only at a significant loss. Thus investments which are more or less *firm-specific* may be classified as completely or partially *irreversible*. It was nearly twenty years before McDonald & Siegel (1986) highlighted the existence of a close analogy between the decision to make an irreversible real investment and the decision to exercise a financial option. Avinash Dixit & Robert Pindyck (1994) provide a systematic exposition of this Neoclassical investment theory.

A *call option* gives its rentier owner the right to buy a financial asset at some predetermined price; once exercised, the option is "killed" and becomes worthless. By analogy, a firm's managers "own" the option to take advantage of an (irreversible) investment opportunity *at any time* after careful analysis of its time-profile of expected net proceeds has shown that re ≥ n% pa or, equivalently, that q ≥ 1. To build or purchase the necessary capital equipment immediately the opportunity becomes apparent "kills" the real "option-value" of *waiting*, e.g. the benefits of postponing the investment until more information concerning future market conditions becomes available.

According to the "bad news principle" of Bernanke (1983), good news is irrelevant to the real option-value of an investment opportunity. In a world of uncertainty, there are positive probabilities of future upward or downward revisions to the expected profitability associated with any eligible investment project. But the option-value of avoiding losses by waiting must increase if there is *bad* news. Good news has *no effect* on the option-value because all it does is confirm the wisdom of investing now – which kills the option anyway. Dixit (1992, p 123) uses the bad news principle to explain why American companies are less aggressive investors than Japanese firms. The former face downside risk – hence their option-value of waiting to invest is always positive – whereas the latter are protected from losses by government supports. With an option-value near zero, any Japanese firm which has identified an investment opportunity *never waits*.

The existence of a real option-value of waiting drives a wedge between the two sides of the "rule" that a firm will maximise its value by investing in projects until re = n% pa or, equivalently, until q = 1. As Dixit & Pindyck (1994, Ch 5) state, "... the simple NPV rule is not just wrong; it is often *very* wrong." For reasonable parameter values, McDonald & Siegel (1986) have shown that it is optimal to defer investing until the present value of a project's benefits are *twice as large* as its capital cost. This represents an upper threshold for investment to occur immediately (e.g. *via* entry of new firms) but the theory also posits a lower threshold (perhaps well below q = 1) for disinvestment to commence. Dixit & Pindyck

(1994, Ch 8, Sect 3) present a good example: in the world copper industry, prices above long-run average cost do not attract new entrants and prices below average variable cost do not induce exit by existing firms.

Both the financial-constraint and option-value theories are valuable in explaining why (*contra* the MM theorem) managers constantly worry about the financial structure of their firms, favour internal finance and continually seek projects for which re >> n% pa or, equivalently, q >> 1. These new approaches, therefore, are simply embellishments of (rather than replacements for) the perfected Jorgenson/Tobin Neoclassical investment theory. As such, the insights they afford also are relevant to all other investment theories that are expressions of the profitability gap genome.

B.8 Origins of the Genome

The Chicago economist, Irving Fisher, was one of the first to realise that net investment is driven by a profitability gap. Keynes (1936, pp 140-1) explained that

> Although he does not call it the 'marginal efficiency of capital', Professor Irving Fisher has given in his *Theory of Interest* (1930) a definition of what he calls 'the rate of return over cost' which is identical with my definition. 'The rate of return over cost', he writes, 'is that rate which, employed in computing the present worth of all the costs and the present worth of all the returns, will make these two equal.' Professor Fisher explains that the extent of investment in any direction will depend on a comparison between the rate of return over cost and the rate of interest. To induce new investment 'the rate of return over cost must exceed the rate of interest'.

The Neoclassical Fisher never committed the common error of conflating the rate of interest (i% pa) and the rate of profit (r% pa); the equilibrium condition that r = i% pa does *not* entail that r ≡ i% pa. Keynes, who accepted the nonergodic world axiom, correctly interpreted Fisher's rate of return over cost as the *expected* rate of profit (re% pa).

Thus the profitability gap genome (as the explanator of net investment) can be traced back directly through Keynes (1936) to Fisher (1930). We already have seen how Kalecki (1933) utilised the same concept, which earlier had been deployed by Spiethoff (1925) in his business cycle theory.

As an explanator of the price level, however, the genome is far older. Keynes (1930a, pp 176-8) credited Knut Wicksell's (1898, 1906) gap between the "natural" and "money" rates of interest (which drove the Swede's "cumulative processes" of deflation and inflation) – as the inspiration for Keynes's own "investment-saving gap" theory of profitability and the price level in his *Treatise on Money*. Wicksell himself identified Henry Thornton (1802) as the ultimate progenitor of this universal "primitive" of investment and capital theory.

B.9 Conclusion

All animals in the gap zoo of investment theory *do* share the same expected profitability gap genome. This justifies using the phylogenic investment function to model real investment behaviour by farmers in the monetised corn economy that is simulated in this thesis. As the investment equation is what principally drives the complex dynamics of the flexprice corn model, including its all-important traverses, a range of investment functions proposed by economists of several schools of thought was examined.

Hopefully the profitability gap investment function will help bring some taxonomic order to the menagerie of specimens that have been collected over many years. The phylogenic investment function is more general; in fact, its genome appears to be one of the few universals of economic science, equally at home as an explanator of investment, inflation/deflation and related cumulative processes.

Depending on how one specifies the expectation function(s) of entrepreneurs, the expected profitability gap is *equally applicable* to the ergodic Neoclassical universe of general equilibrium in logical time and the nonergodic Post-Classical universe of equilibrium stationary states and disequilibrium traverse phenomena in historical time. Thus, the investment function genome holds out the prospect of helping unite, rather than further divide, schools of economic thought.

APPENDIX C

LIST OF PARAMETERS AND VARIABLES

Average Debt	D
Budget Balance	B
Capital Stock	K
Capital Turnover	κ
Capital-Labour Ratio	x
Capital-Output Ratio	v
Consumption	C
Consumption Ratio	c
Corn Price	P
Corn Produced	Q
Cross Elasticity of Demand	χ
D:A Ratio Growth Coefficient	δ
D:A Ratio Growth Rate	gd
Debt:Assets Ratio	d
Dole Wage Fraction	wd
Employment	L
Employment Ratio	e
Employment Wage Coefficient	ε
Foodcorn Capital	Kb
Foodcorn Supplied	Qs
Government Debt	Dg
Government Debt Interest	Gi
Gross Product	Y
Gross Surplus	Rg
Gross Surplus Share	rs
Household Income	Yh
Income Elasticity of Demand	γ
Income Tax Rate	ty
Income Tax Revenue	T
Inflation Rate	gp
Inflation Wage Coefficient	ρ
Intercept Constant	α
Interest Bill	J
Interest Rate	i
Investment	I
Investment Multiplier	k
Labour Productivity	λ
Margin	mn
Markup	m
Money Wage	w
Money Wage Growth Rate	gw
Net Surplus	Rn
Normal Profit Rate	n
Policy Toggle Switch	ts
Price Elasticity of Demand	β
Price Level	p
Prime Cost	pc
Profit	R
Profit Rate	r
Profitability Gap	a
Reaction Coefficient	ϕ

Real Capital Stock	Kr
Real Consumption	Cr
Real Gross Product	Yr
Real Gross Surplus	Rgr
Real Household Income	Yhr
Real Interest Rate	ir
Real Investment	Ir
Real Net Surplus	Rnr
Real Normal Profit Rate	nr
Real Profit	Rr
Real Profit Rate	rr
Real Saving	Sr
Real Wage	wr
Real Wage Bill	Wr
Risk Premium	φ
Saving	S
Saving Ratio	s
Seedcorn Capital	Ka
Seedcorn Invested	Qi
Seedcorn Yield	θ
Unemployment Benefit	Gu
Unemployment Rate	u
Wage Bill	W
Wage Bill Share	ws
Wage Bill Turnover	μ
Workforce	η

APPENDIX D

CD-ROM WITH MODEL FILES

Attached to the inside back cover of this thesis is a pocket containing a CD-ROM, which includes 26 x HTML (*.htm) and 26 x EXCEL (*.xls) spreadsheet files, together with the full text of this dissertation in both WORD (Thesis.doc) and HTML (Thesis.htm) formats.

When the disk is inserted into a CD/DVD-ROM drive, the computer's Web Browser should open and display the Index.htm file, from which all other files are one mouseclick away.

Each *.htm spreadsheet and associated graphs may be viewed, but not manipulated. If the computer also runs Microsoft Excel, the *.xls spreadsheets may be viewed and used for traverse experiments or for modifying the model's structural-form equations.

The following list shows from which spreadsheet files the various Tables and Figures of this thesis were sourced:

<u>Model A</u>

Astat	Table 4.5, Figures 4.2a & b
Asted	Table 4.6, Figures 4.3a & b
Astatmal	Table 4.7, Figures 4.4a & b
Astedmal	Table 4.8, Figures 4.5a & b

<u>Model B</u>

Bstat	Table 5.3
Bsted	Table 5.4, Figures 5.2a & b
Bstatmal	Table 5.5, Figures 5.3a & b
Bstedmal	Table 5.6, Figures 5.4a & b
Bstedpef	Figure 5.2c

<u>Model C</u>

Cstat	Table 5.9
Csted	Table 5.10, Figures 5.6a – c
Cstatmal	Table 5.11, Figure 5.7
Cstedmal	Table 5.12, Figures 5.8a – c

<u>Model D</u>

Dstat	Table 5.15
Dsted	Table 5.16, Figures 5.10a – c
Dstatmal	Table 5.17, Figure 5.11
Dstedmal	Table 5.18, Figure 5.12

<u>Model E</u>

Estat	Table 6.4
Ested	Table 6.5, Figures 6.2a – c
Estatmal	Table 6.6, Figures 6.3a – c
Estat200	Used to generate Table 6.7, Figures 6.4a – f, Figure 6.5, Figure 6.6, Figures 6.7a – f, Figures 6.9a – f
Estatpha	Figures 6.8a – f

<u>Model E*</u>

Fstat0	Gives same results as Estatmal, i.e. policy switch toggled to zero
Fstat1	Table 7.3, Figures 7.2a – c, Figures 7.3a – c
Fsted0	Gives same results as Ested, i.e. policy switch toggled to zero
Fsted1	Table 7.4, Figures 7.4a – c, Figures 7.5a – c

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